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Neural network approach for failure analysis of rectangular plates under wedge impact

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Abstract

The purpose of this work is to establish an empirical relationship that describes the deflection created in a rectangular plate struck by a rigid wedge at the plate centre with sufficient initial kinetic energy to produce large inelastic deformations. A multivariable power series was selected as the form of the mathematical model to develop this empirical relationship. Good agreement between the experimental results and the prediction of maximum deflections for various impact energies has been obtained.

The data used in the development of statistical models was reanalyzed for the prediction of maximum deflection by employing the technique of neural networks with a view towards seeing if better predictions are possible. Neural networks have advantages over statistical models like their data-driven nature, model-free form of predictions, and tolerance to data errors. The neural network models resulted in very low errors and high correlation coefficients as compared to the regression based models.

Keywords: deflection, impact, regression, neural networks, rectangular plate, wedge

1 Introduction

An approximate theoretical method of analysis was developed by Jones and Walters [1, 2, 5] for the response of beams, plates and shells when subjected to dynamic transverse loads which produce large inelastic strains and permanent deformations. This procedure idealises the structural material as rigid, perfectly plastic and retains the influence of large transverse deflections. The method has been used by a number of authors largely to obtain the structural response for dynamic pressure pulses and for impulsive loading, and good agreement has been obtained with experimental work conducted on ductile metal beams, plates, and shells loaded impulsively.

The dynamic plastic response of thin rectangular plates struck transversely by wedge-shape masses has been examined by several authors, such as Zhu [8] and Shen [3]. Zhu reported their nine experimental results [8] and used an energy method proposed by Jones [1] and Wood's

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Nomenclature

B	Half- width of plate
C_1	Multiplication constant of the model
E	Young's modulus of elasticity
E_i	Impact energy $= \frac{1}{2}m_pV^2$
E_s	Strain energy $= \sigma_0^2/2E \times LBT_t$
E_r	Energy ratio
L	Half-length of plate
M_p	Mass of striker
$P1$, $P2$, $P3$	Radicals of dimensionless variables used in the model
S	Half-length of wedge
T_t	Thickness of plate
V	Initial velocity of striker
W	Maximum deflection
σ_0	Static flow stress

model [6] to analyse their tested specimens. The traveling hinge phase was first considered by Yu and Chen [7] for the impulsive loaded rectangular plate. Shen [3] employed a pure membrane model with two traveling hinge phases for wedge impact on rectangular plate. A good agreement was obtained for the permanent maximum deflection of plates between the theoretical predictions proposed in Ref. [3] and the experimental results reported in Ref. [8]. It was found in Ref. [3] that the deflection produced during the two traveling hinge phases might dominate the behaviour for high impact velocity. It was also pointed out in Ref. [3] that the whole response possibly ends before the traveling hinge lines reach their final positions. The problem of a rectangular plate struck by a wedge traveling at a higher speed, is to some extent, similar to the problem of a rectangular plate under an intense impulse, therefore, the traveling hinge phases must be included when the velocity of wedge is higher [3]. Shen [3] proved that the displacement produced during the two traveling hinge phases may dominate the behavior for higher impact velocity.

In the present work, an empirical multivariable power function relationship has been developed for the prediction of maximum deflection created in a rectangular plate struck by a rigid wedge at the centre of plate. The data used in the development of statistical models was reanalyzed for the prediction of maximum deflection by employing the technique of neural networks with a view towards seeing if better predictions are possible. The neural network models resulted in very low errors and high correlation coefficients as compared to the regression based models.

2 Definition of problem and data

When a wedge of the shape shown in Fig. 1 strikes a rectangular fully clamped metallic plate at the centre, the plate undergoes deflection and may even crack depending upon the energy involved in the impact. The prediction of deflection of the plate may help in the prediction of the state cracking.

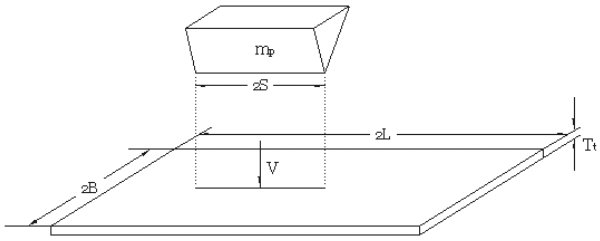

Figure 1: A fully clamped rectangular plate struck by wedge at the centre of the plate

In the present study, a regression and Artificial Neural Network (ANN) models have been developed for the prediction of deflection which has been used in subsequent prediction of the state of damage caused to the plate. The data used for the development of the model is taken from Ref. [4], which consists of sixty data points as given in Table 1. The range of parameters involved in the experimental data are listed in Table 1.

The external dynamic energy imparted to the rectangular plates, $E_i = m_p V^2/2$, in these experiments is much larger than the maximum possible strain energy which could be absorbed by the rectangular plates in a wholly elastic manner, whose upper bound is taken as

$$E_s = \sigma_0^2/2E \times \text{rectangular plate volume} = \sigma_0^2/2E \times LBT_t \tag{1}$$

The strain energy, E_s, given by above equation is defined very conservatively because local plastic deformations would occur at much smaller values of the elastic strain energy than the crude estimate which is used in the above Eq. 1. The ratio of impact energy to the strain energy for the impact loaded rectangular plate gives an energy ratio, as

$$E_r = \frac{m_p E}{4BLT_t}\left(\frac{V}{\sigma_0}\right)^2 \tag{2}$$

The energy ratios according to Eq. 2 range from 8 to 236 for the experimental data. Seventy-five percent of the data points lie in the range of energy ratio between 10 and 70 as seen from Fig. 2. It is observed from the data that the increase in the energy ratio results in the increase in the deflections.

Table 1: Range of parameters for the experimental data of impact of wedge on plate

S. No.	Parameter	Values / Range
Basic Parameters		
1.	Plate length $2L$, (mm)	280
2.	Plate width $2B$, (mm)	130 and 190
3.	Plate thickness, $T_t(mm)$	1.0, 1.5, 2.0, 3.0 and 4.0
4.	Wedge length, S (mm)	40 and 120
5.	Wedge velocity, V (m/s)	2.53-10.45
6.	Plate mass, m (kg)	0.286-0.670
7.	Wedge mass, m_p (kg)	12.16 - 86.65
8.	Max. plate deflection, W (mm)	0.86 - 35.00
9.	Static flow stress, σ_0 (N/mm^2)	153.10 - 357.70
Non-Dimensional Parameters		
1.	W/T_t	3.33 - 35.00
2.	$T_t/2S$	0.008 - 0.100
3.	$m_pV^2/T_t^3\sigma_0$	43.52 - 9135.46
4.	B/L	0.464 and 0.678
5.	m_p/m	19.330 - 34.330

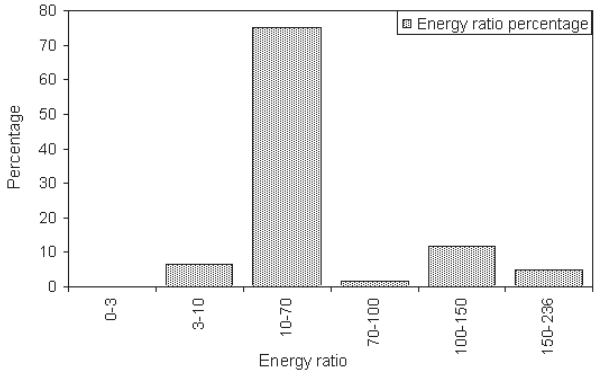

Figure 2: Frequency of percentage energy ratio

3 Regression model

For the prediction of maximum deflection of plate under the concentric normal impact of the wedge, an empirical model has been developed. The model selected is a generic multivariable

power function with the independent and dependent variables applied to the model in such a manner as to maintain non-dimensionality. Failure impact event parameters that are needed, as a minimum, to describe the target deflection are the basic impact geometry (length and width of wedge; length, width and thickness of plate), projectile velocity, and basic material properties (static flow stress of plate and mass of wedge). Static flow stress of plate and mass of wedge were chosen as the material properties of interest for the problem since these are the only material property involved in the basic shock jump relationship. The dimensionless model used for the prediction of maximum deflection of plate is of the form:

$$\frac{W}{T_t} = C_1 \left(\frac{2B}{2L}\right)^{P1} \left(\frac{T_t}{2S}\right)^{P2} \left(\frac{m_p V^2}{\sigma_0 T_t^3}\right)^{P3} \tag{3}$$

where, $C1$, $P1$, $P2$ and $P3$ are the model parameters.

The regression analysis of the data for the above model gives:

$$\frac{W}{T_t} = 0.775 \left(\frac{B}{L}\right)^{0.22} \left(\frac{T_t}{2S}\right)^{0.13} \left(\frac{m_p V^2}{\sigma_0 T_t^3}\right)^{0.5} \tag{4}$$

The mean error in the prediction of results employed for its development is 5.6%, which shows that the model is good for predicting the deflections. A comparison of the experimental and the predicted deflection is presented in Fig. 3. On the other hand, the mean error in the prediction of deflection by Shen [4] is about 11.3%. The parameters involved in the above proposed model in the order of decreasing sensitivity are: V, m_p, σ_0, B/L, T_t and S.

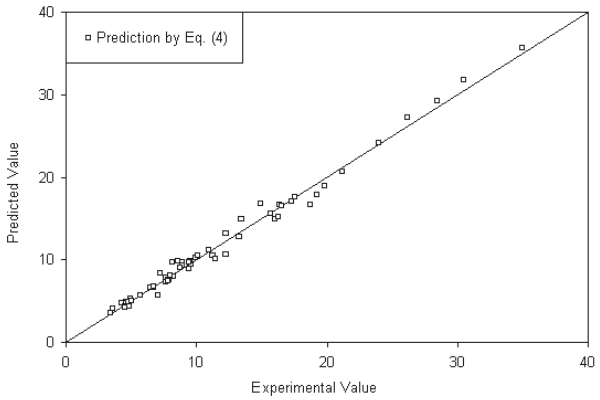

Figure 3: Prediction of dimensionless maximum permanent deflection by regression model (60 data points)

The comparison of mean error in the proposed models and the Shen's prediction is shown in Fig. 4. It is observed from this figure that for about 43.33% data points, the mean error in the prediction by the proposed model given by Eq. 4 is less then 3%, whereas the data giving error less than 3% in Shen's prediction is 20. The model given by Eq. 4 generated better fits of the original experimental data than Shen's prediction.

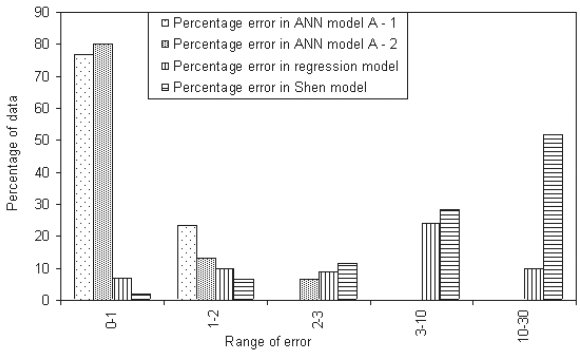

Figure 4: Histogram of percentage error in different models

4 Neural network model

The manner in which the data are presented for training is the most important aspect of the neural network method. Often this can be done in more than one way, the best configuration being determined by trial-and-error. It can also be beneficial to examine the input/output patterns or data sets that the network finds difficult to learn. This enables a comparison of the performance of the neural network model for these different combinations of data. In order to map the causal relationship related to the deflection, two separate input-output schemes (called Model - A1 and Model - A2) were employed, where the first took the input of raw causal parameters while the second utilized their non-dimensional groupings. This was done in order to see if the use of the grouped variables produced better results? The Model - A1 thus takes the input in the form of causative factors namely, L, B, S, T_t, m_p, σ_0 and V yields the output, the deflection, W, while Model - A2 employs the input of grouped dimensionless variables namely, L/B, $T_t/2S$ and $m_pV^2/\sigma_0 T_t^3$ and yields the corresponding dimensionless output W/T_t. Thus, the two models are:

$$\text{Model - A1:} W = f\left(L, B, S, m_p, T_t, V\right) \tag{5}$$

$$\text{Model - A2:} \frac{W}{T_t} = g\left(\frac{L}{B}, \frac{T_t}{2S}, \frac{m_pV^2}{\sigma_0 T_t^3}\right) \tag{6}$$

The input and output variables involved in the above two models were first normalized within the range 0 to 1 as follows:

$$x_N = \frac{x - x_{\min}}{x_{\max} - x_{\min}} \tag{7}$$

where x_N is the normalized value of x; $x_{\max}$ and $x_{\min}$ are the maximum and minimum values of variable, x. This normalization allowed the network to be trained better.

The current study used the data considered above (60 data points) for the prediction of deflection. The training of the above two models was done using 83.33% of the data (i.e. 50

data points) selected randomly. The validation and testing of the proposed models was made with the help of the remaining 16.67% of observations (10 data points), which were not involved in the derivation of the model.

Three neuron models namely, *tansig*, *logsig* and *purelin*, have been used in the architecture of the network with the back-propagation algorithm. In the back-propagation algorithm, the feed-forward (FFBP) and cascade-forward (CFBP) type network was considered. Each input is weighted with an appropriate weight and the sum of the weighted inputs and the bias forms the input to the transfer function. The neurons employed use the following differentiable transfer function to generate their output:

$$\text{Log-Sigmoid Transfer Function:} y_j = f\left(\sum_i w_{ij}x_i + \phi_j\right) = \frac{1}{1+e^{-(\sum w_{ij}x_i+\phi_j)}} \tag{8}$$

$$\text{Linear Transfer Function:} y_j = f\left(\sum_i w_{ij}x_i + \phi_j\right) = \sum_i w_{ij}x_i + \phi_j \tag{9}$$

$$\text{Tan-Sigmoid Transfer Function:} y_j = f\left(\sum_i w_{ij}x_i + \phi_j\right) = \frac{2}{1+e^{-2(\sum w_{ij}x_i+\phi_j)}} - 1 \tag{10}$$

The weight, w, and biases, ϕ, of these equations are determined in such a way as to minimize the energy function. The Sigmoid transfer functions generate outputs between 0 and 1 or -1 and +1 as the neuron's net input goes from negative to positive infinity depending upon the use of *log* or *tansigmoid*. When the last layer of a multilayer network has sigmoid neurons (*log* or *tan*), then the outputs of the network are limited to a small range, whereas, the output of linear output neurons can take on any value.

Further, in order to see if advanced training schemes provide better learning than the basic back propagation, a radial basis function (RBF) network was also used which though requires more neurons but it is sometimes more efficient. The Radial basis transfer function is given by:

$$y_j = f\left(\sum_i \|w_{ij} - x_i\| \phi_j\right) = e^{-(\sum \|w_{ij}-x_i\|\phi_j)^2} \tag{11}$$

The task of identifying the number of neurons in the input and output layers is normally simple, as it is dictated by the input and output variables considered in the model of physical process. Whereas, the appropriate number of hidden layer nodes for the models is not known for which a trial-and-error method was used to find the best network configuration. The optimal architecture was determined by varying the number of hidden neurons. The optimal configuration was based upon minimizing the difference between the neural network predicted value and the desired output. In general, as the number of neurons in the layer is increased, the prediction capability of the network increases in beginning and then becomes stationary.

The performance of all neural network model configurations was assessed on the basis of Mean Percent Error (MPE), Mean Absolute Deviation (MAD), Root Mean Square Error (RMSE), Correlation Coefficient (CC), and Coefficient of Determination, R^2, of the linear regression line between the predicted values from the neural network model and the desired outputs, which were calculated as follows:

$$MPE = \frac{100}{p}\sum_{i=1}^{p}\frac{O_i - t_i}{O_i} \tag{12}$$

$$MAD = \frac{100}{p}\sum_{i=1}^{p}\frac{|O_i - t_i|}{O_i} \tag{13}$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{p}(O_i - t_i)^2}{p}} \tag{14}$$

$$CC = \frac{\sum_{i=1}^{p}(t_i - \bar{t})\left(O_i - \bar{O}\right)}{\sqrt{\sum_{i=1}^{p}(t_i - \bar{t})^2 \sum_{i=1}^{p}\left(O_i - \bar{O}\right)^2}} \tag{15}$$

$$R^2 = 1 - \frac{\sum_{i=1}^{p}(O_i - t_i)^2}{\sum_{i=1}^{p}\left(O_i - \bar{O}\right)^2} \tag{16}$$

where t_i and O_i are target and network output for the ith output; $\bar{t}$ and $\bar{O}$ are the average of target and network output, and p is the total number of data considered.

The training of the neural network models was stopped when either the acceptable level of error was achieved or when the number of iterations exceeded a prescribed maximum. The neural network model configuration that minimized the MAE and RMSE and optimized the R^2 was selected as the optimum and the whole analysis was repeated several times.

5 Relative significance of input neurons

The relative significance of input neurons was assessed by the conducting sensitivity tests on the deflection (output) in both of the models given by Eqs. 5 and 6. In the sensitivity analysis, each input neuron was in turn eliminated from the model and its influence on prediction of deflection was evaluated in terms of the MPE, MAD, RMSE, CC and R^2 criteria. The network architecture of the problem considered in the present sensitivity analysis consists of one hidden layer with ten neurons and the value of epochs has been taken as 100.

The comparison of different neural network models in terms of the five error estimates, with one of the independent parameters removed in each case is presented in Table 2. The results show that for Model - A1, the velocity of strike, V, and mass of projectile, m_p are the two most significant parameters for the prediction of deflection. The variables in the order of decreasing level of sensitivity for Model - A1 are: V, m_p, σ_0, T_t, S, L and B. It is thus seen that the last four parameters have least significant effect when taken independently.

Similarly, Table 3 gives the results of sensitivity analysis for Model - A2. It is apparent that, $m_pV^2/\sigma_0T_t^3$ has most significant effect on the non-dimensional deflection and the other two dimensionless variables, namely L/B, and $T_t/2S$ have least significant effect. In the study of Model - A1, it was observed that V, m_p and σ_0 have significant effect, and thus the influence of $m_pV^2/\sigma_0T_t^3$ is very high in Model - A2. These findings are consistent with existing understanding of the relative importance of the various parameters on deflection.

Table 2: Sensitivity analysis for Model - A1 with Feed Forward Back Propagation*

Input variables	MPE	MAD	RMSE	CC	R^2
All (Eq. 5)	0.048	2.820	0.002	0.991	0.981
No B	-0.135	3.005	0.002	0.989	0.98
No L	-0.500	3.352	0.003	0.985	0.970
No S	-0.120	4.000	0.003	0.985	0.969
No T_t	-0.772	4.000	0.003	0.979	0.957
No σ_0	0.685	5.476	0.005	0.970	0.941
No m_p	0.232	6.111	0.0005	0.964	0.927
No V	-0.823	11.117	0.0007	0.925	0.851

Table 3: Sensitivity analysis for Model - A2 with ANN

Input variables	MPE	MAD	RMSE	CC	R^2
All (Eq.6)	-0.091	3.123	0.000	0.999	0.999
No $\left(\frac{B}{L}\right)$	0.940	6.713	0.000	0.991	0.982
No $\left(\frac{T_t}{2S}\right)$	-0.456	6.157	0.000	0.991	0.983
No $\left(\frac{m_pV^2}{\sigma_0T_t^3}\right)$	-0.467	19.370	0.081	0.913	0.831

In view of the variability in the outcome resulting from the application of different analytical schemes, it is felt that the network which requires all input quantities may be followed for generality.

6 Discussion of results for ANN models

As dictated by the use of Gaussian function all patterns were normalized within range of 0.0 to 1.0 before their use. Similarly all weights and bias values were initialized to random numbers. While the numbers of input and output nodes are fixed, the hidden nodes in the case of FFBP were subjected to trials and the one producing the most accurate results (in terms of the Correlation Coefficient) was selected. The optimization of the training procedure automatically fixes the hidden nodes in the case of the CFBP. The training of these networks was stopped after reaching the minimum mean square error between the network yield and true output over all the training patterns. For the RBF network various values of spread between 0 and 1 were tried out and the one of 0.01 resulting in the best performance on both training and testing data was selected.

The information on number of nodes required to achieve minimum error taken in the case of each training scheme used (i.e. FFBP, CFBP, and RBF) is shown in Table 4 for Model -A1 and A2, respectively. As a matter of general information, which is not of real significance in this study, it can be seen that the cascade correlation algorithm, designed for efficient training, trained the network with fewer epochs than the FFBP network, but the RBF network was trained in a significantly less number of epochs, indicating its training efficiency.

Table 4: Network Architecture∗

Model	Algorithm	Network Configuration			Learning Rate	Momentum Function
		I	H	O	0.5	0.7
	FFBP	7	8	1	0.5	0.7
Model - A1	CFBP	7	9	1	0.5	0.7
	RBF	7	12	1	0.5	0.7
	FFBP	3	6	1	0.5	0.7
Model - A2	CFBP	3	8	1	0.5	0.7
	RBF	3	10	1	0.5	0.7

The network architecture of the two models, given by Eqs. 5 and 6, is given in Figs. 5 and 6 respectively for BP/RBF training scheme. The error estimation parameters (MPE, MAD, RMSE, CC and R^2) on the basis of which the performance of a model is assessed are already given in Tables 2 and 3.

The training and validation of the two models is shown in Figs. 7 and 8. The trained values of connecting weights and bias for the two models are given in Tables 5 and 6 obtained from FFBP training scheme.

The histograms of error in the prediction of deflection for the two models along with the regression model and Shen's model are plotted in Fig. 4. The percentage error in the prediction of deflection for different data sets is plotted in Figs. 9 and 10 for the two models (Model -

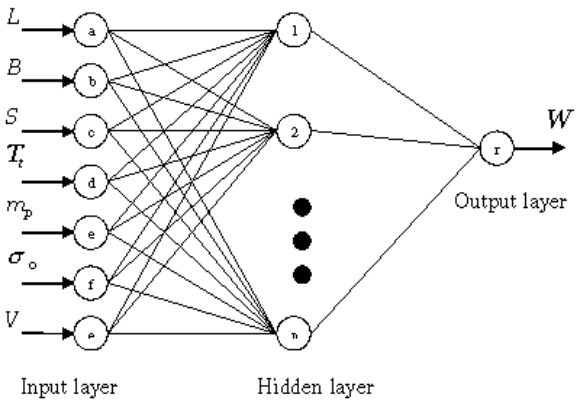

Figure 5: Model - A1: use of raw variables

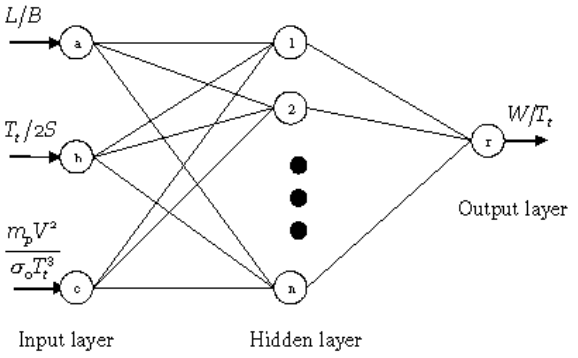

Figure 6: Model - A2: Use of grouped variables

A1 Model - A2). The predicted value of deflection has been plotted against its observed value in Figs. 11 and 12 for the two models. Though the results of non-normalized data are not presented but it has been observed that the normalization considerably improved the training of the model.

The examination of Tables 2 and 3 and Figs. 11 to 12 show that when it comes to overall accuracy of predicting deflection, all error criteria viewed together point out that the simple feed forward network trained using the common BP algorithm is either as good as or even slightly better than more sophisticated networks.

It also shows that the use of grouped variables as input (Model - A2) may be more beneficial than that of the raw variables (Model - A1), provided an appropriate training scheme is chosen, where perhaps grouping of variables had resulted in averaging out their scale effects. The most suitable network, FFBP Model - A2, has the highest CC = 0.999 and $R^2 = 0.999$; and lowest

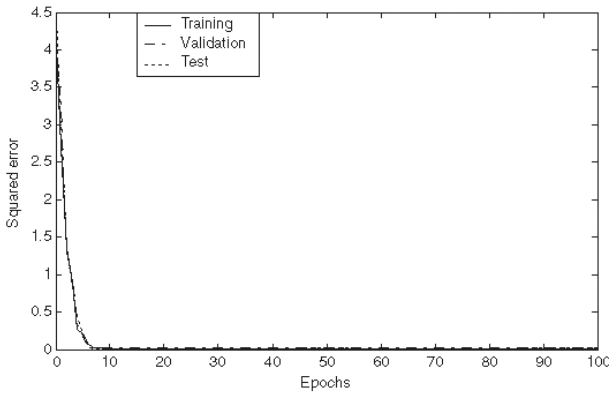

Figure 7: Epochs versus squared error of raw variables by back propagation

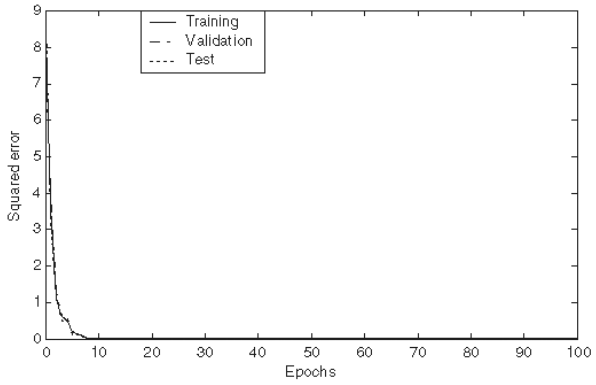

Figure 8: Epochs versus squared error of grouped variables by back propagation

MPE = -0.091, MAD = 3.123, and RMSE = 0.000. All the ANN models featured small RMSE during training; however, the value was slightly higher during validation. The models showed consistently good correlation throughout the training and testing.

In the end therefore the network configuration (FFBP Model - A2) along with corresponding weight and bias matrix given in Table 6 is recommended for general use in order to predict the deflection.

The mean error in the prediction of deflection by proposed model may be compared with the performance of neural network models in Fig. 4, wherein the mean error in prediction by ANN Model - A2 is only 3%. The histogram of percentage error of neural network model shown in Fig. 4 may be compared with the corresponding histogram for the propose model shown in Fig. 4. It is observed from these figure that the percentage error in 100% of the data is less than 3% for the neural network model, whereas the percentage error in the proposed regression model in the same percentage of data is less than 43.3%. This clearly indicates the supremacy

Table 5: Connection Weights and Biases (Refer to Fig. 5) (Output Bias = -0.120)

No. of Neuron	Input Weights							Output Weights	Input Biases
	a	b	c	d	e	f	g	r	
1	-0.096	-0.013	0.996	-1.144	-0.636	0.0192	0.973	-0.805	1.983
2	0.232	0.548	-0.300	-0.735	-1.195	0.039	0.715	1.120	-1.557
3	0.267	-1.285	-0.444	0.848	0.670	0.497	0.043	1.005	-0.249
4	-0.268	1.249	0.526	-1.224	0.549	0.457	1.663	0.859	-0.116
5	-0.483	0.733	1.070	-0.599	-0.518	0.734	-0.627	0.933	0.774
6	-0.010	-0.698	-1.173	-0.911	-1.055	0.432	0.117	1.094	-0.568
7	0.234	0.014	0.807	0.297	0.328	0.377	-0.346	-0.413	-2.205
8	0.422	0.206	0.601	-0.289	0.308	-0.925	1.294	1.127	2.000

Table 6: Connection Weights and Biases (Refer to Fig. 6) (Output Bias= -0.637))

No. of Neuron	Input Weights			Output Weights	Input Biases
	a	b	c	r	
1	0.739	1.648	-1.183	-0.486	-3.912
2	0.686	0.316	1.065	0.718	-3.404
3	-0.562	0.121	0.991	2.260	0.424
4	-0.856	-0.437	2.140	-0.548	-1.109
5	-1.882	0.095	-0.679	-2.236	-2.241
6	-0.641	0.621	0.816	1.158	-4.705

of the neural network model over the regression models.

It may be necessary to modify these transverse velocity fields for other cases, such as conical, hemispherical or other shaped impactors, or for impacts at non-central locations for rectangular plates. If the impact energies are sufficiently large, then a plate might crack. The threshold conditions for cracking have not been examined here, since these have been explored by many authors, including some recent studies on the impact failure of beams and circular plates. With the above proposed models, all information, such as velocities of plate, velocities and locations of traveling hinge lines, displacements of plate, forces between the striker and plate and durations during whole response process, can be obtained.

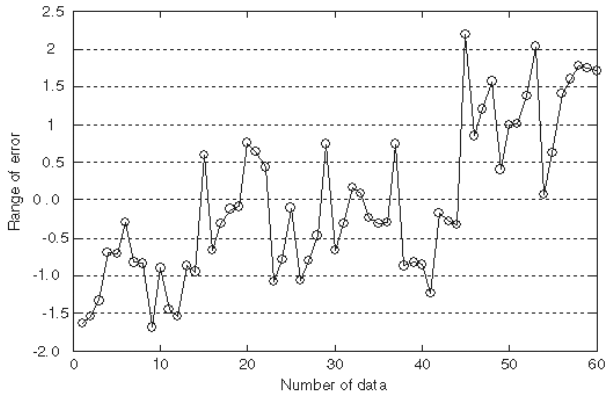

Figure 9: Percentage error for Model - A1

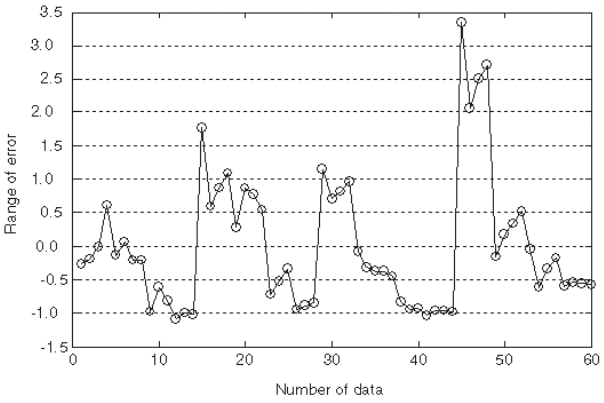

Figure 10: Percentage error for Model - A2

7 Conclusions

A regression based empirical model has been developed for the prediction of maximum permanent deflection created in a rectangular plate struck by a rigid wedge at the centre of plate. The proposed regression model is more accurate predictor of maximum deflection created from wedge than the historical models based upon the data used to generate the model.

The data used in the development of statistical models was reanalyzed for the prediction of maximum deflection by employing the technique of neural networks with a view towards seeing if better predictions are possible. Predictions based on grouped dimensionless forms of the data (L/B, $T_t/2S$ and $m_pV^2/\sigma_0 T_t^3$) were better than those based on the original dimensioned data L, B, S T_t, m_p, σ_0 and V). The neural network with one hidden layer was selected as the optimum network to predict deflection. The network configuration of Model - A2 with FFBP is

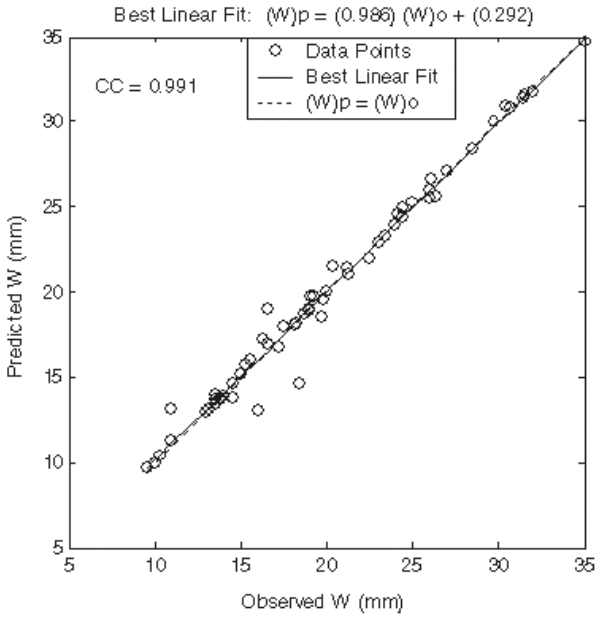

Figure 11: Observed versus predicted W for Model - A1

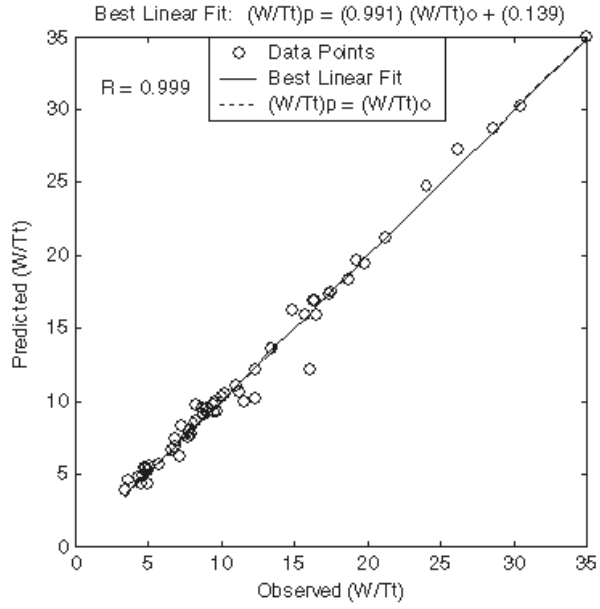

Figure 12: Observed versus predicted W/t_t for Model - A2

recommended for general use in order to predict the deflection.

On the basis of sensitivity analysis of ANN models, it is observed that the set of (L/B and $T_t/2S$) makes it second most significant parameter after $m_p V^2/\sigma_0 T_t^3$. But in view of the variability in the outcome resulting from application of different analytical schemes, it is felt that the network which requires all input quantities may be followed for generality. The neural network model is far better than the regression model for the prediction of the deflection. The recommended ANN model may be used with confidence to study any combination of parameters, which have not been experimentally obtained, but lie within the range of input parameters.

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