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A Cultural Algorithm for Optimal Design of Truss Structures

Abstract

A cultural algorithm was utilized in this study to solve optimal design of truss structures problem achieving minimum weight objective under stress and deflection constraints. The algorithm is inspired by principles of human social evolution. It simulates the social interaction between the peoples and their beliefs in a belief space. Cultural Algorithm (CA) utilizes the belief space and population space which affects each other based on acceptance and influence functions. The belief space of CA consists of different knowledge components. In this paper, only situational and normative knowledge components are used within the belief space. The performance of the method is demonstrated through four benchmark design examples. Comparison of the obtained results with those of some previous studies demonstrates the efficiency of this algorithm.

Keywords

Cultural algorithm, Truss structure design, Size optimization.

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1 INTRODUCTION

In recent decades, various optimization techniques have been applied to optimal design of truss structure with stress and deflection constraints. The optimum design of truss structures are usually categorized into three different optimization problems: size, layout and topology optimizations. In the first category, only the cross sectional areas of the members are considered to minimize the weight of the structure, while the nodal coordinates of the truss are also taken as design variables in the layout optimization of truss structures. In the third category, the number of members of the structure and the connectivity of them are optimized. This paper focuses on the first category of truss optimum design problem, in which only sizing variables are considered as design variables.

Over the last years, the studies on meta-heuristic search methods such as Particle Swarm Optimization (PSO) (Eberhart and Kennedy, 1995), Ant Colony Optimization (ACO) (Dorigo, 1992), Harmony Search (HS) (Geem et al., 2001), Simulated Annealing (SA) (Kirkpatrick et al., 1983) and Big Bang-Big Crunch (BB-BC) (Erol and Eksin, 2006) have shown that these methods can be effi-

ciently used to solve engineering optimization problems characterized by non-convexity, discontinuity and non-differentiability. Most of these stochastic search methods are simulation of the specific phenomenon in the nature such as social behavior of bird flocking or fish schooling, evolution theories of the universe and annealing processes in materials.

Researchers have been applied various meta-heuristic optimization algorithms to the optimal design of truss structures. For example, Camp (2007) employed original BB-BC to size optimization of truss structures with continues and discrete design variables. Lamberti (2008) suggested a heuristic algorithm based on SA. Hasancebi et al. (2009) evaluated the performance of different algorithms in optimal design of pin jointed structures.

As extensions of meta-heuristic algorithms, hybrid algorithms have been developed to improve the performance of original meta-heuristic algorithms. The main aim of developing hybrid algorithms is to provide an adequate balance between the exploration and exploitation mechanisms. The exploration mechanism is related to the ability of algorithm to the performing efficient search in solution space of the optimization problem, while the exploitation mechanism is related to the ability of finding better solutions in the vicinity of the current solutions. Such hybrids have been successfully applied to optimal design of truss structures. For instance, Li et al. (2007) introduced a heuristic particle swarm optimizer (HPSO) for size optimization of truss structures. Kaveh and Talatahari (2009[a]) proposed particle swarm optimizer, ant colony strategy and harmony search scheme. This method is based on the particle swarm optimizer with passive congregation (PSOPC), ant colony optimization and harmony search scheme. Degertekin (2012) presented an efficient harmony search algorithm (EHS) and self-adaptive harmony search algorithm (SAHS) for sizing optimization of truss structures. In another work, Degertekin and Hayalioglu (2013) used teaching-learning-based optimization (TLBO) method for sizing truss structures which is a new meta-heuristic search method. TLBO simulates the social interaction between the learners and teacher. Kaveh and Talatahari (2009[b], 2010[a]) proposed a hybrid big bang-big crunch (HBB-BC) by combining BB-BC algorithm and with Sub Optimization Mechanism (SOM) for size optimization of space trusses and ribbed domes. In this method, SOM is an auxiliary tool which works as a search-space updating mechanism.

In some cases, researchers utilized novel optimization algorithms. For example, Kaveh and Talatahari (2010[b,c,d], 2012) utilized Charged System Search (CSS) to optimal design of frame, grillage and skeletal structures and Imperialist Competitive Algorithm (ICA)) for size optimization of skeletal structures. And recently, Kaveh and Khayatazad (2013) employed Ray Optimizer (RO) to size and shape optimization of truss structures. Sonmez (2011) used Artificial Bee Colony (ABC) algorithm to sizing of truss structures.

In this study, a specific version of Cultural Algorithms (CAs) is utilized to size optimization of truss structures. CA is a population based random search method which simulates the social evolution process. Compared with other meta-heuristic methods, CA uses a belief space beside the population space. The belief space is divided into distinct categories. In this paper, only two categories, called Normative knowledge, and Situational knowledge are used within the belief space. The algorithm is an iterative process in which new populations are obtained using influence function based on knowledge components of belief space. Four truss design examples are utilized with stress and

deflection constraints and results are compared with different methods in order to show the efficacy of present approach.

The remainder of this paper is organized as: mathematical description of the optimum design problem is first reviewed in Section 2. Then, Section 3 presents a brief review of the CA. In Section 4, the effectiveness of CA is verified by four design examples. Finally, conclusions are presented in Section 5.

2 OPTIMUM DESIGN PROBLEM

The main aim of optimal design of a truss structure is to minimize the weight of the structure while satisfying some constraints on stresses and deflections. In this class of optimization problems, cross sectional areas are taken as design variables. The optimal design of a truss structure can be formulated as:

$$\begin{aligned} &\text{Find: } A=[\, A_1, A_2, \ldots, A_{nd}\,] \\ &\text{To minimize: } W(\{A\}) = \sum_{i=1}^{m} \gamma_i A_i L_i \\ &\text{Subjected to:} \\ &\sigma_i^c \le \sigma_i \le \sigma_i^t, \qquad i = 1,2,\ldots,m \\ &A_{\min} \le A_j \le A_{\max} \qquad j = 1,2,\ldots,nd \\ &\delta_{\min} \le \delta_k \le \delta_{\max}, \qquad k = 1,2,\ldots,n \end{aligned} \tag{1}$$

Where A is the vector containing the design variables; m is the number of members making up the structure; W(.) is the weight of the structure; γ_i is the material density of member i; A_i is the cross-sectional area of the member i which is between A_{min} and A_{max}; L_i is the length of the member i; nd is the number of design variables; n is the number of nodes; σ_i^t and σ_i^c are the allowable tension and compressive stresses for member i, respectively; δ_i is the displacement of node i and δ_{min} and δ_{max} are corresponding lower and upper limits.

Optimal design of truss structure should satisfy the above mentioned constraints. In this study, the constraints are handled by using a simple penalty function method. Thus, a fitness function must be given to evaluate the quality of a solution candidate. For each solution candidates, following cost function is defined:

$$F_{fitness} = W(\{A\}) \times f_{penalty} \tag{2}$$

$$f_{penalty} = (1 + \varepsilon_1 . \varphi)^{\varepsilon_2}, \quad \varphi = \sum_{i=1}^{q} \varphi_i \tag{3}$$

Where $f_{penalty}$, is the penalty function represented by solution A, q is the number of constraints and φ is the penalty factor which is related to the violation of constraints. In order to obtain the values

of φ_i, the stresses and nodal displacements of the structure are compared to the corresponding upper or lower bounds as follow.

$$\begin{cases} \varphi_i = \left| \dfrac{\sigma_i^{t,c} - \sigma_i}{\sigma_i^{t,c}} \right| & \text{for} \quad \sigma_i \le \sigma_i^c \ \ or \ \sigma_i \ge \sigma_i^t \\ \varphi_i = 0 & \text{for} \quad \sigma_i^c \le \sigma_i \le \sigma_i^t \end{cases} \tag{4}$$

As it can be seen from Eq. (4), if the constraints are not violated, the value of the penalty function will be zero. In Eq. (3), the values of parameters ε_1 and ε_2 are selected considering the exploration and the exploitation rate of the search space. In this study ε_1 is taken as unity, and ε_2 starts from 2 and gradually increases. The value of ε_2 for tth iteration is calculated as follow:

$$\varepsilon_2{}^{(t)} = \varepsilon_2{}^{(t-1)} + 10^{-3}.t \tag{5}$$

3 CULTURAL ALGORITHM

CA is a stochastic optimization technique originally developed by Reynolds (1991, 1999) inspired by theories of cultural evolution in sociology and archaeology. In fact, each society has a population and the individuals are the members of this population. The individuals of a society have cultural experiences that acquired by the previous generations. Culture can be seen as a set of ideological phenomena shared by a population (Peter et al., 2004), which consists of the beliefs, art and other things that acquired and transformed to the current generation by the previous generations. Sociologists believe that that the most of those forms of culture might be symbolically encoded and shared among the individuals of the society as a inheritance mechanism, and this mechanism may enhance the adaptability of the societies as well as accelerate the evolution speed of the society by making use of the domain knowledge obtained from generation to generation and spreading those useful information among all the individuals of the society (Youlin et al., 2011). Based on the described mechanism, the CA simulates the social interactions between the individuals of the population to develop a new optimization method. This algorithm uses the domain knowledge extracted during the optimization in order to bias the search process. As illustrated in Figure 1, CA utilizes two population and belief spaces which influence each other based on influence and acceptance functions. The population space consists of possible solution candidates to the optimization problem and the belief space records the cultural information about the behaviors and experiences of elites in the population space.

3.1 Belief Space

As mentioned before, the belief space consists of different knowledge components. The types of knowledge components depend mainly on the optimization problem being solved. Generally, the belief space consists of the following two knowledge components (Reynolds and Chung, 1997):

1. A Situational knowledge component, which is includes the best experience or solution gained by whole individuals in population space. This knowledge component is like global best in particle swarm optimization.
2. A Normative knowledge component, which is record the behaviors and experiences of accepted individuals from the population space and provide a set of intervals, one for each dimension of the problem. These intervals specify the ranges of search space which is good to search, and eliminate undesirable parts.

In addition, another three knowledge components can be added, such as domain knowledge, historical knowledge and topographical knowledge components (Reynolds and Saleem, 2000; Peng et al., 2003). But in this paper, only two knowledge components (Situational and Normative) are used in the belief space. Thus, the belief space expressed as the tuple:

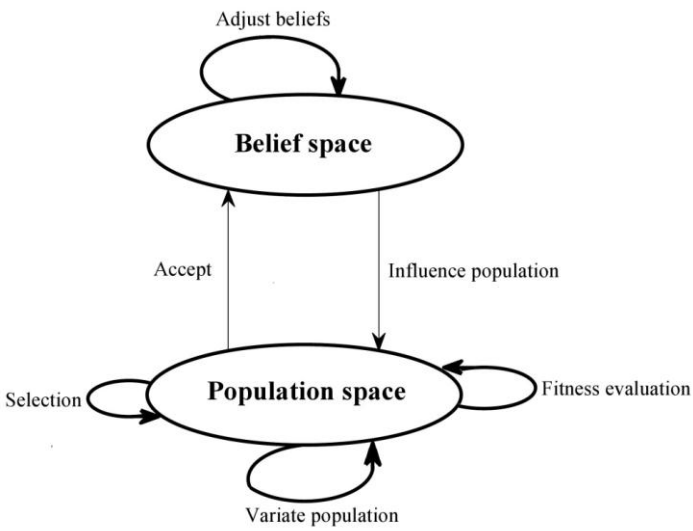

Figure 1: Illustration of components of the Cultural algorithm.

$$B(t) = (S(t), N(t)) \tag{6}$$

Where S(t) and N(t) is the Situational and Normative knowledge components, respectively, and can be expressed as:

$$S(t) = \{A^{*}(t)\} \tag{7}$$

$$N(t) = \{X_1,...,X_n\} \tag{8}$$

For each dimension following information is stored:

$$X_j=\{I_j(t),L_j(t),U_j(t)\} \quad , j=1,2,...,nd \tag{9}$$

Where, {A*(t)} is the vector of Situational design variables and nd is the number of design variables. $I_j(t)$ denotes the interval $\{A_j^{min}(t), A_j^{max}(t)\}$ for design variable j, which is assigned $A_j^{min} = +\infty$ and $A_j^{max} = -\infty$ for all design variables at the beginning time t=0. L_j (t) and U_j (t) represents the scores for the lower and upper bounds of design variable *j*. L_j (t) and U_j (t) initialized to $+\infty$.

3.2 Acceptance Function

The acceptance function determines the number of solution candidates from the population space to adjust knowledge components of belief space. For this purpose there are two static and dynamic methods. In static method, the number of individuals that accepted to shape beliefs is fixed during the time, while in the dynamic methods changes with respect to time. In this paper, static method is employed to accept individuals to shape beliefs. Thus, the top %N of individuals based on fitness values is accepted to adjust the belief space.

3.3 Adjusting the Belief Space

The knowledge components of belief space are adjusted by selected individuals as follows:

3.3.1 Situational Knowledge

$$\begin{cases} S(t+1) = \{A^k(t)\} & if\ W(\{A^k(t)\}) < W(\{A^*(t)\}) \\ S(t+1) = \{A^*(t)\} & otherwise \end{cases} \qquad k=1,2,\ldots,na \tag{10}$$

Where W(.) is the weight of the structure, $\{A^k(t)\}$is the vector of kth accepted individual and na is the number of accepted individuals to adjust the belief space.

3.3.2 Normative Knowledge

$$A_j^{min}(t+1) = \begin{cases} A_j^k(t) & if\ A_j^k(t) \le A_j^{min}(t)\ or\ W(\{A^k(t)\}) < L_j(t) \\ A_j^{min}(t) & otherwise \end{cases} \tag{11}$$

$$A_j^{max}(t+1) = \begin{cases} A_j^k(t) & if\ A_j^k(t) \ge A_j^{max}(t)\ or\ W(\{A^k(t)\}) < U_j(t) \\ A_j^{max}(t) & otherwise \end{cases} \tag{12}$$

$$L_j(t+1) = \begin{cases} \mathrm{W}(\{A^k(t)\}) & \text{if } A_j^k(t) \le A_j^{min}(t)\ or\ W(\{A^k(t)\}) < L_j(t) \\ L_j(t) & otherwise \end{cases} \tag{13}$$

$$U_j(t+1) = \begin{cases} \mathrm{W}(\{A^k(t)\}) & \text{if } A_j^k(t) \ge A_j^{max}(t)\ or\ W(\{A^k(t)\}) < U_j(t) \\ U_j(t) & otherwise \end{cases} \tag{14}$$

Where $A_j^k(t)$ is the *j*th variable of the *k*th accepted individual to adjust belief space.

3.4 Influence Function

The positions of individuals in the population space are updated by influence function. Reynolds and Chung (1997) proposed four influence function to update positions. In this paper, only following influence function is used:

$$A_{ij}(t+1) = \begin{cases} A_{ij}(t) + |\sigma_{ij} N(0,1)| & if\ A_{ij}(t) < A^*_{ij}(t) \\ A_{ij}(t) - |\sigma_{ij} N(0,1)| & if\ A_{ij}(t) > A^*_{ij}(t) \\ A_{ij}(t) + \sigma_{ij} N(0,1) & otherwise \end{cases} \qquad i=1,2,..,ni\ ,\ j=1,2,..,nd \tag{15}$$

Where $A_{ij}(t+1)$ is the new solution at time t for individual i and variable j, $N(0,1)$ is a normally distributed random variable with a mean of 0 and a standard deviation of 1, ni is the number of individuals used in population space, nd is the number of design variables and σ_{ij} is the strategy parameter for individual i and design variable j which is calculated as follow:

$$\sigma_{ij} = \beta\,(A_j^{max}(t) - A_j^{min}(t)) \tag{16}$$

Where β is the user defined parameter. Finally, the optimal design of truss structures with CA can be summarized as following steps:

Step 1: Initialization
In this step, the initial population space is randomly generated between the lower and upper bounds for each design variable.

Step 2: Create and initialize the belief space.
The initial belief space created and initialized as explained in Section 3.1.

Step 3: Evaluation
Evaluation of each individual in population space and selecting the top %N of individuals based on the fitness values for adjust the belief space.

Step 4: Adjust Beliefs
In this step, the knowledge components of belief space are adjusted by accepted individuals as described in Section 3.3.

Step5: Influence population
In this step, the positions of individuals are updated by influence function as explained in Section 3.4.

Step 6: Finish or redoing
Steps 3, 4 and 5 repeated until a terminating criterion is fulfilled.

4 DESIGN EXAMPLES

In this section, four design examples have been conducted to assess the performance of the CA approach for the optimal design of truss structures with stress and deflection constraints: 10-bar planar truss, 25-bar spatial truss, 72-bar spatial truss and 120-bar dome truss. The performance of present algorithm is compared with some simple and improved algorithms from literature.

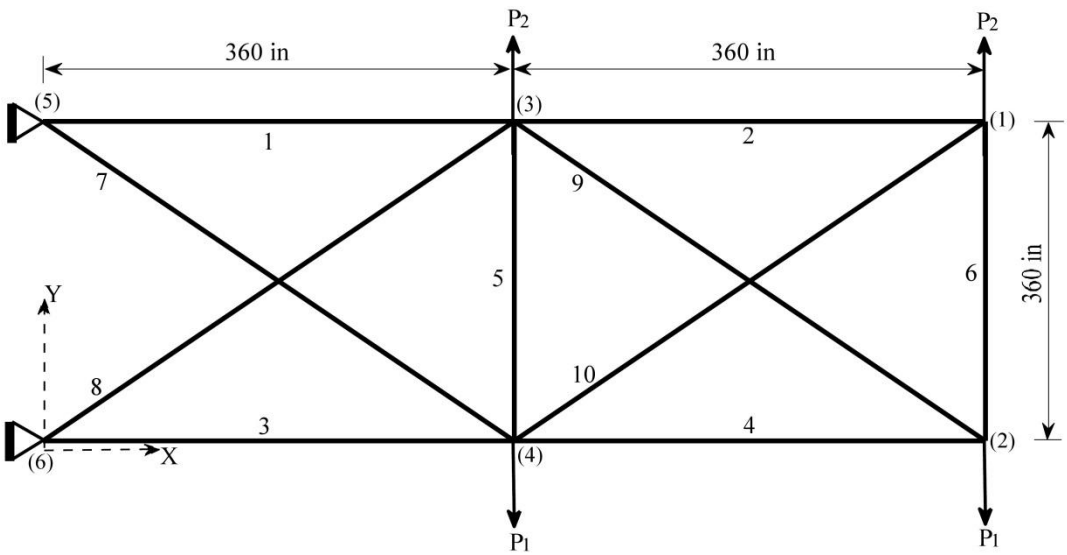

Figure 2: Scheme of the 10-bar planar truss.

In the all design examples, the population sizes of the algorithm are taken as 10, the number of accepted individuals to adjust belief space is 4 and the value of β parameter is chosen as uniformly random number between 0 and 1. Due to stochastic nature of algorithm, the algorithm carries out independently for 10 times for each design example. Each run stops when the maximum structural analyses are reached. The maximum number of the structural analyses for each design example is different and it is depends on the dimension of the optimization problem. Therefore, the maximum structural analyses are set to 24,000 for example 1 and 20,000 for examples 2 and 3. For the last example, 12,000 structural analyses are considered.

The CA algorithm and direct stiffness method for analysis of truss structures have been implemented in MATLAB program and run in Dell Vostro 1520 with Intel CoreDuo2 2.66 GHz processor and 4 GB RAM memory.

4.1 A 10-bar Planar Truss

The 10-bar planar truss shown in Figure 2 is the first design example. The Young's modulus and material density of truss members are 10^4 ksi and 0.1 lb/in^3 , respectively. The members are subjected to the stress limits of ± 25 ksi. The maximum nodal displacements in X and Y directions are limited to ± 2 in for all free nodes. The minimum allowable cross sectional area of each member is

taken as 0.1 in^2. In this design example, the loading condition is considered as: P1=150 kips and P2=50 kips.

In Table 1, the results obtained by the CA are compared with those reported in the literature like PSO, PSOPC, HPSO, ABC-AP, EHS, SAHS and TLBO. From Table 1, it can be concluded that CA gives lightest design as compared to the results obtained by PSO, EHS, SAHS and TLBO, but heavier design than PSOPC, HPSO and ABC-AP methods. However, it is clear from Table 1 that the CA required significantly less structural analyses than PSOPC, HPSO and ABC-AP methods. In addition, TLBO obtained 4678.31 lb after 14,875 structural analyses, while CA found the same weight after 10,510 structural analyses.

In addition, the convergence behaviors of the best solution and the average of 10 independent runs are shown in Figure 3.

Design variables (in^2)	Li et al. (2007)			Sonmez (2011)	Degertekin (2012)		Degertekin and Hayalioglu (2013)	This study
	PSO	PSOPC	HPSO	ABC-AP	EHS	SAHS	TLBO	CA
A_1	22.935	23.473	23.353	23.4692	23.589	23.525	23.524	23.1610
A_2	0.102	0.101	0.100	0.1005	0.100	0.100	0.100	0.1000
A_3	25.355	25.287	25.502	25.2393	25.422	25.429	25.441	25.9465
A_4	14.373	14.413	14.250	14.3540	14.488	14.488	14.479	14.4840
A_5	0.100	0.100	0.100	0.1001	0.100	0.100	0.100	0.1000
A_6	1.990	1.969	1.972	1.9701	1.975	1.992	1.995	1.9699
A_7	12.346	12.362	12.363	12.4128	12.362	12.352	12.334	12.2929
A_8	12.923	12.694	12.984	12.8925	12.682	12.698	12.689	12.9209
A_9	20.678	20.323	20.356	20.3343	20.322	20.341	20.354	20.0708
A_{10}	0.100	0.103	0.101	0.1000	0.100	0.100	0.100	0.1000
Weight (lb)	4679.47	4677.70	4677.3	4677.077	4679.02	4678.84	4678.31	4678.01
Average weight (lb)	N/A	N/A	N/A	N/A	4681.61	4680.08	4680.12	4681.5
Standard deviation (lb)	N/A	N/A	N/A	N/A	2.51	1.89	1.016	3.86
No. of analyses	150,000	150,000	125,000	500×10^3	11,402	7267	14,857	17,600

Table 1: Optimized designs for the 10-bar planar truss.

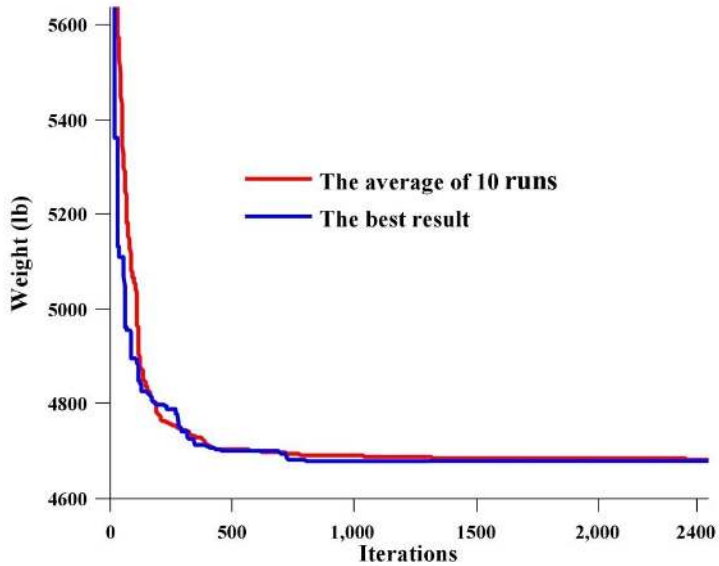

Figure 3: Convergence diagrams for the 10-bar planar truss.

4.2 A 25-bar Spatial Truss Structure

The second design example deals with the size optimization of a twenty-five-bar spatial truss structure shown in Figure 4. The Young's modulus and material density of truss members are 10^4ksi and 0.1 lb/in^3, respectively. Twenty five members are categorized into eight groups, as follows: (1) A_1, (2) $A_2 - A_5$, (3) $A_6 - A_9$, (4) $A_{10} - A_{11}$, (5) $A_{12} - A_{13}$, (6) $A_{14} - A_{17}$, (7) $A_{18} - A_{21}$, and (8) $A_{22} - A_{25}$.

The spatial truss structure is subjected to the multiply loading condition as shown in Table 2. The maximum nodal displacements in all directions are limited to ± 0.35 in for all free nodes. The allowable tension stresses are the same for the all design groups, but the allowable compressive stresses depend to the length of the members and it is different for each design group as shown in Table 3. The range of cross sectional areas varies from 0.01 in^2 to 3.4 in^2.

The optimization results obtained by the CA are presented in Table 4 and are compared with those of the PSO, PSOPC, HPSO, BB-BC, EHS, SAHS and TLBO approaches. From Table 4, it is evident that CA yields lighter structural weight than other methods. The best result of the CA approach is 545.05, while it is 545.19, 545.38, 545.15, 545.49, 545.12 and 545.09 lb for the HPSO, BB-BC, EHS, SAHS and TLBO algorithm, respectively. In addition, it is observed that TLBO found minimum weight of 545.09 lb after 15,318 structural analyses while CA obtained the same weight after 7000 structural analyses. Also the convergence behaviors of the best solution and the average of 10 independent runs are presented in Figure 5.

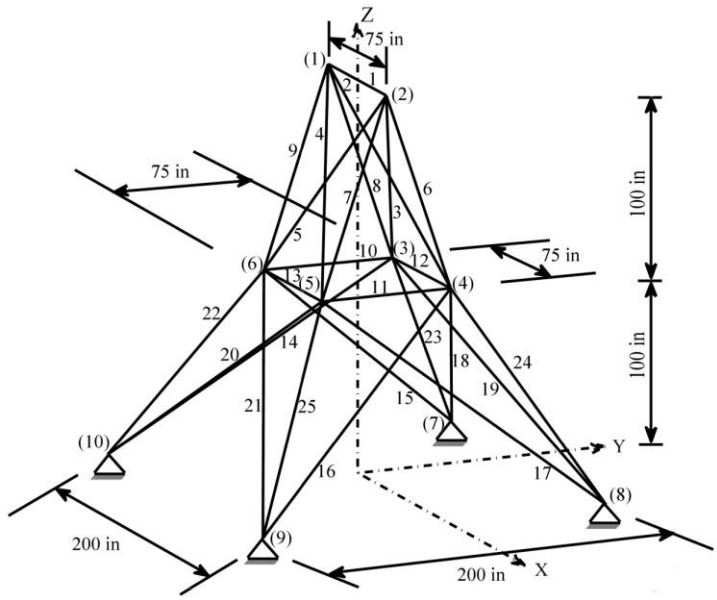

Figure 4: Scheme of the 25-bar spatial truss structure.

Node	Condition 1			Condition 2		
	P_x	P_y	P_z	P_x	P_y	P_z
1	0.0	20.0	-5.0	1.0	10.0	-5.0
2	0.0	-20.0	-5.0	0.0	10.0	-5.0
3	0.0	0.0	0.0	0.5	0.0	0.0
6	0.0	0.0	0.0	0.5	0.0	0.0

Note: loads are in kips.

Table 2: Loading conditions for the 25-bar spatial truss.

Element group	Allowable compressive stress (ksi)	Allowable tension stress (ksi)
1	35.092	40.0
2	11.590	40.0
3	17.305	40.0
4	35.092	40.0
5	35.092	40.0
6	6.759	40.0
7	6.959	40.0
8	11.082	40.0

Table 3: Allowable stress values for the 25-bar spatial truss.

Design variables (in^2)		Li et al. (2007)	Camp (2007)			Degertekin (2012)		Degertekin and Hayalioglu (2013)	This study
		PSO	PSOPC	HPSO	BB-BC	EHS	SAHS	TLBO	CA
1	A_1	9.863	0.010	0.010	0.010	0.010	0.010	0.0100	0.010000
2	$A_2 - A_5$	1.798	1.979	1.970	2.092	1.995	2.074	2.0712	2.020640
3	$A_6 - A_9$	3.654	3.011	3.016	2.964	2.980	2.961	2.9570	3.017330
4	$A_{10} - A_{11}$	0.100	0.100	0.010	0.010	0.010	0.010	0.0100	0.010000
5	$A_{12} - A_{13}$	0.100	0.100	0.010	0.010	0.010	0.010	0.0100	0.010000
6	$A_{14} - A_{17}$	0.596	0.657	0.694	0.689	0.696	0.691	0.6891	0.693830
7	$A_{18} - A_{21}$	1.659	1.678	1.681	1.601	1.679	1.617	1.6209	1.634220
8	$A_{22} - A_{25}$	2.612	2.693	2.643	2.686	2.652	2.674	2.6768	2.652770
Weight (lb)		627.08	545.27	545.19	545.38	545.49	545.12	545.09	545.05
Average weight (lb)		N/A	N/A	N/A	545.78	546.52	545.94	545.41	545.93
Standard deviation (lb)		N/A	N/A	N/A	0.491	1.05	0.91	0.42	1.55
No. of analyses		150,000	150,000	125,000	20,566	10,391	9051	15,318	9380

Table 4: Optimized designs for the 25-bar spatial truss.

4.3 A 72-bar Spatial Truss Structure

A 72-bar spatial truss shown in Figure 6 is the third design example. The Young's modulus and material density of truss members are 0.1 lb/in^3 and 10^4 ksi, respectively. The 72 members of this spatial truss are divided into 16 groups using symmetry, as follows:

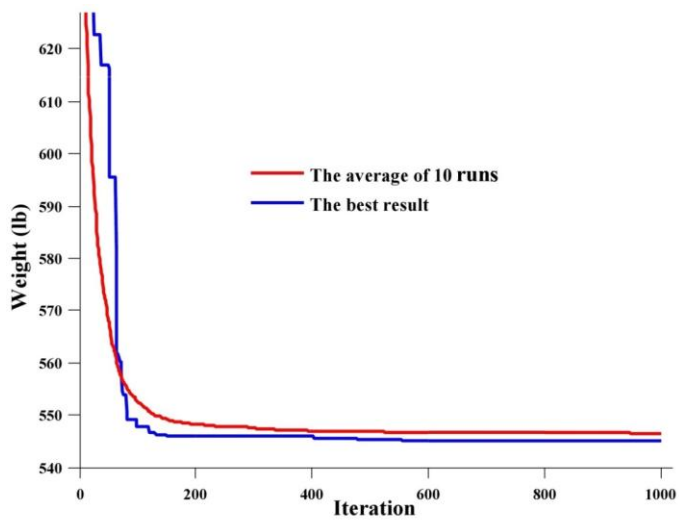

Figure 5: Convergence diagrams for the 25-bar spatial truss.

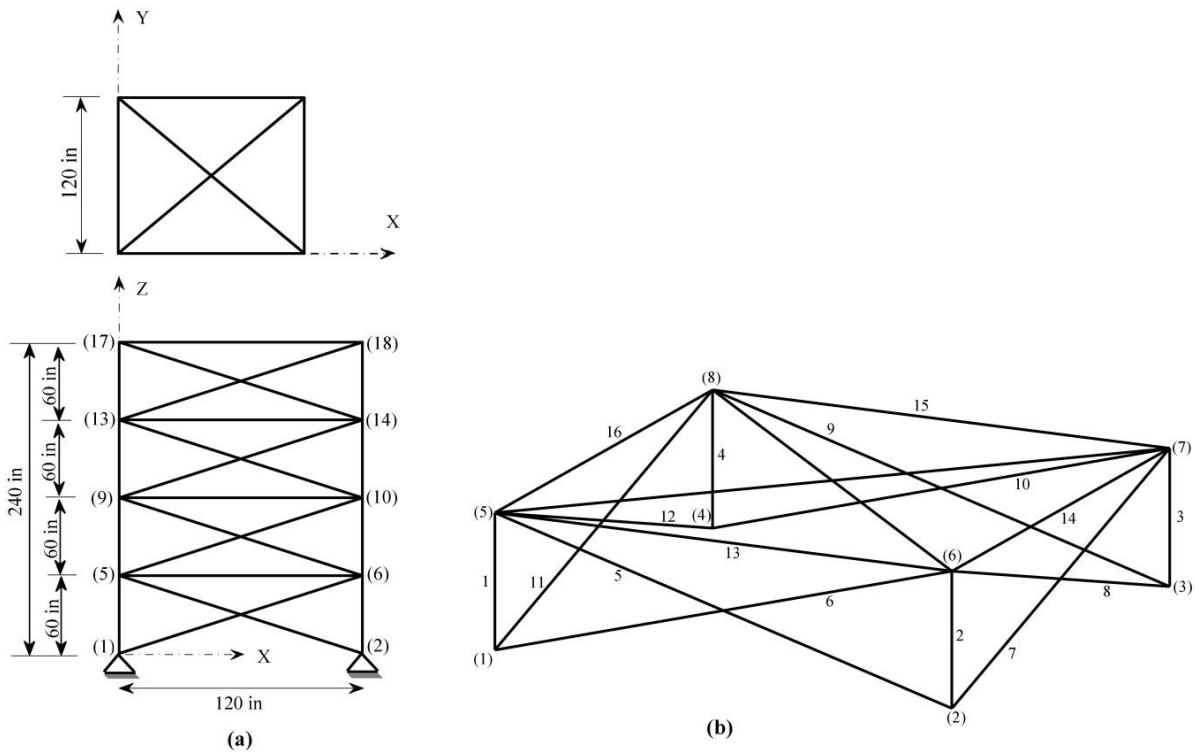

Figure 6: Scheme of the 72-bar spatial truss: (a) top and side view, (b) element and node numbering pattern for first story.

Node	Condition 1			Condition 2		
	P_x	P_y	P_z	P_x	P_y	P_z
17	5.0	5.0	-5.0	0.0	0.0	-5.0
18	0.0	0.0	0.0	0.0	0.0	-5.0
19	0.0	0.0	0.0	0.0	0.0	-5.0
20	0.0	0.0	0.0	0.0	0.0	-5.0

Note: loads are in kips.

Table 5: Loading conditions for the 72-bar spatial truss.

	Design Variables (in^2)	Camp (2007)	Kaveh and Talatahari (2009^b)	Kaveh and Khayatazad (2013)	Degertekin (2012)		Degertekin and Hayalioglu (2013)	This study
		BB-BC	HBB-BC	RO	EHS	SAHS	TLBO	CA
1	$A_1 - A_4$	1.8577	1.9042	1.83649	1.967	1.860	1.90640	1.860930
2	$A_5 - A_{12}$	0.5059	0.5162	0.502096	0.510	0.521	0.50612	0.509300
3	$A_{13} - A_{16}$	0.1000	0.1000	0.100007	0.100	0.100	0.10000	0.100000
4	$A_{17} - A_{18}$	0.1000	0.1000	0.10039	0.100	0.100	0.10000	0.100000
5	$A_{19} - A_{22}$	1.2476	1.2582	1.252233	1.293	1.271	1.26170	1.262910
6	$A_{20} - A_{30}$	0.5269	0.5035	0.503347	0.511	0.509	0.51110	0.503970
7	$A_{31} - A_{34}$	0.1000	0.1000	0.100176	0.100	0.100	0.10000	0.100000
8	$A_{35} - A_{36}$	0.1012	0.1000	0.100151	0.100	0.100	0.10000	0.100000
9	$A_{37} - A_{40}$	0.5209	0.5178	0.572989	0.499	0.485	0.53170	0.523160
10	$A_{41} - A_{48}$	0.5172	0.5214	0.549872	0.501	0.501	0.51591	0.525220
11	$A_{49} - A_{52}$	0.1004	0.1000	0.100445	0.100	0.100	0.10000	0.100010
12	$A_{53} - A_{54}$	0.1005	0.1007	0.100102	0.100	0.100	0.10000	0.102540
13	$A_{55} - A_{58}$	0.1565	0.1566	0.157583	0.160	0.168	0.15620	0.155962
14	$A_{59} - A_{62}$	0.5507	0.5421	0.52222	0.522	0.584	0.54927	0.553490
15	$A_{63} - A_{70}$	0.3922	0.4132	0.435582	0.478	0.433	0.40966	0.420260
16	$A_{71} - A_{72}$	0.5922	0.5756	0.597158	0.591	0.520	0.56976	0.561500
Best weight (lb)		379.85	379.66	380.458	381.03	380.62	379.63	379.69
Average weight (lb)		382.08	381.85	382.5538	383.51	382.42	380.20	380.86
Standard deviation (lb)		1.912	1.201	1.2211	1.92	1.38	0.41	1.8507
No. of analyses		19,621	13,200	19,084	15,044	13,742	19,778	18,460

Table 6: Optimized designs for the 72-bar spatial truss.

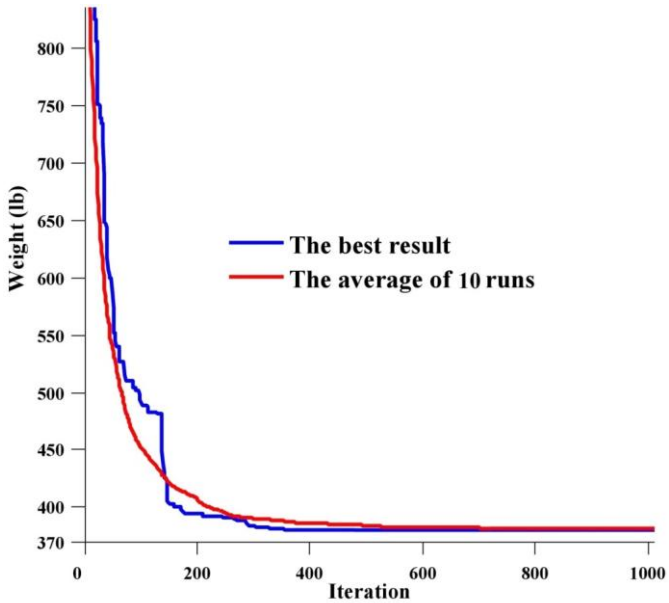

Figure 7: Convergence diagrams for the 72-bar spatial truss.

(1)$A_1 - A_4$, (2) $A_5 - A_{12}$, (3) $A_{13} - A_{16}$, (4) $A_{17} - A_{18}$, (5) $A_{19} - A_{22}$, (6) $A_{20} - A_{30}$, (7) $A_{31} - A_{34}$, (8) $A_{35} - A_{36}$, (9) $A_{37} - A_{40}$, (10) $A_{41} - A_{48}$, (11) $A_{49} - A_{52}$, (12) $A_{53} - A_{54}$, (13) $A_{55} - A_{58}$, (14) $A_{59} - A_{62}$, (15) $A_{63} - A_{70}$, (16) $A_{71} - A_{72}$.

The spatial truss structure is subjected to the loading conditions given in Table 5. The maximum nodal displacements in all directions are limited to ± 0.25 in for all free nodes. The minimum and maximum cross sectional areas for each member are 0.1 in^2 and 4 in^2, respectively.

The optimization results obtained by the CA are presented in Table 6 and are compared with those of the BB-BC, HBB-BC, RO, EHS, SAHS and TLBO approaches. From Table 6, it can be concluded that CA gives lightest design as compared to the results obtained by BB-BC, RO, EHS, SAHS and TLBO, but slightly heavier design than HBB-BC and TLBO methods. Moreover, the convergence diagrams of the best solution and the average of 10 independent runs are presented in Figure 7.

4.4 A 120-bar Dome Truss

The fourth design example is the size optimization of a 120-bar dome truss shown in Figure 8. Table 7 presents the nodal coordinates of this structure. The members of the structure are divided into 7 groups using symmetry as shown in Figure 8. The modulus of elasticity is 30,450 ksi, and the material density is 0.288 lb/in^3. The yield stress of steel is taken as 58.0 ksi. The dome is subjected to the vertical loading at all free nodes. These loads are taken as -13.49 kips at node 1, -6.744 kips at nodes 2 through 14, and -2.248 kips at the rest of the nodes. The minimum cross sectional area of all members is 0.775 in^2 and the maximum cross sectional area is taken as 20.0 in^2. The stress and displacement constraints are considered as:

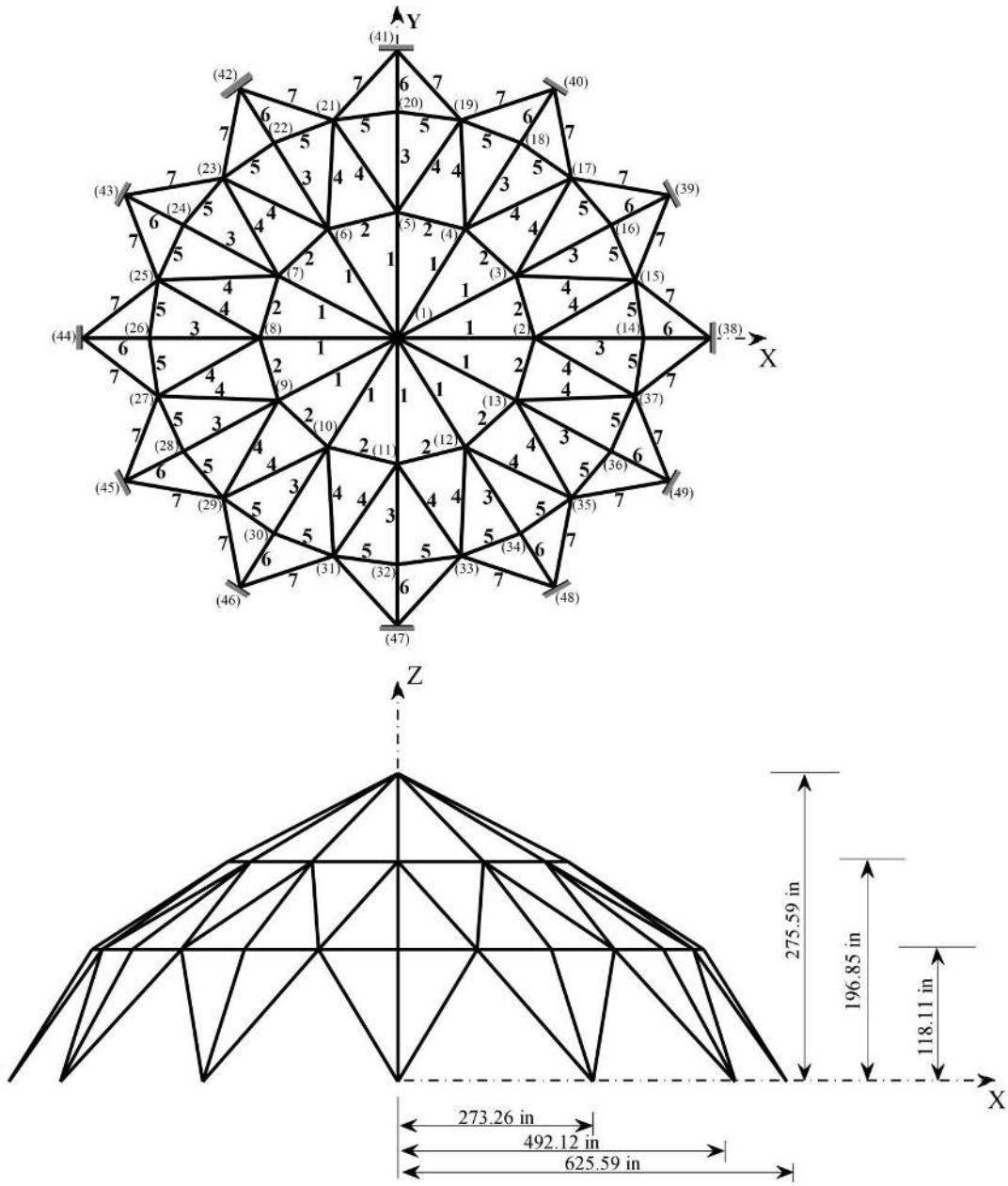

Figure 8: Scheme of the 120-bar dome truss.

(1) Stress constraint (according to the AISC ASD (1989) code):

$$\begin{cases} \sigma_i^+ = 0.6Fy & for\ \sigma_i \geq 0 \\ \sigma_i^- & for\ \sigma_i < 0 \end{cases} \tag{17}$$

Where σ_i^- is calculated according to the slenderness ratio:

$$\begin{cases} [(1-\frac{\lambda_i^2}{2C_c^2})F_y]/(\frac{5}{3}+\frac{3\lambda_i}{C_c}-\frac{{\lambda_i}^3}{8{C_c}^3}) & \text{for } \lambda_i < C_c \\ \frac{12\pi^2 E}{23\lambda_i^2} & \text{for } \lambda_i \geq C_c \end{cases} \quad (18)$$

	Coordinate (in)				Coordinate (in)		
Node	X	Y	Z	Node	X	Y	Z
1	0.000	0.000	275.590	26	-492.120	0.000	118.110
2	273.260	0.000	196.850	27	-475.350	-127.370	118.110
3	236.650	136.630	196.850	28	-426.190	-246.060	118.110
4	136.630	236.650	196.850	29	-347.980	-347.980	118.110
5	0.000	273.260	196.850	30	-246.060	-426.190	118.110
6	-136.630	236.650	196.850	31	-127.370	-475.350	118.110
7	-236.650	136.630	196.850	32	0.000	-492.120	118.110
8	-273.260	0.000	196.850	33	127.370	-475.350	118.110
9	-236.650	-136.630	196.850	34	246.060	-426.190	118.110
10	-136.630	-236.650	196.850	35	347.981	-347.980	118.110
11	0.000	-273.260	196.850	36	426.188	-246.060	118.110
12	136.630	-236.650	196.850	37	475.351	-127.370	118.110
13	236.650	-136.630	196.850	38	625.590	0.000	0.000
14	492.120	0.000	118.110	39	541.777	312.795	0.000
15	475.351	127.370	118.110	40	312.795	541.777	0.000
16	426.188	246.060	118.110	41	0.000	625.590	0.000
17	347.981	347.981	118.110	42	-312.800	541.777	0.000
18	246.060	426.188	118.110	43	-541.780	312.795	0.000
19	127.370	475.351	118.110	44	-625.590	0.000	0.000
20	0.000	492.120	118.110	45	-541.780	-312.800	0.000
21	-127.370	475.351	118.110	46	-312.800	-541.780	0.000
22	-246.060	426.188	118.110	47	0.000	-625.590	0.000
23	-347.980	347.981	118.110	48	312.795	-541.780	0.000
24	-426.190	246.060	118.110	49	541.777	-312.800	0.000
25	-475.350	127.370	118.110				

Table 7: The nodal coordinates of the 120-bar dome truss.

Where E = the modulus of elasticity; F_y= the yield stress of steel; C_C= the slenderness ratio (λ_i) dividing the elastic and inelastic buckling regions $(C_C = \sqrt{2\pi^2 E/F_y})$; λ_i= the slenderness ratio $(\lambda_i = kL_i/r_i)$; k = the effective length factor; L_i= the member length; and r_i= the radius of gyration. In addition, the radius of gyration (r_i) can be expressed in terms of cross-sectional areas as $r_i = \alpha A^{\beta}$ (Saka, 1990), in which α and β are the constants depending on the types of selected sections for the truss members. In this example, similar to the previous works, the pipe section (a=0.4993 and b=0.6777) is selected.

(2) The maximum nodal displacements are limited to 0.1969 in for all free nodes.
In this example, four cases of constraints are considered as follows:

Case (1): with stress constraints and without any limitations of nodal displacement.
Case (2): with stress constraints and displacement limitations of ±0.1969 in imposed on all nodes in x- and y-directions.
Case (3): displacement limitation of ±0.1969 in only in z-direction and without stress constraints.
Case (4): all constraints explained in cases 1, 2 and 3 are considered together.

Design variables (in^2)	Kaveh and Talatahari (2009[a])	Kaveh and Khayatazad (2013)		This study
	PSO	PSOPC	RO	CA
1	3.147	3.235	3.128	3.122897
2	6.376	3.37	3.357	3.353849
3	5.957	4.116	4.114	4.111981
4	4.806	2.784	2.783	2.782138
5	0.775	0.777	0.775	0.775000
6	13,798	3.343	3.302	3.300503
7	2.452	2.454	2.453	2.445793
Weight (lb)	32432.9	19618.7	19476.193	19454.49
No. of analyses	N/A	125,000	19,950	4270

Table 8: Optimized designs for 120-bar dome truss (Case 1).

Design variables (in^2)	Kaveh and Talatahari (2009[a])		Kaveh and Khayatazad (2013)	This study
	PSO	PSOPC	RO	CA
1	15.978	3.083	3.084	3.0831690
2	9.599	3.639	3.360	3.3526138
3	7.467	4.095	4.093	4.0927515
4	2.790	2.765	2.762	2.7612602
5	4.324	1.776	1.593	1.5922992
6	3.294	3.779	3.294	3.2927453
7	2.479	2.438	2.434	2.4335890
Weight (lb)	41052.7	20681.7	20071.90	20064.69
No. of analyses	N/A	125,000	19,950	7600

Table 9: Optimized designs for 120-bar dome truss (Case 2).

Design variables (in^2)	Kaveh and Talatahari (2009 [a])		Kaveh and Khayatazad (2013)	This study
	PSO	PSOPC	RO	CA
1	1.773	2.098	2.044	1.97582
2	17.635	16.444	15.665	15.47210
3	7.406	5.613	5.848	5.58273
4	2.153	2.312	2.29	2.20966
5	15.232	8.793	9.001	9.46333
6	19.544	3.629	3.673	3.75880
7	0.8	1.954	1.971	1.97027
Weight (lb)	38273.83	31776.2	31733.2	31680.60
No. of analyses	N/A	125,000	19,850	11,930

Table 10: Optimized designs for 120-bar dome truss (Case 3).

Design variables (in^2)	Kaveh and Talatahari (2009[a,b])			Kaveh and Talatahari (2010[c])		Kaveh and Khayatazad (2013)	This study
	PSOPC	PSACO	HBB-BC	ICA	CSS	RO	CA
1	3.040	3.026	3.037	3.02750	3.027	3.030	3.02591
2	13.149	15.222	14.431	14.45960	14.606	14.806	14.7652
3	5.646	4.904	5.130	5.24460	5.044	5.440	5.08463
4	3.143	3.123	3.134	3.14130	3.139	3.124	3.13569
5	8.759	8.341	8.591	8.45410	8.543	8.021	8.43852
6	3.758	3.418	3.377	3.35670	3.367	3.614	3.35678
7	2.502	2.498	2.500	2.49447	2.497	2.487	2.49627
Weight (lb)	33481.2	33263.9	33287.9	33256.2	33251.9	33317.8	33253.95
No. of analyses	150,000	32,600	10,000	6000	7000	19,800	5800

Table 11: Optimized designs for 120-bar dome truss (Case 4).

The optimal cross sectional areas obtained by the CA and the other optimization methods recently published in literature are reported in Table 8, Table 9, Table 10 and Table 11 for all cases. In Cases 1, 2 and 3, it is quite evident that CA gives the lightest designs than other techniques in the literature based on Table 8, Table 9 and Table 10. For case 4, from Table 11, it can be concluded that CA gives the lightest design as compared to the results obtained by PSOPC, PSACO, HBB-BC, ICA and RO, but slightly heavier design than CSS method.

For all cases, Figures 9-12 compare the existing values (the member's stresses corresponding to the best solution) and allowable values for stress and displacement constraints. Based on these figures, it can be concluded that the stress and displacement constraints of the structure are not violated and the presented optimum designs are completely feasible. In addition, it can be seen that the axial stresses in the most of the members of the structure are very close to the allowable values, which show the optimality of the presented designs.

Finally the convergence characteristics of the CA are shown in Figure 13 for all cases.

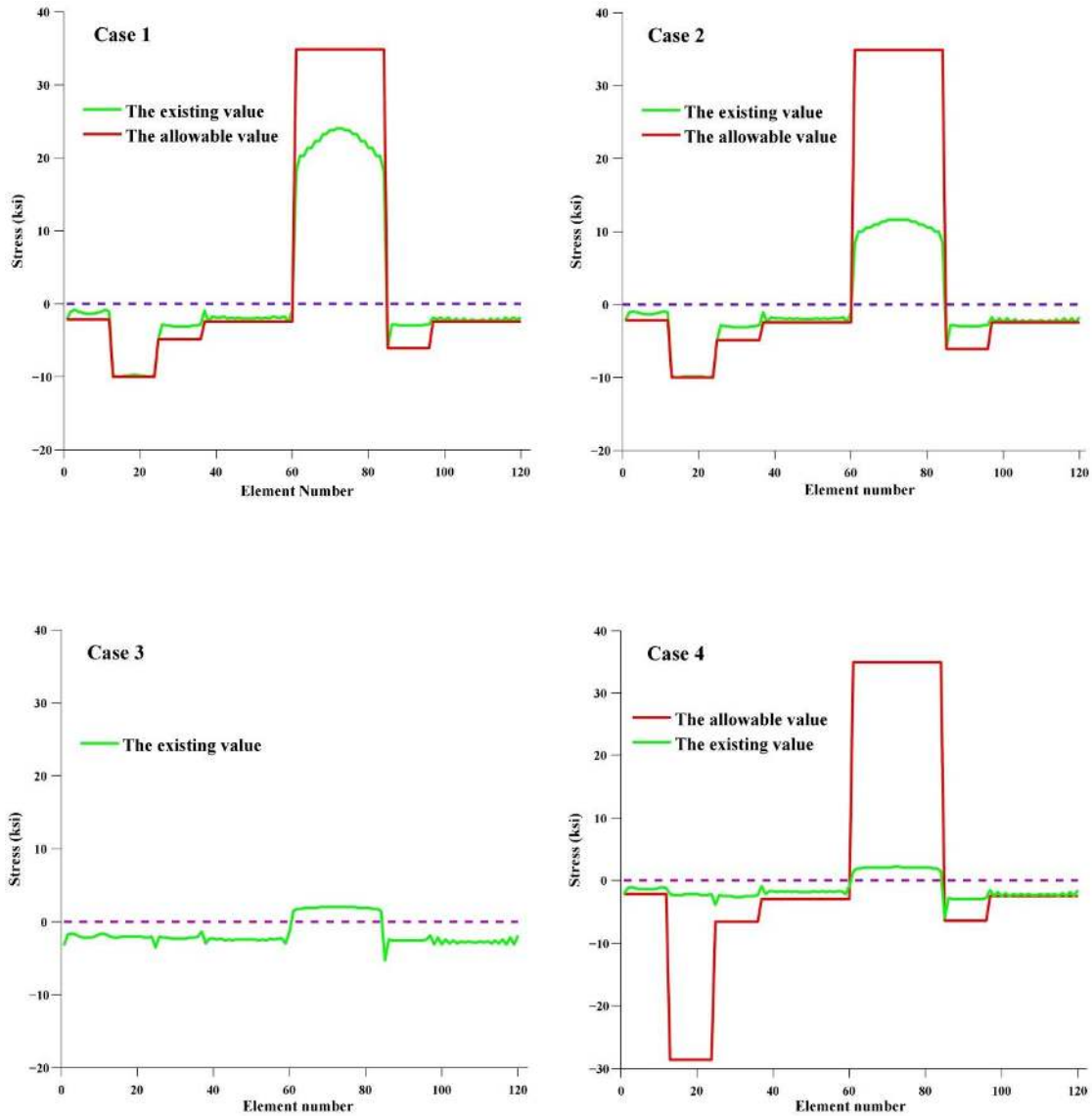

Figure 9: Comparison of existing and allowable stresses (four Cases).

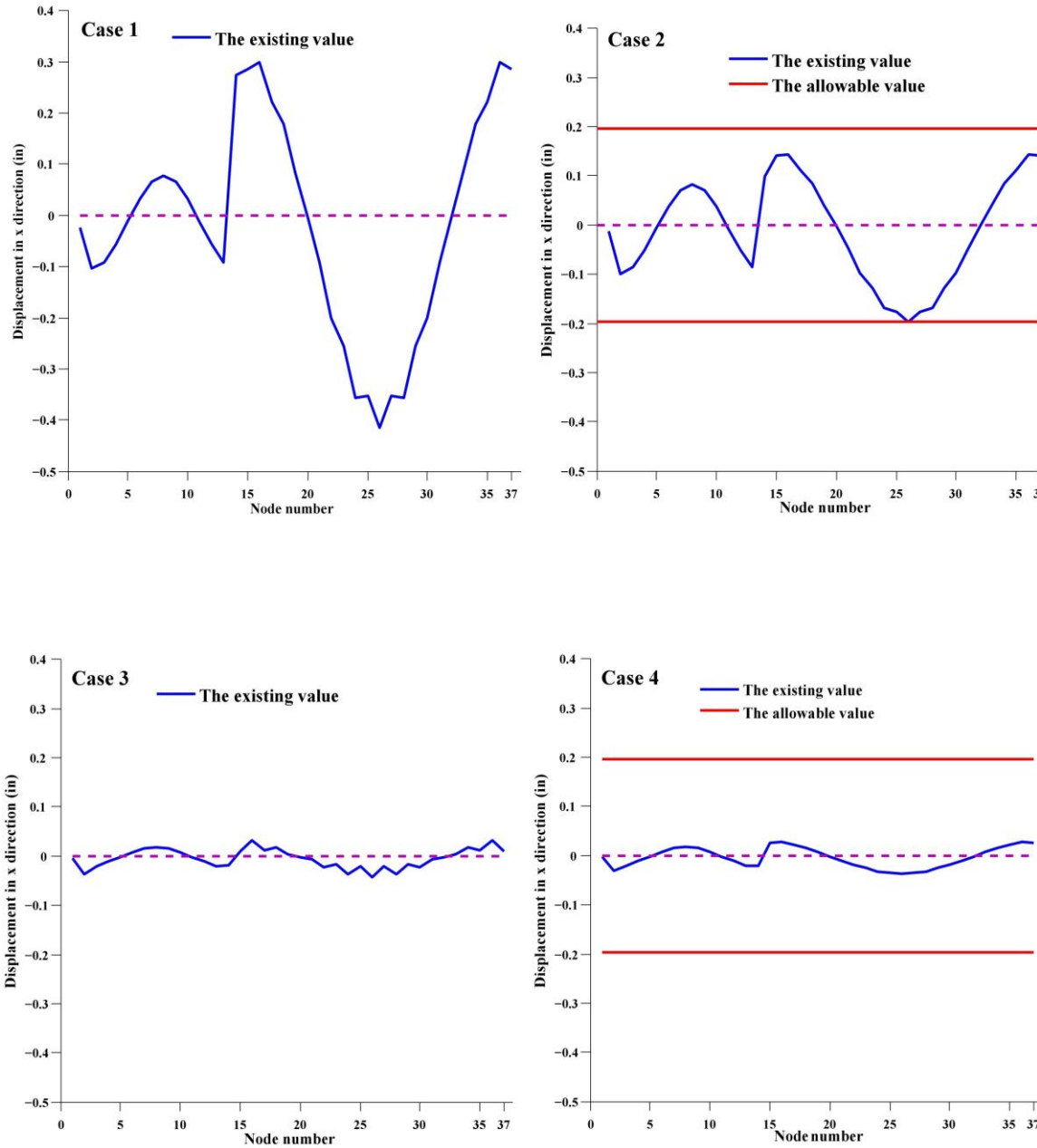

Figure 10: Comparison of existing and allowable displacements in x direction (four Cases).

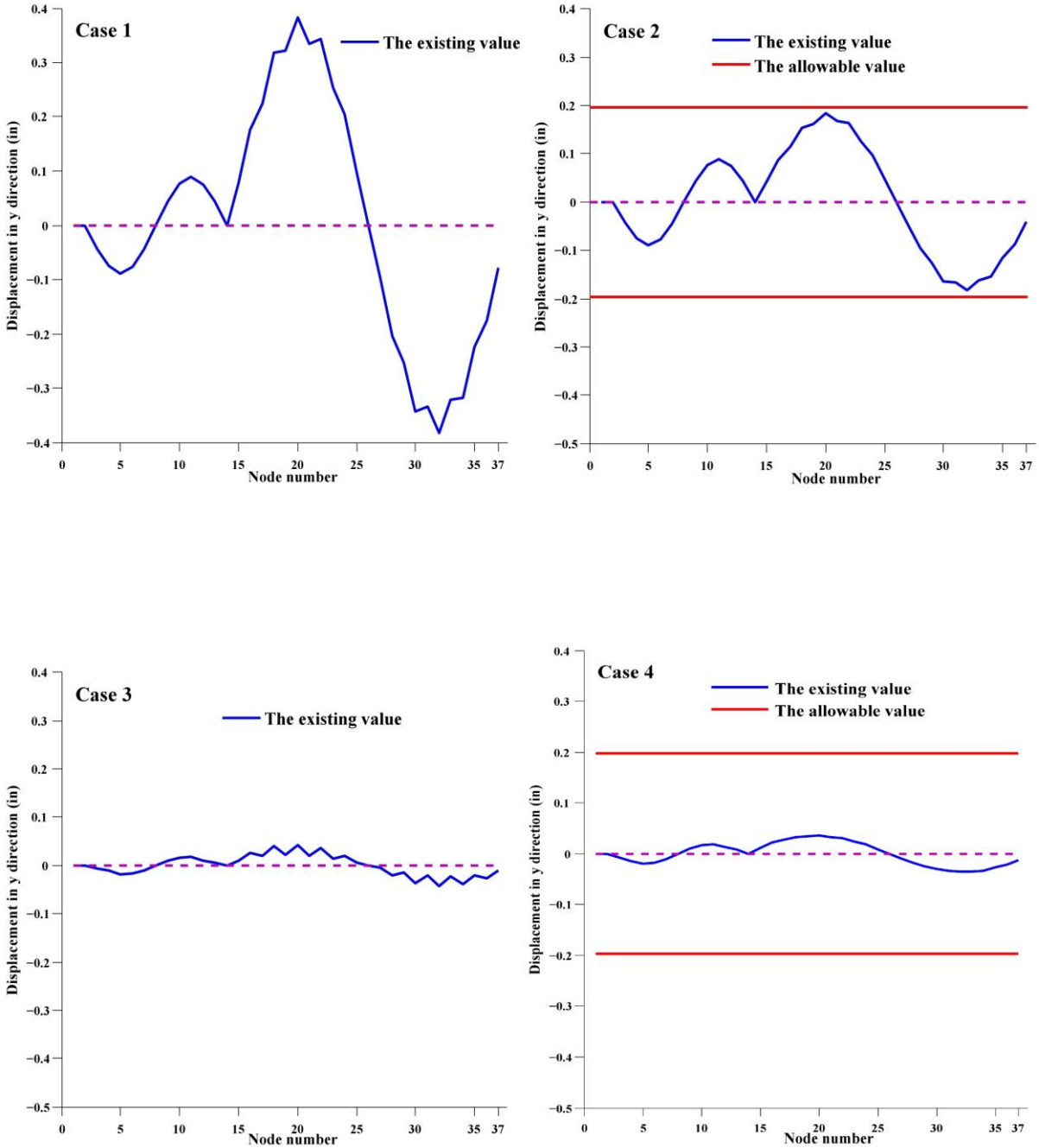

Figure 11: Comparison of existing and allowable displacements in y direction (four Cases).

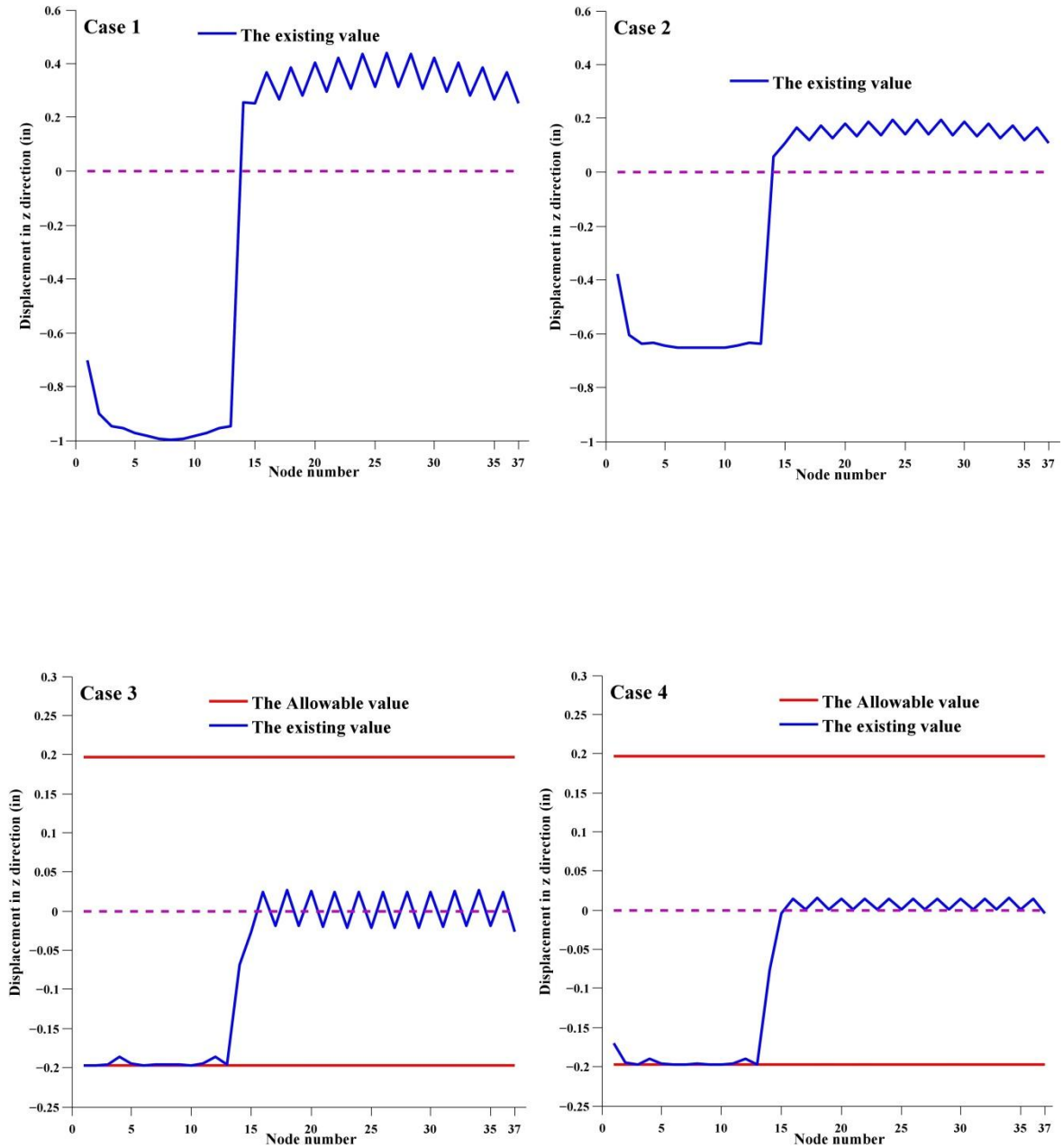

Figure 12: Comparison of existing and allowable displacements in z direction (four Cases).

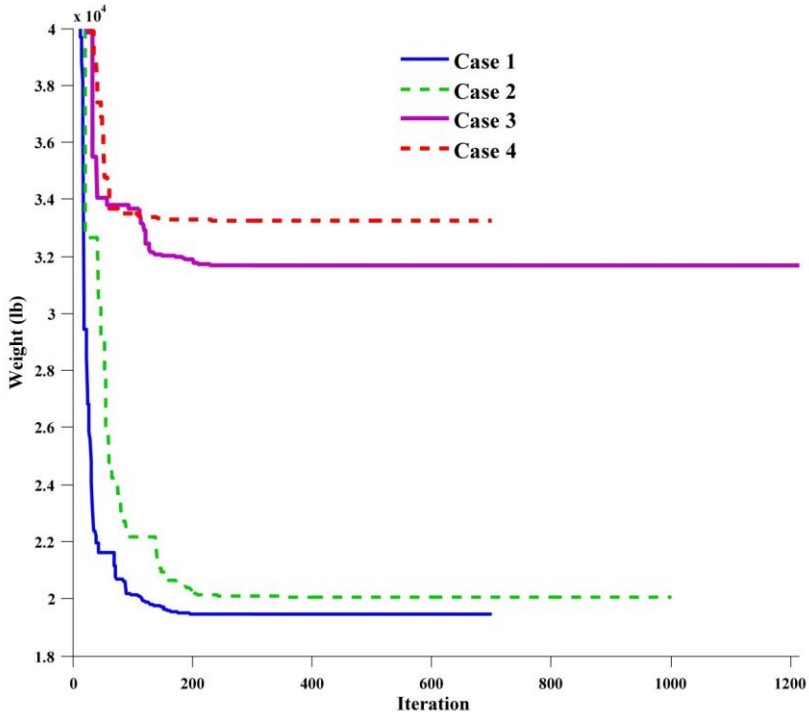

Figure 13: Convergence diagrams of the best results for the 120-bar dome truss (four Cases).

5 CONCLUSIONS

This work addresses application of Cultural Algorithm (CA) to optimal design of truss structures under stress and deflection constraints. CA is a population based meta-heuristic algorithm which uses the belief space beside the population space. The belief space of the CA has different knowledge components. In this paper, only two normative and situational knowledge components are used in the belief space. In belief space, the behaviors and experiences of elite individuals are recorded and then used to bias the search process of the algorithm. The performance of the CA is evaluated using a set of four well-known truss design examples. The numerical results show the efficiency and capabilities of the CA in finding the optimal designs for truss structures. The comparisons of the results obtained by the CA and other optimization methods show that the CA obtains relatively light structural weights with less structural analyses. Moreover, the same parameters are used for the all design examples and the separate sensitivity analyses of internal parameters are not required for the each design example. Furthermore, the feasibility of the obtained optimum designs are investigated in the last design example and it is shown that the stress and displacement constraints are not violated at the optimum designs. In order to enhance the exploration and exploitation mechanisms and provide more stable results with smaller standard deviations, future works should be focus on presenting of hybrid versions of this algorithm with other optimization techniques to increase the efficiency.

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