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# Parametric study on average stress–average strain curve of composite stiffened plates using progressive failure method

**Abstract**

The post-buckling behavior of composite ships' stiffened plate panels has been studied. In this study, the average strain- average stress curves for these panels are derived using progressive failure method as well as nonlinear finite element method. The boundary conditions are appropriate for the continuous plate panels used in shipbuilding. The effects of the aspect ratio, initial geometrical imperfection and stiffener size on the post-buckling of these stiffened panels are evaluated.

**Keywords**

Progressive failure analysis, hat stiffened plate panel, initial imperfection, Finite Element Method (FEM).

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## 1 INTRODUCTION

The application of composite materials in marine industries is so ever-increasing that nowadays high- and medium-length ships are being built of them. Besides, assessment of the ultimate strength of composite ships is of great concern owing to their great lengths. It is clear that the composite stiffened plate panels are the main building components of such ships and thus, derivation of their average stress – average strain curves plays an important role when estimating the ultimate strength of composite ships.

Investigation of the behavior of composite panels during failure is a very complicated task because of the complexity in calculating failure modes as well as the responses of composite materials. Many studies have been performed on mitigating the risks embedded in designing these structures, and numerous methods have been put forth for modeling the failure of composite materials one of which is progressive failure method. As known, in this method, the applied load is increased incrementally and the state of the structure is then examined at each stage of loading by using one of the failure criteria. If a failure occurs in one laminate, the mechanical properties of that certain lam-

inate are reduced according to the applied model. Since the failure criteria of composite materials can be applied providing that the stress distribution in structure is precisely given, it is necessary to compute stress distribution by finite element or finite strip methods in progressive failure method.

In addition, the study on the behavior of the composite materials by using progressive failure method has attracted many researchers. Figure 1 illustrates a history of the major studies carried out in this field based on different stress analysis methods such as finite element and finite strip ones. In these researches, various failure criteria like Tsai-Wu, Maximum stress, Hashin, etc. are utilized.

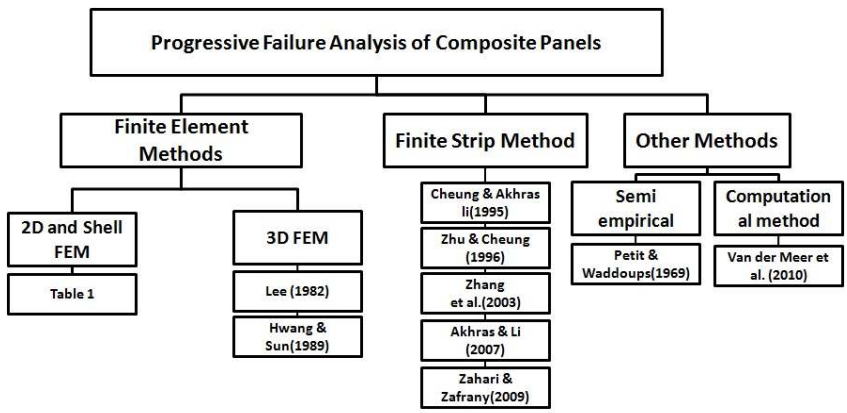



**Figure 1:** History of the major studies carried out on progressive failure assessment of composite panels.

As mentioned earlier, the use of failure criteria for composite materials necessitates the accurate stress distribution. Table 1 lists some research studies conducted by using progressive failure analysis based on FEM.

| Researcher | Structure [Loading] | Theory (LG[1] or NLG[2]) | Failure Criteria |
|---|---|---|---|
| Sandhu (1974) | Laminated plate | | Total energy criterion |
| Sandhu *et al*. (1983) | Composite laminates containing a pin loaded hole | | Total strain energy failure criterion |
| Ochoa and Engblom (1987) | Plate+ beam [Tension + Bending] | LG | Piecewise smooth failure criteria |
| Pandey and Reddy (1987) | Plate with a cutout [In plane + Transverse] | FSDT3, LG | Maximum stress, maximum strain, Hoffman, Tsai-Wu, Tsai-Hill |
| Chang *et al.* (1984) | Laminated plat with a cut out [Tension] | CLPT4, LG | Yamada and Sun , Hahn and Tsai |
| Chang *et al.* (1988) | Notched laminated plate [Tension + Compression] | CLPT | Yamada and Sun |
| Tan (1991) | Damage lamina [In-plane] | | Tsai-Wu |

**Table 1:** A summary of key previous studies carried out on composite structures using progressive failure and finite element methods

| Researcher | Structure [Loading] | Theory (LG[1] or NLG[2]) | Failure Criteria |
|---|---|---|---|
| Tolson and Zarabas (1991) | Plate [Biaxial and Transverse] | Mindlin Plate theory, LG | Hoffman, Hashin and Lee, Lee and Maximum stress criteria |
| Averill and Reddy (1992) | Laminated shell structures | Micromechanics based model, NLG | Micromechanics based model |
| Kim and Hong. (1992) | Flat plate with/ without hole | | Macroscopic Failure criteria |
| Tan and Perez (1993) | Laminated composites with a hole [Compression] | | Tsai-Wu |
| Shahid and Chang (1995) | Plate [Tension + Shear] | LG | Modifying the existing failure criteria |
| Daudeville *et al.* (1995) | Plate with a hole [Tension] Plate [Tension +Compression] | | Fracture Mechanics |
| Eason. and Ochao (1996) | Generate an element for ABAQUS for progressive failure Analysis | Shear deformable element [ABAQUS] | Hashin, Greszczuk, Lee, Maximum Stress, Ochao |
| Moas and Griffin (1997) | Curved frame [Transverse] | NLG | Phenomenological failure criteria |
| Kim *et al.* (1998) | A pin loaded laminated composite [Shear+ Tension + bearing] | | Hashin |
| Singh and Kumar (1998) | Thin Plate [Shear ($PB^5$)] | FSDT , NLG | Tsai–Hill, Maximum stress |
| Kong *et al.* (1998) | Stiffened structure [Compression (PB)] | | Maximum stress |
| Baranski and Biggers (1999) | Plate [Compression (PB)] | NLG [ABAQUSE] | Modified Hashin's |
| Gummadi (1999) | Laminated beams and arches | Total Lagrangian method, NLG | Lee , Hashin , Maximum Stress |
| Spottswood and Anthony N (2001) | A thin, curved composite panel [Transverse] | simplified large displacment/rotation theory, NLG | Hashin |
| Xie and Biggers (2003) | Flat plates and curved panels with a central cutout [Compression (PB)] | LG/ NLG, [ABAQUSE] | Modified Hashin's |
| Damodar *et al.* (2004) | Curved panels with/ without a circular cutout [Compression (PB)] | NLG, Imperfection [ABAQUS] | Hashin |
| Goyal *et al* (2004) | Plate with circular hole [Compression (PB)] | NLG, [ABAQUS] | Modified Hashin by Chang and Lessard . |

**Table 1:** A summary of key previous studies carried out on composite structures using progressive failure and finite element methods (continued).

| Researcher | Structure [Loading] | Theory (LG[1] or NLG[2]) | Failure Criteria |
|---|---|---|---|
| Key *et al.* (2004) | Rib-stiffened panels [Tensile] | Multicontinuum technology, NGL | Mayes and Hansen |
| Oh *et al.* (2006) | Hat-stiffened curved panels [Compression (PB)] | | Maximum Stress, Tsai-Hill, Tsai-Wu |
| Chen and Soares(2007) | Stiffened curved panels [compression (PB)] | NLG, [in house code] | Tsai-Wu |
| Zhang *et al.* (2008) | Grid stiffened composite plates/shells [Compressive (PB)] | Beam and plate, NLG [Integrated FEM+ Finite difference] | Modified Hashin by Chang and Lessard, Chai–Gadke's criteria for delamination |
| Wagner and. Balzani (2010) | laminated shell structures [Compression (PB)] | NLG | Extended Hashin |

[1] Linear Geometry
[2] Nonlinear Geometry
[3] First Order Shear Deformation Theory
[4] Classical Laminated Plate Theory
[5] Post-buckling

**Table 1:** A summary of key previous studies carried out on composite structures using progressive failure and finite element methods (continued)

Moreover, the studies carried out on progressive failure by using Finite Strip Method (FSM) are very sparse. To name a token, FSM was, for the first time, used in progressive failure analysis by Cheung *et al.* (1995) aiming at investigating the behavior of aniotropic panels by using Li failure criterion. In another research, Akhras and Li (2007) used FSM and progressive failure method in order to analyze thick composite plates and imposed Li failure criterion.

Furthermore, Zahari and El-Zafrany (2009) proposed a model for analyzing the failure of stiffened plates and composite panels by using FSM on the basis of Mindlin theory. They, also, compared the results with those elicited from ABAQUS FEM and found good conformity.

Petit and Waddoups (1969) used the progressive failure method for the first time and applied Classical Laminated Plate Theory (CLPT) for stress analysis. They first calculated the compression and shear values by means of tensile and compression test. The stiffness matrix was then updated at each loading step and finally, the failure point occurred when the stiffness matrix became singular. Van der Meer *et al.* (2010) developed a computational method for progressive failure analysis in which fiber failure and matrix cracks were analyzed using continuum mechanics.

To the best of authors' knowledge, the studies on the analysis of the stiffened plates and the investigation of their boundary conditions in marine industries are scarce. Only, Chen and Soares used the progressive failure method for analyzing average stress–average strain curves of stiffened marine plates. Given the lack of research in this field, the present study aims at the parametric investigation of these curves. First, the average stress – average strain curves are obtained for stiffened composite plates of ships by using progressive failure method and the effect of different geometric parameters on these curves are then examined for the case of stiffened composite plates.

## 2 PROGRESSIVE FAILURE ANALYSIS METHOD

Progressive failure of composite materials is considered in the current analyses for study of post-buckling strength of composite panels utilizing Tsai-Wu failure criterion. After modeling, meshing and initial loading, the values of stresses and deformations are calculated. Then, the magnitude of the failure parameter is computed based on failure criterion. Afterwards, all layers of each of the components are examined aiming at controlling the occurrence of the failure. If failure occurs, the mechanical properties of the materials approach almost zero in the relevant layer.

At the next step, the force is increased incrementally and the above mentioned procedure is repeated until the final failure occurs and the ultimate strength or buckling strength of composite panels is eventually acquired using progressive failure method.

If the stacking of plate and stiffeners are not the same or there are several types of stacking in the structure, the states of all layers should be investigated for the applied finite elements and the inferred stresses in each layer have to be transferred to the relevant coordinate system and the magnitude of stresses is considered in the certain failure criterion.

## 3 CHARACTERISTICS OF FINITE ELEMENT MODELS

In this section, boundary and loading conditions, the material of the plates and its properties, the applied software, the type of finite element in use and the reason for its selection are discussed.

The plate models examined in the present study are of the structural components of the ships that can be found in such sections as deck, bottom, sides, bulkheads and tanks. These structures are analyzed to infer their average stress – average strain curves for calculating the final strength of the ship. Note that the average stress – average strain curves of these models are obtained by using progressive failure analysis method.

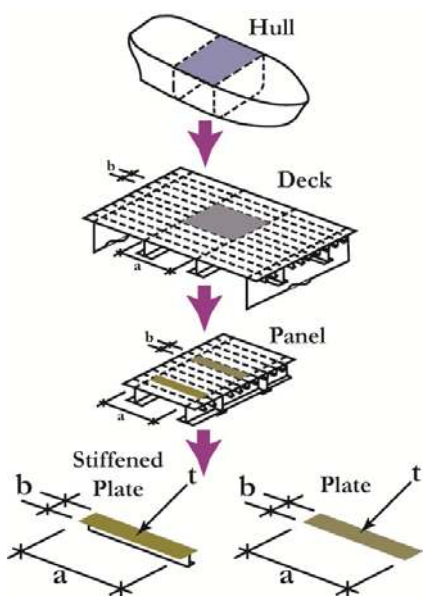



**Figure 2:** The plate panel and stiffened plate panel in ship structure.

The plates of a ship structure are strengthened by transverse and/or longitudinal stiffeners. The extent of plate models with stiffeners and its relationship with other structural areas of a ship is presented in Figure 2. The strength of stiffened and unstiffened plate panels can be separately studied by applying real boundary conditions. The extent the models is somehow chosen that each model includes two transverse and longitudinal stiffeners as shown in Figure 3.

All modeled plates have the aspect ratio of 2 to 3 which is exactly same as that of the real plates used in the structure of composite ships. The geometric and dimensional characteristics of the selected stiffened plates are given in Table 2.

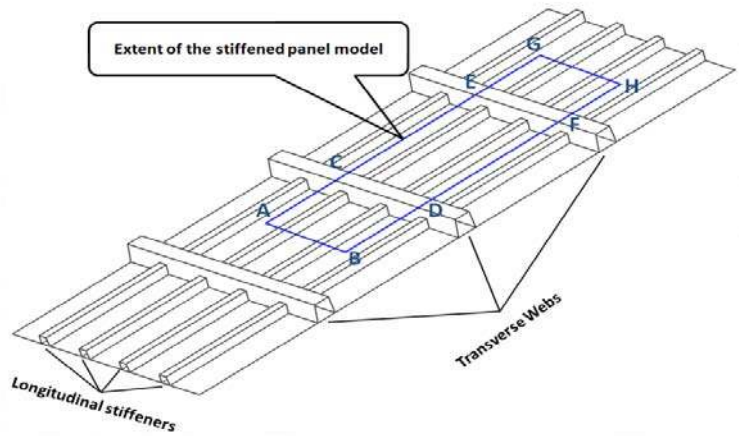



**Figure 3:** Extent of Finite Element Model.

| Mechanical Properties | Symbols | Magnitude |
|---|---|---|
| Module of Elasticity in main direction of the material | $E_1$ | 15 GPa |
| Module of Elasticity in direction normal to main direction of the material | $E_2$ | 13.5 GPa |
| Shear Modulus in direction 12 and 13 | $G_{12}, G_{13}$ | 3.45 GPa |
| Shear Modulus in direction 23 | $G_{23}$ | 3.45 GPa |
| Tensile strength in direction 1 | $X_T$ | 238 MPa |
| Compression strength in direction 1 | $X_C$ | 210 MPa |
| Tensile strength in direction 2 | $Y_T$ | 204 MPa |
| Compression strength in direction 2 | $Y_C$ | 224 MPa |
| Shear strength in direction 12 | $S_{12}$ | 84 MPa |
| Shear strength in direction 13 | $S_{13}$ | 84 MPa |
| Shear strength in direction 23 | $S_{23}$ | 84 MPa |

**Table 2:** Mechanical properties of the material used in this study (Direction 1 is the laminate main direction and direction 2 is the direction normal to direction 1).

The proper selection of boundary conditions and the correct way to apply them on the model is one of the most significant parts of the modeling. If the conditions do not match the reality, then the solutions results will not be reliable, generalizable and valid. The boundary conditions strictly depend on the aspect ratio of the plates in the investigation of the strength of the stiffened plates.

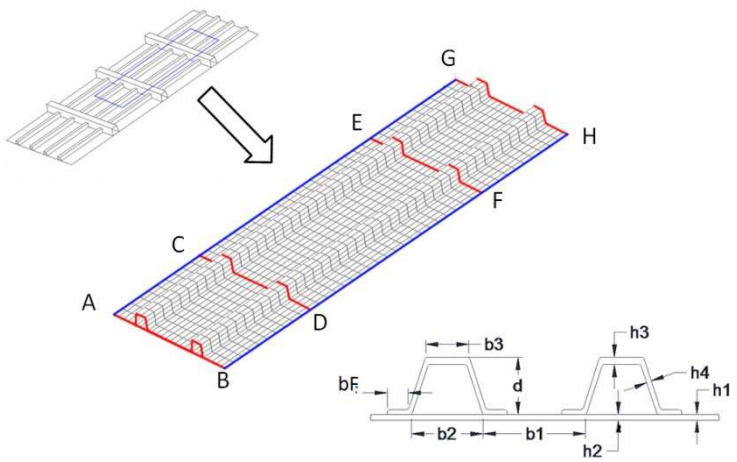



**Figure 4:** Boundary conditions and dimensions of the Finite Element Model.

Figure 4 shows a finite element model of a stiffened plate. The area of the model is so that it includes two longitudinal and two transverse stiffeners. Lines AB, BH, GH and AG represent the boundaries of the model and lines CD and EF demonstrate where web frames or transverse stiffeners are located. Given that transverse stiffeners are stiffer than the longitudinal ones; their lateral deformation is assumed to be zero.
The boundary conditions of the stiffened plates are as follows:

- In the case of symmetrical stiffeners, the symmetrical conditions are applied along the lines AG and BH.
- In the case of odd and even dimensional ratios, the periodic continuous or symmetry conditions are applied in transverse lines AB and GH in Figure 4, respectively.
- Since transverse beams are not modeled, the deformation of plate points along z axis is constrained to transverse web frame location. Therefore, the deformation of the points is bounded to the lines CD and EF along z. Furthermore, the longitudinal deformation of the plate is coupled in the place of the effect of compressive force.

In this research, it is attempted to present the results as practical as possible; thus, the dimensions and materials used in parametric study are same as those prevalent in shipbuilding. Table 3 tabulates the mechanical characteristics of the applied composite.

The modeling and simulation have been performed by using ANSYS software whose structural section was of concern; because every structural problem, regardless of the diversity in components of the structure, can be solved using this section. Note that all pre- and post-processing steps of the

modeling were carried out by a macro code written in the ANSYS Programming Design Language (APDL).

Owing to the fact that the stiffened plate elements of ships are thin-walled, their out-of-plane stress is negligible but their in-plane stress is determinant. Hence, the modeling of these components is precise enough with shell elements. Therefore, the element Shell 181, that is appropriate for modeling the thin and relatively thick plates and is constructed based on Classic Plate Strain and Mindlin theories, was used in the analyses. These elements are composed of 4 nodes and each node has 6 degrees of freedom. This element is appropriate for linear and nonlinear solutions with large deformations and great angle variations. Also, this element can be used for modeling the composite and laminated materials.

| **Stiffened Panel** | **a** | **b** | **bf** | **d** | **b2** | **b3** | **h1** | **h2** | **h3** | **h5** |
|---|---|---|---|---|---|---|---|---|---|---|
| Geo 1 | 300 | 1000 | 18 | 62 | 60 | 50 | 10 | 10 | 12 | 22 |
| Geo 2 | 325 | 1000 | 18 | 62 | 60 | 50 | 10 | 10 | 12 | 22 |
| Geo3 | 333 | 1000 | 18 | 62 | 60 | 50 | 10 | 10 | 12 | 22 |
| Geo 4 | 350 | 1000 | 18 | 62 | 60 | 50 | 10 | 10 | 12 | 22 |
| Geo 5 | 375 | 1000 | 18 | 62 | 60 | 50 | 10 | 10 | 12 | 22 |
| Geo 6 | 400 | 1000 | 18 | 62 | 60 | 50 | 10 | 10 | 12 | 22 |
| Geo 7 | 425 | 1000 | 18 | 62 | 60 | 50 | 10 | 10 | 12 | 22 |
| Geo 8 | 450 | 1000 | 18 | 62 | 60 | 50 | 10 | 10 | 12 | 22 |
| Geo 9 | 475 | 1000 | 18 | 62 | 60 | 50 | 10 | 10 | 12 | 22 |
| Geo 10 | 500 | 1000 | 18 | 62 | 60 | 50 | 10 | 10 | 12 | 22 |
| Geo 11 | 300 | 1000 | 18 | 103 | 100 | 83 | 10 | 10 | 12 | 22 |
| Geo 12 | 400 | 1000 | 18 | 103 | 100 | 83 | 10 | 10 | 12 | 22 |
| Geo 13 | 500 | 1000 | 18 | 103 | 100 | 83 | 10 | 10 | 12 | 22 |
| Geo 14 | 300 | 1000 | 18 | 31 | 30 | 25 | 10 | 10 | 12 | 22 |
| Geo 15 | 400 | 1000 | 18 | 31 | 30 | 25 | 10 | 10 | 12 | 22 |
| Geo 16 | 500 | 1000 | 18 | 31 | 30 | 25 | 10 | 10 | 12 | 22 |

**Table 3:** Dimensions of the studied stiffened panel models.

One of the important parameters in finite element analysis is the density of meshing which should be so selected to give rise to acceptably precise solutions. Excessive increase in the density of meshing will dramatically increase the time required for the solution. Therefore, the density should be determined accurately. Based on the results of numerous analyses, the density of 40 longitudinal

meshes and 15 transverse meshes is finally chosen for the plates. Also, the density of 2 meshes in the height of web, 2 ones in the width of flange and 40 meshes in the length is considered for stiffeners.

Given the fact that the mechanical properties of composites vary in different directions, in contrary to isotropic materials, the failure can take place by a combined effect of the stresses. Thus, various states of failure may occur in the structure and obviously, each criterion per se is not able to predict these states. There are a lot of failure criteria to predict the occurrence of failure. The formulations of some of these criteria are presented as follows:

At a glance, it can be said that the criteria of the failure for composite materials are the same as those of the failure for isotropic ones which include such criteria as Maximum Stress criterion, Maximum Strain criterion, and Polynomial criterion. The use of a precise mathematical equation for estimating the failure is strictly constrained to the number of the possible states of failure. It is also noted that the required parameters for two-dimensional failure criteria include longitudinal, transverse and tensile as well as shear strength. As mentioned above, this study uses Tsai-Wu failure criterion which is in good harmony with experimental results. This criterion is reflected as Equation (1). $\sigma_1$, $\sigma_2$ and $\tau_{12}$ are the normal stress in direction 1, normal stress in direction 2 and shear stress in direction 12, respectively.

$$\left(\frac{1}{X_T}+\frac{1}{X_C}\right)\sigma_1+\left(\frac{1}{Y_T}-\frac{1}{Y_C}\right)\sigma_2+\frac{\sigma_1^2}{X_T X_C}+\frac{\sigma_2^2}{Y_T Y_C}+\left(\frac{\tau_{12}}{S_{12}}\right)^2+2f_{12}\sigma_1\sigma_2\geq 1 \quad (1)$$

## 4 VALIDATION

To validate the proposed method, the corresponding results were compared with those of similar studies and the results of strength tests of three kinds of composite structures under compressive load, namely plates stiffened by blade stiffeners, plates stiffened by I-type stiffeners, and plates stiffened by hat-type stiffeners. The first two tests were done by Kong *et al.* (1998) and the last one was done by Smith and Dow (1985).

### 4.1 Plates with blade and I-type stiffener

Kong *et al.* (1998) studied composite plates stiffened by I-type and blade stiffeners under the pressure. The details of the models are shown in Figure 5. The other specifications of the models can be listed as follows:

- The length of the plates was 280 mm out of which 15 mm from both ends were being fitted inside the test machine; therefore, the effective length equals 250 mm.
- The loading was in-plane compressive type imposed on both ends as illustrated in Figure 6.
- The ends were considered as perfectly clamped.
- Stacking was in the form of [0.9,±45]s in plate and [0.9,+45,-45]s in stiffeners. Mechanical properties and composite failure strength are given in Table 4.

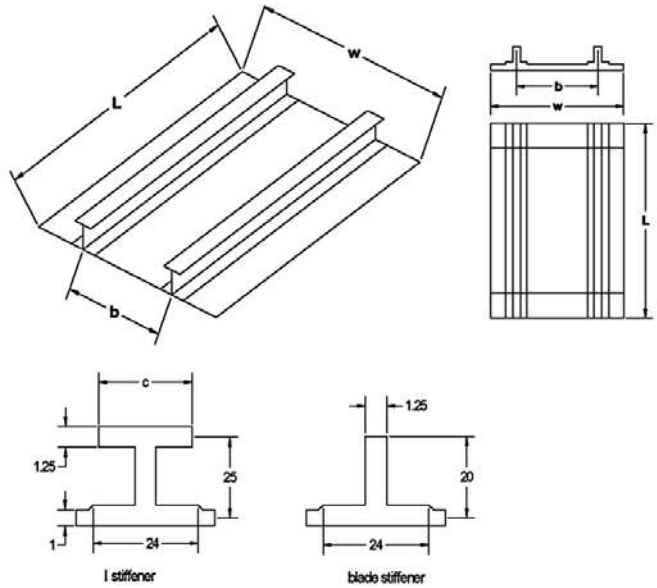



**Figure 5:** Kong *et al.* (1998) test model.

| Mechanical Properties | Symbols | Magnitude |
|---|---|---|
| Module of Elasticity in main direction of the material | $E_1$ | 130GPa |
| Module of Elasticity in direction normal to main direction of the material | $E_2$ | 10 GPa |
| Shear Modulus in direction 12 and 13 | $G_{12}, G_{13}$ | 4.85 GPa |
| Shear Modulus in direction 23 | $G_{23}$ | 3.62GPa |
| Tensile strength in direction 1 | $X_T$ | 1933 MPa |
| Compression strength in direction 1 | $X_C$ | 1051 MPa |
| Tensile strength in direction 2 | $Y_T$ | 51 MPa |
| Compression strength in direction 2 | $Y_C$ | 141MPa |
| Shear strength in direction 12 | $S_{12}$ | 61 MPa |
| Shear strength in direction 13 | $S_{13}$ | 130GPa |
| Shear strength in direction 23 | $S_{23}$ | 10 GPa |

**Table 4:** Material Properties of the Kong *et al.* (1998) test model.

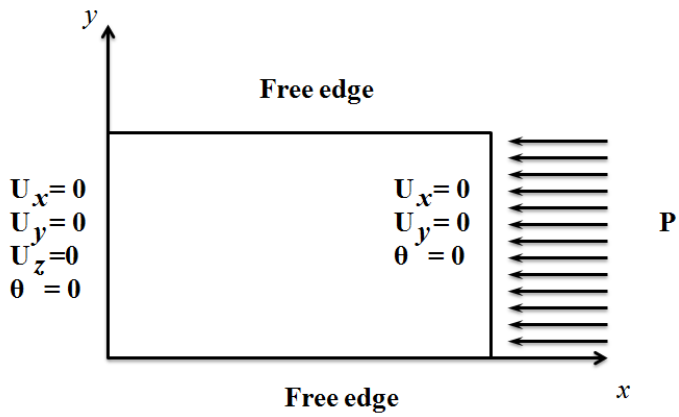



**Figure 6:** Loading and boundary conditions of the Kong *et al.* test model (1998).

First, the results of the plates stiffened by blade stiffeners are discussed. As nonlinear analysis is done in which both progressive failure methodology and failure criterion are applied simultaneously. The first failure in panel occurs at the load of 26 kN. Consequently, the decrease of mechanical properties is carried out in components having failed laminate and this process continues until the complete failure of the panel. Note that the panel finally collapses under the load of 27.2 kN.

Figure 7 shows the compressive load – end shortening curve and out-of-plane deformation distribution for the panel with blade stiffener. As can be observed in out-of-plane deformation distribution during progressive failure, the third buckling mode occurs in panel after the first failure. Buckling, first, happens under the load of about 5 kN and then, at the load greater than buckling initiation load, the first failure happens in panel. Eventually, panel collapses after some stages of failure and progressive failure and the death of some failed layers due to the excessive deformation caused by failure and buckling. In this analysis, it is concluded that the greatest failure occurs in the layers of stiffener in such panels and the period from the initiation of failure until final failure and collapse is very short.

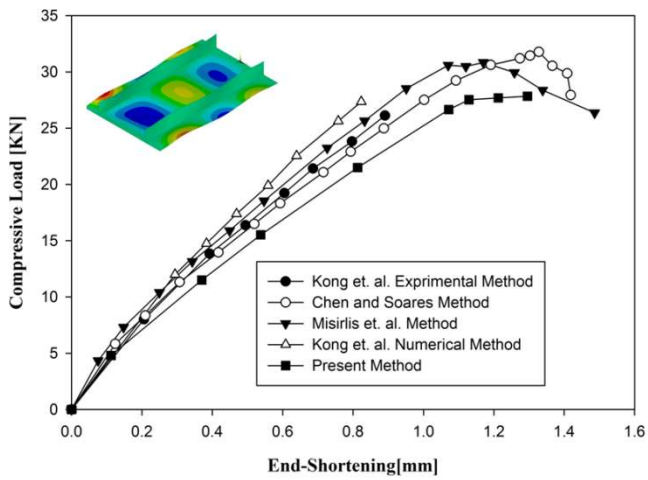



**Figure 7:** Comparison of the present method results for Kong *et al.* test with previous works (Kong *et al.* (1998), Misirlis *et al.* (2009), Chen and Soares (2007)).

At next stage, the response of plates stiffened by I-type stiffeners is investigated. The width of stiffener wing is 20 mm in this panel. Also, the whole set at the junction of stiffener to the plate is thicker than the plate, which is considered in the analysis. Given the geometry described for the plate stiffened by I-type stiffener, the meshing process was performed by using meshes with different sizes. It is noted that some or all layers of the components may undergo failure at the stage of load increase from failure initiation until the final failure.

Figures indicate that in contrary to panels with blade stiffener, the failure in panel with I-type stiffener occurs more in lamina, and at the moment of failure, panel suffers failure and collapse in central part and in the region lacking stiffener. Load – end shortening curve resulted from the analysis of this type of panel and the results of the other studies (Kong. *et al.* (1998)) are shown in Figure 8.

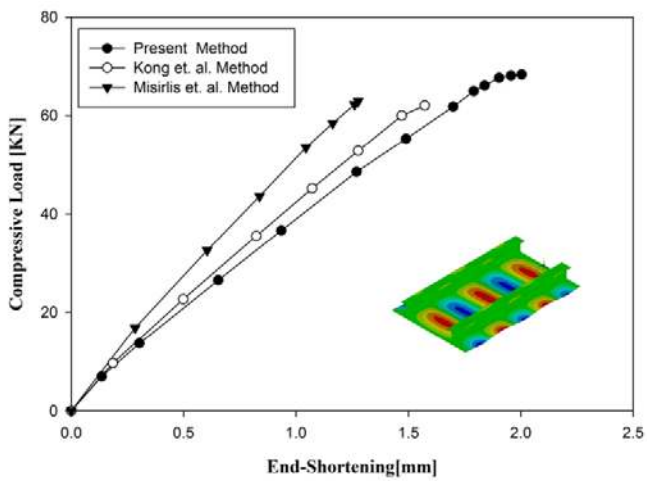
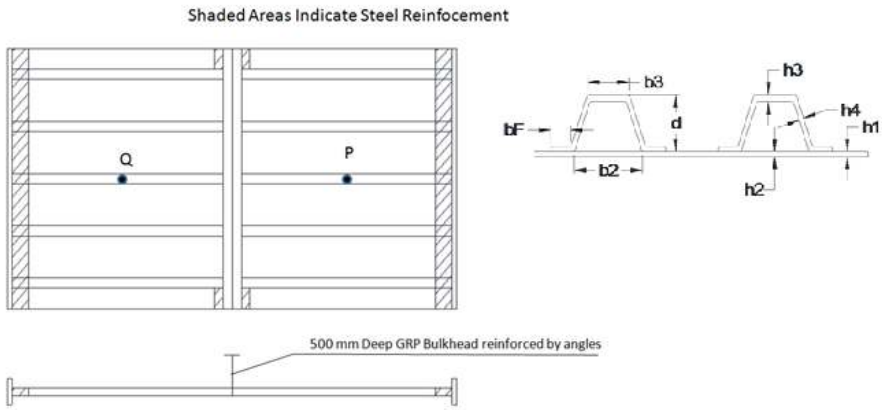



**Figure 8:** Comparison of the present method results for Kong *et al.* test with previous works (Kong *et al.* (1998), Misirlis *et al.* (2009))

As seen in the Figure 8, buckling initiation load is very close to each other in different methods, but as the load is increased, the difference amongst the methods intensifies. In the investigated panels, the progressive failure method does not result in great differences in the results such that the load of failure initiation and that of final collapse are very close to each other and slightly after the first failure, the structure collapses completely. Also, the time for analysis is very short in present method which indicates that it is capable to be used with an acceptable precision. It is finally concluded that the failure mostly occurred in the stiffening layers and their conjunction with the plate.

### 4.2 Plate with hat-type stiffener

In previous research, Smith and Dow (1985) studied longitudinally stiffened GRP panels with the real scale. The geometric details of the panel are presented in Figure 9. As well; Table 5 represents the properties of the materials.




| B | L | $d$ | $h_4$ | $h_3$ | $h_2$ | $h_1$ | $b_F$ | $b_4$ | $b_3$ | $b_2$ | $b$ | $a$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3200 | 6120 | 132 | 4 | 8.6 | 12.7 | 12.7 | 54 | 123 | 92 | 108 | 640 | 3060 |

**Figure 9:** Dimensions of Smith and Dow (1985) test model.

| Mechanical Properties | Symbols | Magnitude |
|---|---|---|
| Module of Elasticity in main direction of the material | $E_1$ | 15 GPa |
| Module of Elasticity in direction normal to main direction of the material | $E_2$ | 13.5 GPa |
| Shear Modulus in direction 12 and 13 | $G_{12}, G_{13}$ | 3.45 GPa |
| Shear Modulus in direction 23 | $G_{23}$ | 3.62GPa |

**Table 5:** Material Properties of the Smith and Dow model test.

According to Smith and Dow (1985), the experiment was conducted in a specific steel grillage machine. Also, the force was progressively increased and then was decreased to zero in some stages in order to make it possible to investigate the failures.

As the authors reported, the force was increased to the stress of 25 $kg.m^{-2}$, while deformation was invisible, but after exceeding this stress threshold, a whole deformation started to occur in pillar pole mode. Also, the longitudinal stiffeners started to deform upwardly in one side of the bulkhead and to deform downwardly in the other side.

In present study, the panel was nonlinearly analyzed in order to compare the results of the proposed method with those of Smith and Dow's experiment (1985). Figure 10 shows the finite element model of the panel. This model is composed of five hat-type longitudinal stiffeners ranging along two spans. In addition, the edges of ACE and BDF are free and the edge AB is imposed by an external force whereas the deformation of edge EF is limited in all directions and the edge AB did not deform transversely. The transverse deformation of line CD is constrained in order to model the central bulkhead which is located along line CD. Given that the hatched regions in the main model are built of steel, the materials are defined as steel in the corresponding regions of the model, too.

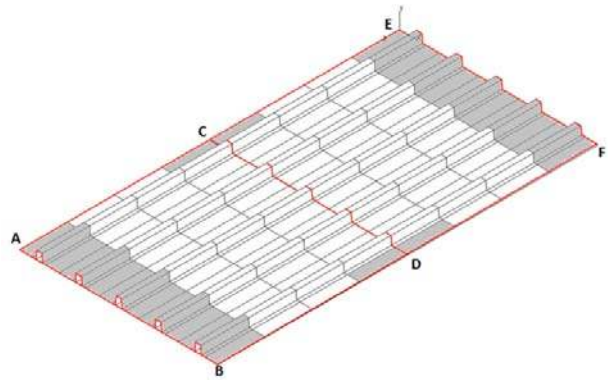



**Figure 10:** Finite Element model of the Smith and Dow test

It is assumed that the properties of materials are constant during all stages of the solution until the time before the failure, whereas after the failure, the properties of material start to decrease in the layers and components related to the location of failure. It is also supposed that the initial deformations with the range of $w_{op}$ in local buckling mode and with the range of $w_{os}$ in the total buckling mode occur in the structure concurrently. It is noted that the range of these deformations matches the maximum deformations measured in laboratorial model which are equal to $\frac{w_{op}}{a} = 0.0005$ and $\frac{w_{os}}{a} = 0.0013$, respectively.

Obviously, the deformations are slight until the stress threshold of >29 MPa and over this threshold, total buckling occurs and finally, a downward span and another upward hatch buckle. Also, the results report that the failure starts in the hatch that buckles downwardly and quickly proceeds in the width of longitudinal stiffeners. It should be mentioned that the failure does not proceed toward the inside of the plate. Figure 11 shows the diagram of vertical deformation of two points P and Q in terms of average stress which is compared with Smith's analysis. Note that point P moves upwardly and point Q moves downwardly. Therefore, the deformation of point P is shown on the right of diagram and that of Q on the left side.

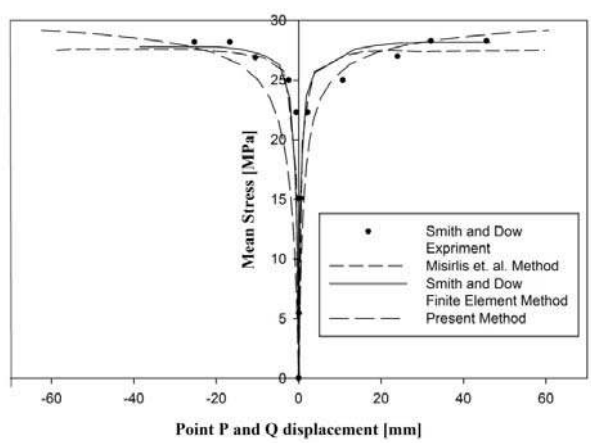



**Figure 11:** Comparison of the present method results for Smith and Dow test with previous works.

The failed elements at the threshold of ultimate strength achievement calculated by the proposed method and the spot of failure in the experiment are represented in Figure 12. To make a conclusion, as explained earlier, the results of calculations are in agreement with the results of the experiment carried out by Smith and Dow (1985).

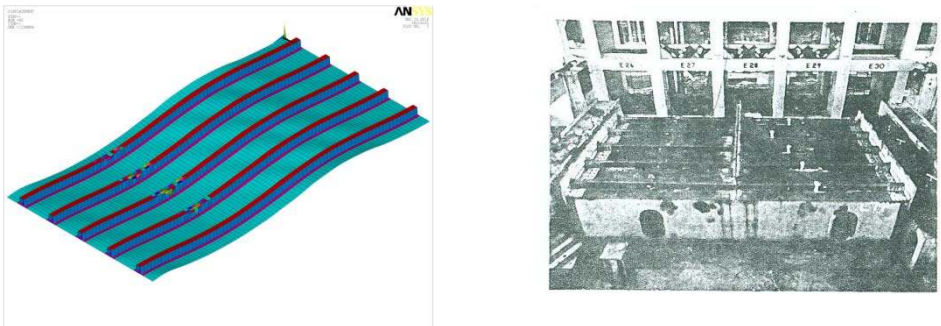


**Figure 12:** Fractured element in present model and Smith and Dow test model.

## 5 BUCKLING AND POSTBUCKLING BEHAVIOR OF STIFFENED PLATES

### 5.1 Effect of dimensional ratio on buckling and post-buckling behavior of stiffened flat plates

To study the effect of aspect ratio of stiffened plates on buckling and post-buckling behavior, 10 rectangular stiffened plates with the aspect ratios used in shipbuilding were analyzed. The geometric characteristics of these models named Geo1 to Geo10 are presented in Table 2. It is worth mentioning that the behavior of stiffened plates depends on their aspect ratio. Also, the behaviors are determined on the basis of the boundary conditions explained in the previous sections.

In stiffened plates with odd or close to odd aspect ratios, the deformation was in the form of one half-wave occurred along each span. In these stiffened plates, in the half-wave where the deformation is downward, the first failure occurs in stiffener and grows within the width, length and layers of the stiffener and stops as soon as it reaches the plate. At this stage, stiffened plate approaches its final failure limit. In addition, no considerable differences were observed in average stress – average strain curve for the plates with different odd aspect ratios.

On the other hand, in stiffened plates with even or close to even aspect ratios, the deformation resembles two half-waves along a span. In these plates, the first failure of the plate occurs in the half-wave whose deformation is downward. As the force is increased, failure occurs in stiffeners and grows in the width, length and layers of the stiffener. Also, the failure, similar to the previous state, stops as soon as it reaches the plate. At this stage, stiffened plate reaches its final failure limit. Again, there is nuance in average stress – average strain curves of the plates in terms of different even aspect ratios. However, this difference was considerable between plates with odd and even aspect ratios. It is noted that the strength value of plates with even aspect ratio was up to 54% more than that of plates with odd aspect ratio. To sum up, average stress – average strain curve and an example of the deformations are given in Figures 13-14.

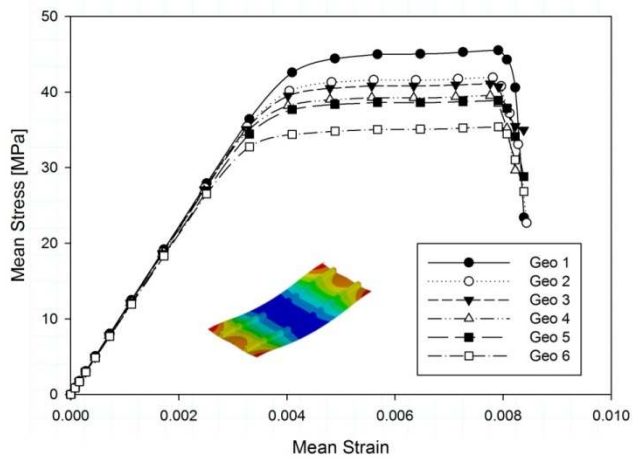



**Figure 13:** Average stress – average strain curves for stiffened plates having different aspect ratios.

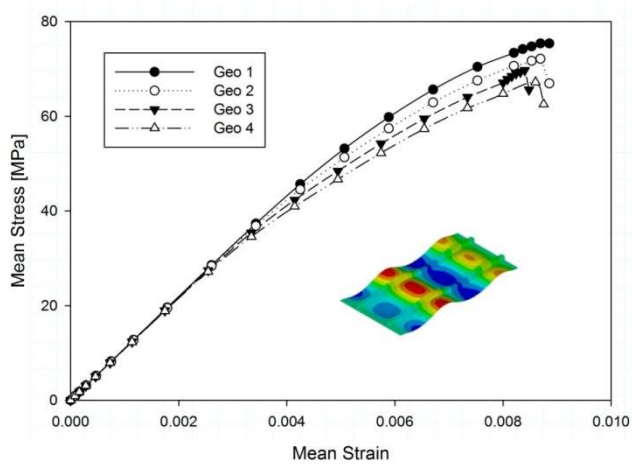



**Figure 14:** Average stress – average strain curves for stiffened plates having different aspect ratios.

Figure 15 shows the magnitude of maximum average stress as well as that of energy absorbed until the failure in this process (which is equal to the area under average stress – average strain curve).

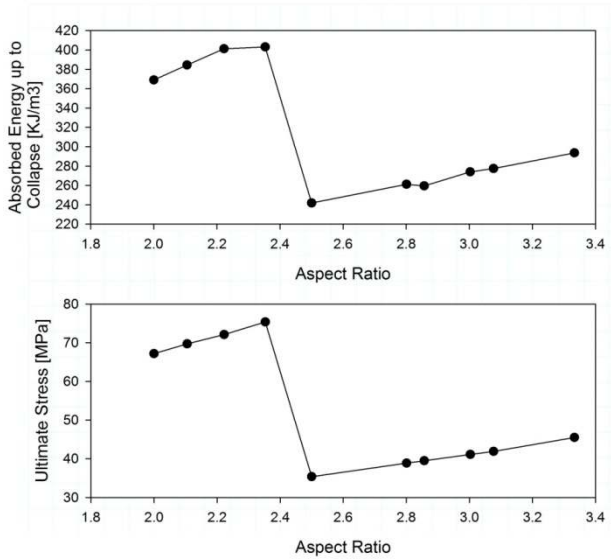



**Figure 15:** Absorbed energy up to collapse and ultimate strength of the stiffened panel with different aspect ratios.

### 5.2 Effect of dimensional ratio on buckling and post-buckling behavior of stiffened flat plates

Considering the approximately similar behavior of stiffened plates with odd aspect ratios to each other as well as the almost analogous behavior of those with even aspect ratios to each other, three geometries of Geo 1, Geo 6 and Geo 10 were selected in order to study the effect of initial deformation on the buckling and post-buckling responses in stiffened plates. These geometries were chosen because it is generally tried, in designing ship structures, to somehow choose the spacing between stiffeners that facilitates the building process too.

The deformation function for plates with odd aspect ratios is considered to be a sinusoidal one with one half-wave in a span and the corresponding function for plates with even aspect ratios is supposed to be, again, a sinusoidal function, but, with two half-waves along the length. The magnitude of maximum initial deformation equals $\Delta_{\mathrm{Im}}$. The magnitude of $\Delta_{\mathrm{Im}}$ is regarded as 1, 3, 5 and 7 mm.

Figures 16-18 show average stress – average strain diagrams for three geometries with various magnitudes of initial deformations in which *Im* is the variable introducing the maximum initial deformation. Figures 17-19 indicate average stress – average strain curves for composite stiffened plate with aspect ratio of 3.33 under four initial deformations. As can be seen, the higher the initial deformation, the greater the strength of the composite plate. Figures 17-19 show average stress – average strain curve for composite stiffened plate with aspect ratios of 5 and 2 under four initial deformations. In these cases too, the trend is as can be observed in Figure 17.

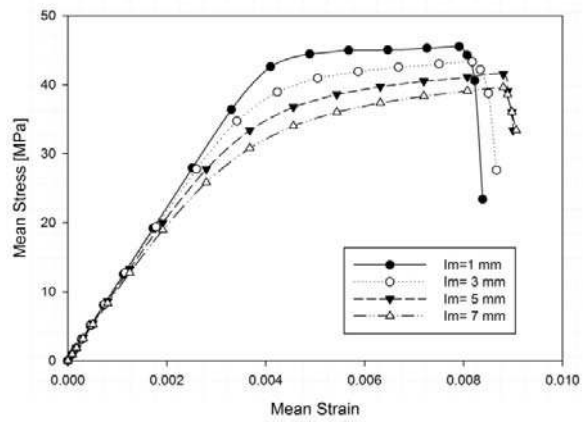



**Figure 16:** Average stress – average strain curve for plate panel with aspect ratio of 3.33 and different magnitudes of initial imperfection.

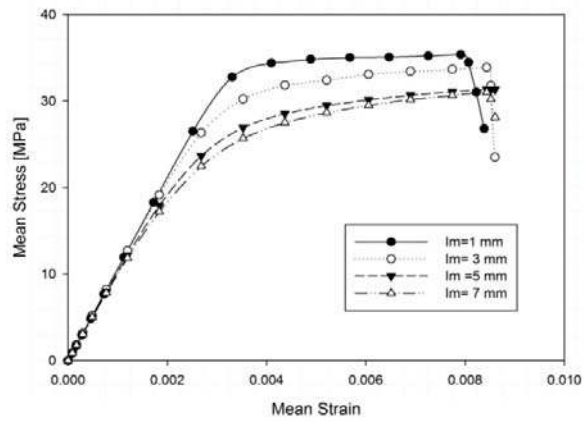



**Figure 17:** Average stress – average strain curve for plate panel with aspect ratio of 2.5 and different magnitudes of initial imperfection.

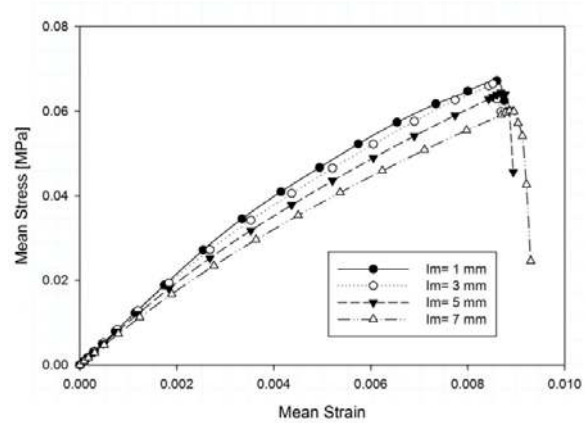



**Figure 18:** Average stress – average strain curve for plate panel with aspect ratio of 2 and different magnitudes of initial imperfection.

The figures reveal that the total behavior of plates with different magnitudes of initial deformations is not much varied and average stress – average strain curves have similar forms within each group of plates. The failure trend of plates with odd aspect ratios is quite similar. Note that the failure, firstly, occurs in stiffener and as the load is increased, the failed region starts to be quickly extended along the thickness and width of the stiffener and the plate reaches its ultimate strength. The state of ultimate strength happens in stiffener and develops around it.

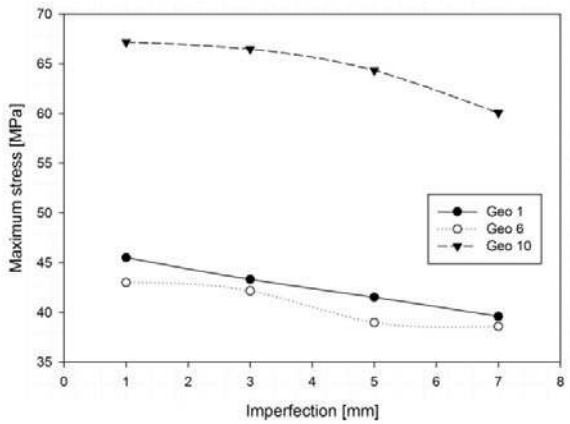



**Figure 19:** Ultimate strength of the stiffened panel with initial imperfection.

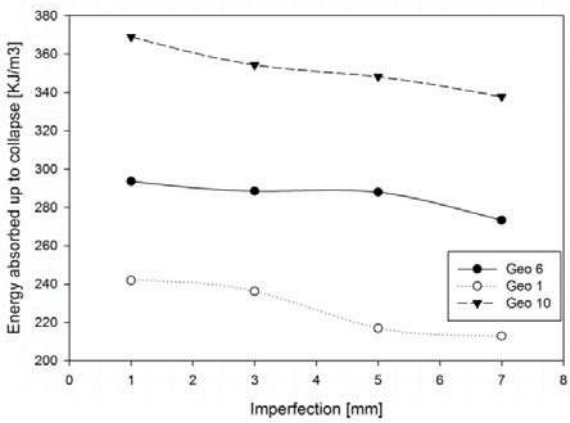



**Figure 20** Absorbed energy up to collapse by stiffened panel with initial imperfection.

Although the general behavior of stiffened plates did not change with the increase in initial imperfection, the strength magnitude of stiffened composite plates decreased with the increase in the ultimate strength. Figures 19-20 show a summary of the effect of initial deformation on the strength of stiffened plates. As can be seen, initial deformation greatly affects the ultimate strength and when $\Delta_{\mathrm{Im}} = 7mm$, the magnitude of the maximum stress applied to the stiffened can be decreased by 13% and the magnitude of the energy absorbed by stiffened plate until the failure can be reduced by 12%, whereas previous studies have underestimated or even have not dealt with the impact of initial deformation on ultimate strength of composite plates.

### 5.3 Effect of stiffness of stiffeners on buckling and post-buckling behavior of stiffened flat plates

In order to examine the impact of the stiffness of the longitudinal stiffeners on ultimate strength of composite ships, three categories of stiffened panels with the aspect ratios of 2, 2.5 and 3.33 were analyzed. These stiffened panels can be categorized into three groups of low, medium and high strength stiffened panels. The models' categories of these stiffened panels are presented in Table 6.

| Aspect Ratio | Stiffened Panel Strength | Model Name |
|---|---|---|
| 3.33 | High Strength Stiffened Panel | Geo 11 |
| | Medium Strength Stiffened Panel | Geo 1 |
| | Low Strength Stiffened Panel | Geo 14 |
| 2.5 | High Strength Stiffened Panel | Geo 12 |
| | Medium Strength Stiffened Panel | Geo 6 |
| | Low Strength Stiffened Panel | Geo 15 |
| 2 | High Strength Stiffened Panel | Geo 13 |
| | Medium Strength Stiffened Panel | Geo 10 |
| | Low Strength Stiffened Panel | Geo 16 |

**Table 6:** Categories of these stiffened panels.

The average stress – average strain curves of these stiffeners are given in Figures 21-23 which represent the responses of stiffened plates with the aspect ratios of 3.33, 2.5 and 2, respectively.
Note that Geo 11, Geo 14 and Geo 1 indicate plates with high, low and medium strength, respectively. The behavior of stiffened plates Geo 1 and Geo 14 are similar to each other while high strength stiffened panel (Geo 11) exhibits a quite different behavior. Furthermore, in high strength stiffened panel (Geo 11) model, the stiffener remained quite un-deformed and only the plate buckled. The buckling mode of plate was alike the buckling of the plate with simple supports and was independent of the boundary conditions defined for the plate.

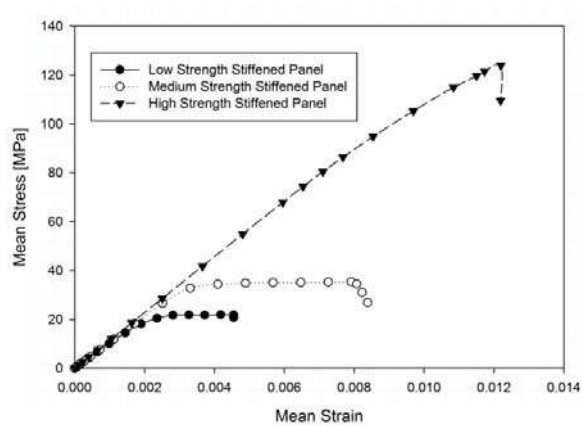



**Figure 21:** Average stress – average strain curve for plate panel with aspect ratio 3.33 and three different stiffener dimensions.

In Figure 23, the behaviors of low strength stiffened plates (Geo 15) and medium strength stiffened plates (Geo 6) are similar, but high strength stiffened plate (Geo 12) shows completely different behavior. In high strength stiffened plate model, the stiffener remains quite undeformed and only the plate buckles. The buckling behavior of the plate is similar to that of a plate with simple support and is independent of boundary conditions defined for this plate.

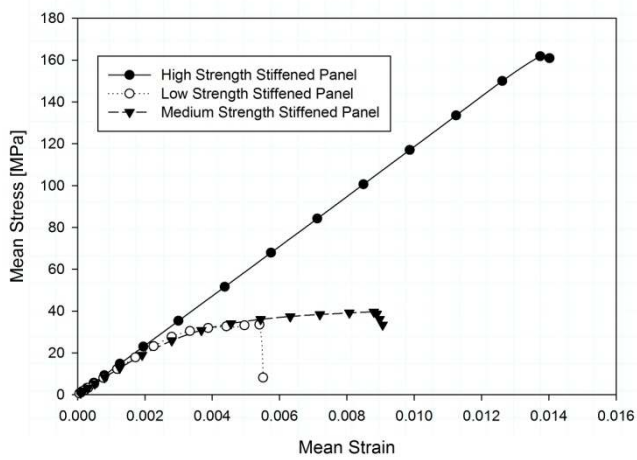



**Figure 22:** Average stress – average strain curve for plate panel with aspect ratio 2.5 and three different stiffener dimensions.

In Figure 24, the behaviors of low strength stiffened plates (Geo 16) and medium strength stiffened plates (Geo 10) are similar, whereas high strength stiffened plate (Geo 13) shows a completely different behavior. This was confirmed in Figure 23 and Figures 24 for other aspect ratios of the plate.

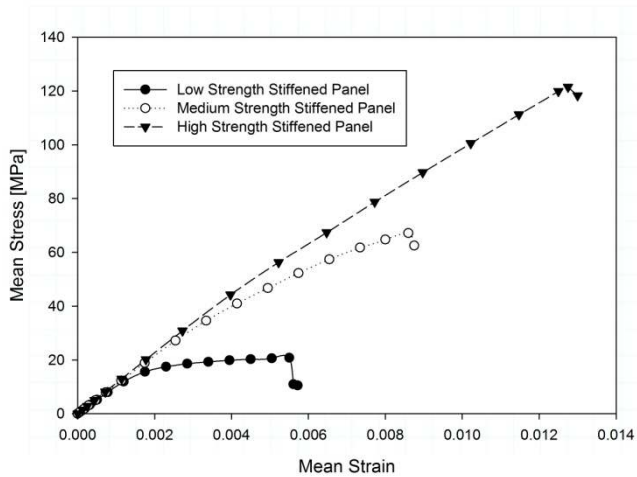



**Figure 23:** Average stress – average strain curve for plate panel with aspect ratio 2 and three different stiffener dimensions.

## 6 CONCLUSIONS

The previous studies report sparse researches in the area of stiffened plates used in shipbuilding. In addition, the effect of initial deformation on the strength of composite stiffened plates has not been studied yet.

In this research, the progressive failure method for composite materials was studied and the model was developed and then validated. Next, different forms of the composite stiffened plates used in marine industries were analyzed in terms of different aspect ratios, stiffener dimensions and initial deformation. A set of analyses was carried out based on nonlinear progressive failure method. Afterwards, average stress – average strain curves were derived.

The investigation of the effect of aspect ratio on buckling and post-buckling behavior of stiffened flat plates of a ship revealed that this parameter considerably affects the buckling and post-buckling behavior of stiffened flat plates; because the behaviors of stiffened plates with odd and even aspect ratios are very different. It was also found that the ultimate strength of stiffened plates with odd aspect ratio was almost half of those of stiffened plates with even aspect ratios. However, in the selected range of aspect ratio, the buckling and post-buckling behaviors of stiffened plates with different odd aspect ratios are very similar to each other and this phenomenon is also true for stiffened plates with different even aspect ratios.

Finally, within investigation of the initial deformation impact on buckling and post-buckling behavior of composite stiffened plates, the form of initial deformation was considered to be same as the first mode of initial deformation of stiffened plates. It was also concluded that initial deformation considerably influences ultimate strength and that when $\Delta_{\mathrm{Im}} = 7mm$. Note that previous studies have not dealt with the impact of initial deformation on ultimate strength of composite plates.

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