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Effect of Open Crack on Vibration Behavior of a Fluid-Conveying Pipe Embedded in a Visco-Elastic Medium

Abstract

In this paper vibration behavior of a fluid-conveying cracked pipe surrounded by a visco-elastic medium has been considered. During this work, the effect of an open crack parameters and flow velocity profile shape inside the pipe on natural frequency and critical flow velocity of the system has been analytically investigated. An explicit function for the local flexibility of the cracked pipe has been offered using principle of the fracture mechanics. Comparison between the results of the present study and the experimental data reported in the literature reveals success and high accuracy of the implemented method. It is demonstrated that the existence of the crack in the pipe, decreases the natural frequency and the critical flow velocity so that the system instability onsets at a lower flow velocity in comparison with the intact pipe. Results indicate that the flow velocity profile shape inside the pipe caused by the viscosity of real fluids, significantly affects the critical flow velocity of both intact and fluid-conveying cracked pipe. For instance, as the flow-profile-modification factor decreases from 1.33 to 1.015, the dimensionless critical flow velocity of intact clamped-clamped pipe increases from 5.45 to 6.24.

Keywords

Cracked Fluid-conveying Pipe, Vibration Analysis, Natural Frequency, Velocity Profile, Critical Flow Velocity.

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NOMENCLATURE

A, $A_i, i = 1,..,8$	constant	t	time
a	crack depth	U	mean fluid velocity
a_x	crack depth of the strip (in Figure 1)	u	dimensionless fluid velocity
a_c	dimensionless crack depth	x	coordinate
b	half width of crack	X	axial coordinate
C	damping coefficient of foundation	W and $\bar{W}$	transverse deflection

NOMENCLATURE (continuation)

c	local flexibility	w	dimensionless transverse deflection
E	Young module	w_L, w_R	left and right side of the crack transverse deflection
F_{ext}	external force	y	coordinate
$F(\xi/h_x)$	crack effect function		
h	height of rectangular section	α	flow-profile-modification factor
h_x	height of the strip (in Figure 1)	β	dimensionless frequency
I	second moment of area	ζ	dimensionless axial coordinate
K_I	stress intensity factor	ζ_c	dimensionless crack location
K_t	equivalent torsional spring stiffness	η	Dimensionless damping factor
K_s, k_s	foundation constant, dimensionless	λ, $\lambda_i, i = 1,..,4$	constants
K_m, k_m	Winkler stiffness, dimensionless	ξ	local coordinate
l	length of pipe	ν	Poisson's ratio
M	bending moment	φ	dimensionless parameter related to the effect of crack depth
m_f and m_p	mass per unit length of pipe and fluid	ω	natural frequency
m	dimensionless mass		
$p_i, i = 1,..,4$	constants		
P, Q, R	matrix		
$R_o$and R_i	Outer and inner radius of the pipe		
T	matrix		

1 INTRODUCTION

Dynamic behavior of the fluid-conveying pipes has significant importance due to its extensive application in different industries such as petrol and gas transportation systems (Païdoussis 1998, Ibrahim 2010, Ghaitani and Esmaeili 2013). These pipes often lie on a foundation and would be cracked as a result of various internal and external loads while conveying fluid. So, analyzing the dynamic behavior of the cracked pipes would be practically important.

Inspired by the work of Tada et al. (1973), many researchers have performed the vibration analysis of the cracked structures such as thin walled tubes during the last five decades. Recently, some reviews on vibration of cracked structures were reported by Dimarogonas (1996) and Kumar et al. (2010). In some researches the crack has been simulated by a local reduced section or stiffness of the structure around the crack (Sinha, Friswell et al. 2002). The local flexibility of the structure is reduced due to the crack and this concept has been used to model the vibration behavior of the cracked pipe in some studies (Xie 1998, Mahjoob and Shahsavari 2007, Hu et al. 2011, Yumin et al. 2011, liu et al. 2012, Bai et al. 2013). Literature review indicates that available studies about the vibration behavior of the cracked pipes can be divided into three groups;

- Having no fluid inside,
- Filled with fluid without flow (zero flow velocity)
- Having fluid flow inside (fluid-conveying pipe).

There are fairly large amount of researches dedicated to the first two groups; Liu et al. (2003) examined the stability of thin-walled pipe due to circumferential cracks analytically and experimentally based on the coupled-response measurements. Ye et al. (2010) investigated the vibration behavior of the cracked pipe in the absence of the fluid flow using the finite element method. Their results show that the presence of the crack in the pipe reduces the natural frequency, and its effect on the upper natural frequencies is more than the lower ones. Murigendrappa et al. (2004a; 2004b) studied the vibration behavior of the cracked pipe theoretically and experimentally. In their works, the crack has been considered as a torsional spring and the transfer matrix method with Rayleigh's quotient has been used for vibration analysis. Naniwadekar et al. (2008) developed an experimental method for crack detection based on changing the natural frequency and presented a numerical method to analyze the vibration behavior of a straight horizontal steel hollow pipe filled with an incompressible fluid (without flow).

In contrast to the first two groups, there are fairly limited amount of investigation about the vibration behavior of the cracked fluid-conveying pipes (third group). Yoon and Soo are two researchers that have contributed in most papers published about vibration behavior of the fluid-conveying cracked pipes. They investigated the effects of the open crack parameters and the moving mass on the dynamic behavior of simply supported pipe conveying fluid using Timoshenko beam model. Numerical methods based on the transfer matrix have been preferred in their works (Son, Lee et al. , Yoon and Son 2004, Yoon, Son et al. 2004, Yoon and Son 2005, Yoon and Son 2006, Son, Cho et al. 2007, Yoon and Son 2007, Yoon, Son et al. 2007, Son and Yoon 2008, Son, Yoon et al. 2010). So, available research in literature did not consider the effects of the crack parameters on the critical flow velocity which is our main objective in this paper.

In present paper the vibration behavior of the cracked fluid-conveying pipe is examined via Euler-Bernoulli beam model focusing on natural frequency and critical flow velocity. Also, in order to have a more realistic model, the surrounding elastic medium is modeled via a visco-elastic foundation. Our study is carried out through an analytical method. The crack is considered using a massless torsional spring. An explicit function for obtaining the local flexibility of the cracked pipe is presented using the theory of fracture mechanics. Also, we demonstrate that how the fluid flow velocity profile would influence the critical flow velocity of the system.

2 LOCAL FLEXIBILITY OF THE CRACKED PIPES

Dimarogonas and Papadopoulos (1983) proposed the massless torsional spring model of the cracked beam and shaft, and derived the additional flexibility due to the crack using the principle of fracture mechanics. Following this procedure, a small vertical sectional strip is considered in the cracked section of the pipe. The strain energy of the strip is obtained using the theory of fracture mechanics. By integrating of the strain energy over the cracked section, for the local flexibility, an explicit function is obtained as a function of the crack depth.

The additional strain energy due to the crack can be considered in the form of a flexibility coefficient expressed in terms of the stress intensity factor, which can be derived by Castigliano's theorem in the linear elastic range. Therefore, the local flexibility in the presence of the crack is defined by (Zou et al. 2003):

$$c = \frac{\partial^2 U}{\partial M^2} = \frac{1-\nu^2}{E}\int_{-b}^{b}\int_{0}^{a}\frac{\partial^2 (K_I^2)}{\partial M^2}\,dxdy \tag{1}$$

where E is Young module, υ is Poisson's ratio, a and b are the crack dimensions as shown in Figure 1. K_I is the stress intensity factor corresponding to the first fracture mode due to the bending moment M.

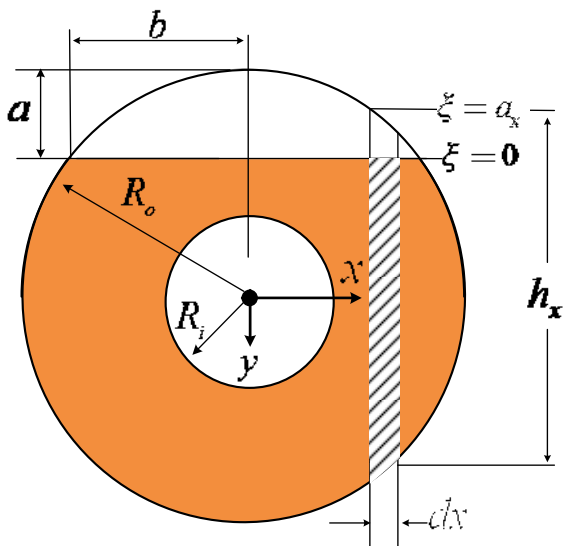


Figure 1: Cross section of the cracked pipe.

It is seen from Eq. (1) that the local flexibility depends on the stress intensity factor. In order to calculate the stress intensity factor for the cracked pipes, consider a small vertical rectangular strip, having a small cross-sectional width dx and height h_x with an arbitrary offset distance x, as shown in Figure 1. In Ref. (Tada, Paris et al. 2000) the stress intensity factor for a rectangular section is presented as:

$$K_I = \frac{Ma}{2I^2}\sqrt{\pi a}F(a/h) \tag{2}$$

where, a is the crack depth, I and $h$are the second moment of area and height of the rectangular cross-section, respectively and $F(a/h)$ is a mathematical function of crack dimension. Employing Eq. (2) for the rectangular strip shown in Figure 1 leads:

$$K_I = \frac{M\xi}{2I^2}\sqrt{\pi\xi}\,F(\xi/h_x) \tag{3}$$

in which, ξ is local coordinate which varies from $\xi = 0$ to $\xi = a_x$ swiping the crack depth in the given strip, and $F(\xi/h_x)$ is defined as (Tada et al. 1973):

$$F\left(\frac{\xi}{h_x}\right) = \sqrt{\frac{2h_x}{\pi\xi}\tan\left(\frac{\pi\xi}{2h_x}\right)}\frac{0.923 + 0.199\left(1 - \sin\left(\frac{\pi\xi}{2h_x}\right)\right)^4}{\cos\left(\frac{\pi\xi}{2h_x}\right)} \tag{4}$$

Substituting Eq. (3) into Eq. (1) leads to:

$$c = \frac{1-\nu^2}{E}\int_{-b}^{b}\int_{0}^{a_x}\frac{\partial^2(K_I^2)}{\partial M^2}dydx = \frac{\pi(1-\nu^2)}{2EI^2}\int_{-b}^{b}\int_{0}^{a_x} h_x^2\,\xi F^2\,d\xi dx \tag{5}$$

The equivalent rotational spring stiffness, K_t, is defined as (Dimarogonas and Papadopoulos, 1983):

$$K_t = \frac{1}{c} \tag{6}$$

Taking into account the coordinate system illustrated in Figure 1, h_x and b calculated as:

$$h_x = 2\sqrt{R_o^2 - x^2} \tag{7}$$

$$b = \sqrt{R_o^2 - (R_o - a)^2} \tag{8}$$

and the upper boundary of the crack,a_x, can be expressed as:

$$a_x = \frac{h_x}{2} - (R_o - a) = \sqrt{R_o^2 - x^2} - (R_o - a). \tag{9}$$

By defining the dimensionless variables; $\bar{x} = x/R_o$, $\bar{h_x} = h_x/R_o$, $\bar{b} = b/R_o$, $\bar{a_x} = a_x/R_o$ and $\alpha_c = a/R_o$, the dimensionless form of Eqs. (7)- (9) becomes:

$$\bar{h}_x = 2\sqrt{1 - \bar{x}^2} \tag{10}$$

$$\bar{b} = \sqrt{1 - (1 - \bar{a})^2} \tag{11}$$

$$\bar{a}_x = \frac{\bar{h}_x}{2R_o} - \left(1 - \frac{a}{R_o}\right) = \sqrt{1 - \bar{x}^2} - (1 - a_c). \tag{12}$$

Substitution of Eqs. (10)-(12) into Eq. (5) leads:

$$c = \frac{2\pi(1-\nu^2)R_0^5}{EI^2}\int_{-\bar{b}}^{\bar{b}}\int_{0}^{\bar{a}_x}(1-\bar{x}^2)\,\bar{\xi}F^2\left(\frac{\bar{\xi}}{\bar{h}_x}\right)d\bar{\xi}d\bar{x} \tag{13}$$

where $\bar{\xi} = \xi/R_0$. For the flexibility coefficient, the dimensionless form introduced as:

$$\bar{c} = \frac{EI^2}{2\pi(1-\nu^2)R_0^5}c \tag{14}$$

it can be derived from Eq. (13):

$$c = \int_{-\bar{b}}^{\bar{b}} \int_{0}^{\bar{a}_x} (1-\bar{x}^2)\,\bar{\xi} F^2\left(\frac{\bar{\xi}}{\bar{h}_x}\right) d\bar{\xi} d\bar{x} \tag{15}$$

Eq. (15) represents an integral form for the local flexibility of the cracked pipes. This integral expression is only a function of the dimensionless crack depth, $\alpha_c = a/R_o$. By numerical integration of Eq. (15) and fitting an appropriate curve to the numerical values of $\bar{c}$ and then using the least square method, an explicit expression for the dimensionless flexibility coefficient of the cracked pipe is obtained as:

$$\begin{aligned}\bar{c}(a_c) = {} & -5.6\times10^{-6}(a_c) + 0.0335(a_c)^2 + 16.19(a_c)^3 - 1047.91(a_c)^4 + 5.98\times10^4(a_c)^5 \\ & - 2.34\times10^6(a_c)^6 + 6.03\times10^7(a_c)^7 - 1.00\times10^9(a_c)^8 + 1.02\times10^9(a_c)^9 \\ & - 5.83\times10^9(a_c)^{10} + 1.42\times10^{11}(a_c)^{11}\end{aligned} \tag{16}$$

It should be noted that according to the main assumption in the fracture mechanics, Eq. (16) is satisfied up to middle of the pipe wall thickness i.e. $a = (R_0 - R_i)/2$.

This explicit function may be applied for any pipe with specified mechanical properties and geometrical dimensions. Considering Eqs. (16) and (14), the equivalent stiffness of the cracked pipe, K_t is calculated straightforwardly from Eq. (6). In order to verify the accuracy of this model, the predicted values of K_t and the experimental results reported by Murigendrappa et al. (2004) for a cracked pipe with a parameters $R_o = 16.5\text{mm}$, $R_i = 10\text{mm}$, $E = 65\text{GPa}$ and $\nu = 0.33$ have been compared in Table 1. The comparison shows that there is a good agreement between the equivalent stiffness obtained from the proposed method and the experimental data. So, the maximum difference between the predicted results and the experimental data of Murigendrappa et al. (2004) is less than 5%.

Crack depth, a (mm)	Local stiffness, k_t (MNm rad^{-1})		Error (%)
	Presented model, Eq. (7)	Experimental data (Murigendrappa et al. 2004)	
1.238	16.10	16.06	0.25
1.650	9.56	9.32	2.58
2.475	3.62	3.71	2.43
4.125	1.10	1.05	4.76

Table 1: The local stiffness due to crack, Comparison between the calculated results and the experimental data of Murigendrappa et al. (2004).

The effect of the crack depth on the dimensionless local flexibility coefficient is shown in Figure 2. The figure indicates that an increase in the crack depth enhances the flexibility coefficient of the cracked pipe because of the reduction in equivalent stiffness (Eq. 6).

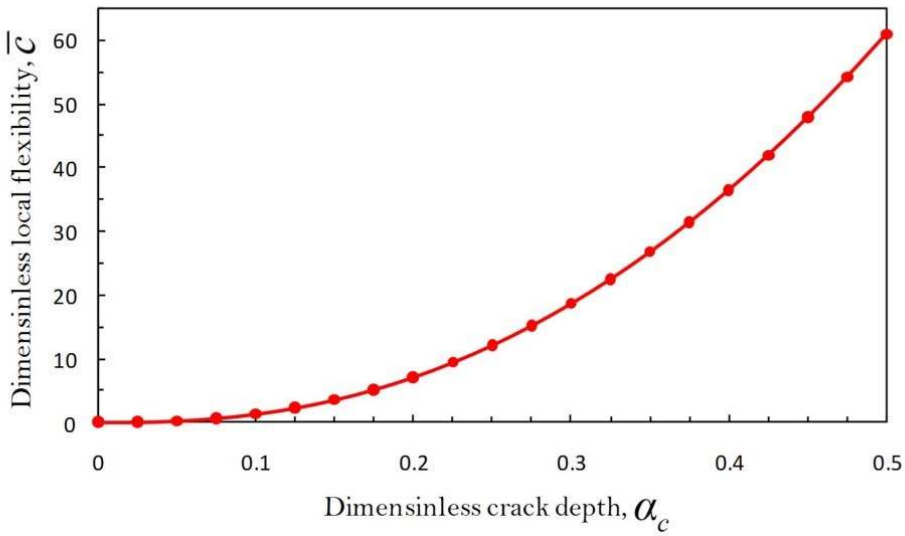


Figure 2: Effect of the crack depth on the dimensionless local flexibility coefficient.

3 MATHEMATICAL MODEL

Figures 3a and 3b illustrate schematically a fluid-conveying cracked pipe resting on a visco-elastic foundation and its corresponding mathematical model, respectively. Using the Euler-Bernoulli theory, the modified governing equation of motion for the uniform fluid-conveying pipe which is subjected to an external force, F_{ext} , can be written as (Guoa et al. 2010):

$$EI\frac{\partial^4 W(X,t)}{\partial X^4}+\alpha m_f U^2\frac{\partial^2 W(X,t)}{\partial X^2}+2m_f U\frac{\partial^2 W(X,t)}{\partial X\partial t}+\left(m_f+m_p\right)\frac{\partial^2 W(X,t)}{\partial t^2}=F_{ext} \qquad (17)$$

where m_p and m_f are the mass per unit length of pipe and fluid, respectively. t stands for time, X is axial coordinate, $W(X,t)$ and EI are the transverse deflection and the flexural stiffness of the pipe, respectively. αdenotes a dimensionless parameter named as the flow-profile-modification factor dependent on fluid velocity profile inside pipe. Considering the power-law profile for the time-average flow velocity profile (Streeter et al. 1998)as:

$$U(r)=u_{max}(\frac{y}{R})^{\frac{1}{n}}$$

α can be determined by (Guoa et al. 2010):

$$\alpha=1+\frac{5n+1}{\left(2n\right)^2\left(n+2\right)}$$

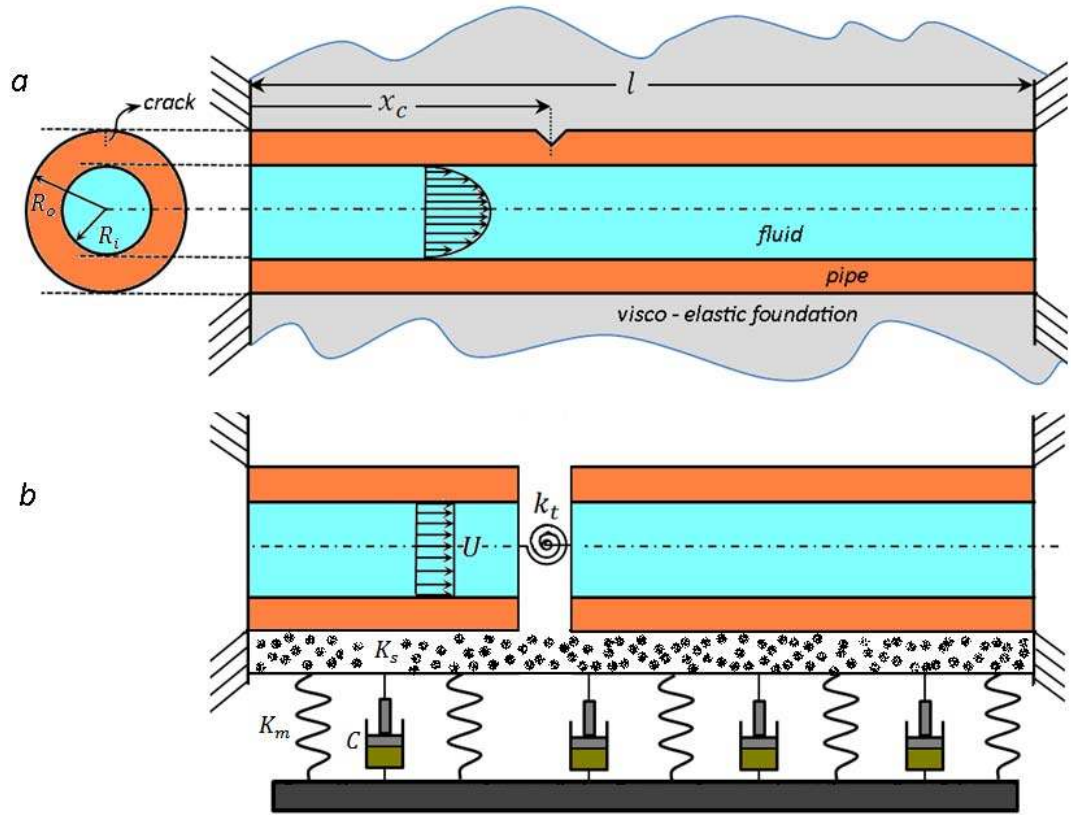


Figure 3: **(a)** Schematic view of the cracked clamped-clamped pipe, **(b)** Mathematical model.

Where n is dependent on the Reynolds number. The well-known one-seven power-law velocity profile introduced by Prandtl can be achieved for n=7 that is valid for the smooth-wall circular pipes in the range $3\times10^3 \leq Re \leq 3\times10^5$ (Re denotes the Reynolds number). Also, n=4 and n=5 provide the velocity profile of a rough-wall circular pipe based on the experiments of Nikuradse. For n=7, α =1.020; and for n=4 and n=5, $\alpha \approx 1.037–1.055$. We will discuss more about the impact of α on the vibration behavior of the intact and cracked fluid-conveying pipes in subsection 6.2.

In the present work, the external force related to the visco-elastic foundation modeled using two-parameter Kelvin-Voigt model. Based on this model F_{ext} can be written as:

$$F_{ext} = K_s \frac{\partial^2 W(X,t)}{\partial X^2} - C \frac{\partial W(X,t)}{\partial t} - K_m W(X,t) \tag{18}$$

where, K_m and C stand for the Winkler stiffness constant and the damping coefficient of the visco-elastic foundation, respectively. K_s is an additional parameter defining the foundation, usually termed as the shear constant of the foundation (Kargarnovin et al. 2005, Nguyen and Duhamel 2008, Ansari et al. 2011, Basu and Kameswara 2013). By substituting of Eq. (18) into Eq. (17), the governing equation of the fluid-conveying pipe resting on a visco-elastic foundation is obtained as:

$$EI\frac{\partial^4 W(X,t)}{\partial X^4}+(\alpha m_f U^2-K_s)\frac{\partial^2 W(X,t)}{\partial X^2}+2m_f U\frac{\partial^2 W(X,t)}{\partial X\partial t}+(m_f+m_p)\frac{\partial^2 W(X,t)}{\partial t^2}+C\frac{\partial W(X,t)}{\partial t}+K_m W(X,t)=0 \tag{19}$$

It is assumed that the solution of Eq. (19) is:

$$W(X,t)=\bar{W}(X)e^{i\omega t} \tag{20}$$

where $\bar{W}(X)$ and ω indicate the amplitude and natural frequency, respectively. The dimensionless variables are defined as:

$$\zeta=\frac{X}{l},w=\frac{\bar{W}}{l},m=\frac{m_f}{m_f+m_p},u=\sqrt{\frac{m_f}{EI}}Ul,\qquad \beta=\frac{\omega}{\sqrt{\frac{EI}{(m_f+m_p)l^4}}},$$
$$k_m=\frac{l^4}{EI}K_m,\qquad k_s=\frac{l^2}{EI}K_s,\eta=\frac{Cl^2}{\sqrt{EI(m_f+m_p)}} \tag{21}$$

Substituting Eq. (20) into Eq. (19) and using the above dimensionless variables, Eq. (19) can be rewritten as follows:

$$\frac{d^4w}{d\zeta^4}+(\alpha u^2-k_s)\frac{d^2w}{d\zeta^2}+2i\beta\sqrt{mu}\frac{dw}{d\zeta}+(i\beta\eta-\beta^2+k_m)w=0 \tag{22}$$

The solution of the Eq. (22) is:

$$w(\zeta)=Ae^{\lambda\zeta} \tag{23}$$

where A and λ are constants. Substituting this expression in Eq. (22) leads to the following characteristic equation:

$$\lambda^4+(\alpha u^2-k_s)\lambda^2-2i\beta\sqrt{mu}\lambda+(i\beta\eta-\beta^2+k_m)=0 \tag{24}$$

The Eq. (24) is a quartic polynomial equation, which has generally four complex roots (consider a typical real root as a complex root with zero imaginary part). The explicit algebraic formulas for a general solutions of Eq. (24) are expressed as (Persidis 2007):

$$\lambda_1=-\frac{1}{2}(p_1+\sqrt{p_2-p_3}),\quad \lambda_2=-\frac{1}{2}(p_1-\sqrt{p_2-p_3}),\quad \lambda_3=\frac{1}{2}(p_1+\sqrt{p_2-p_3}),$$
$$\lambda_4=-\frac{1}{2}(p_1-\sqrt{p_2-p_3}) \tag{25}$$

where the parameters p_1, p_2 and p_3 which depend on the dimensionless natural frequency β, have been defined in appendix. The general solution of Eq. (22) is

$$w(\zeta)=A_1e^{\lambda_1\zeta}+A_2e^{\lambda_2\zeta}+A_3e^{\lambda_3\zeta}+A_4e^{\lambda_4\zeta} \tag{26}$$

In which A_i, $i=1,2,3,4$ are unknown constants and will be obtained by applying the boundary conditions.

4. FREQUENCY EQUATION

Consider the corresponding mathematical model of the cracked pipe conveying fluid resting on a visco-elastic medium as shown in Figure 3b. For analyzing the free vibration of the cracked pipe, the entire pipe is divided from the cracked section into the two pipe segments. The crack is represented by a torsional mass-less spring that its stiffness was calculated previously in section 2. The two pipe segments can be treated separately. The equations of motion for two mentioned intact segments are obtained from Eq. (26) as follows:

$$w_L(\zeta) = A_1 e^{\lambda_1 \zeta} + A_2 e^{\lambda_2 \zeta} + A_3 e^{\lambda_3 \zeta} + A_4 e^{\lambda_4 \zeta} \tag{27}$$

$$w_R(\zeta) = A_5 e^{\lambda_1 \zeta} + A_6 e^{\lambda_2 \zeta} + A_7 e^{\lambda_3 \zeta} + A_8 e^{\lambda_4 \zeta} \tag{28}$$

where $w_L(\zeta)$ and $w_R(\zeta)$ are the vibration mode of each segment in the left and the right side of the crack, respectively. These vibration modes have eight unknown constants A_i, $i = 1,2,...,8$, which can be obtained by applying both boundary conditions and compatibility conditions at the crack location. The compatibility of displacement, moment and shear force of both segments at the dimensionless crack location (i.e. $\zeta = \zeta_c$), are respectively given by:

$$w_L(\zeta_c) = w_R(\zeta_c),\ \ w_L''(\zeta_c) = w_R''(\zeta_c)\ ,\ w_L'''(\zeta_c) = w_R'''(\zeta_c) \tag{29}$$

and the discontinuity in the slope of the pipe at the cracked location may be implemented as

$$w_L'(\zeta_c) - w_R'(\zeta_c) = \varphi w_R''(\zeta_c). \tag{30}$$

where $\varphi = \frac{EI}{K_t l}$ is the dimensionless parameter indicating the effect of crack depth. General boundary conditions can be applied for the present model. In this paper, clamped–clamped supported pipe is considered. Thus the four boundary conditions may be written as:

$$w_L(\zeta)|_{\zeta=0} = 0,\ \ \left.\frac{dw_L(\zeta)}{d\zeta}\right|_{\zeta=0} = 0, \qquad w_R(\zeta)|_{\zeta=1} = 0,\ \ \left.\frac{dw_R(\zeta)}{d\zeta}\right|_{\zeta=1} = 0. \tag{31}$$

Substituting Eqs. (27) and (28) into the boundary conditions Eq. (31) and into the compatibility conditions Eqs. (29) and (30), gives a set of eight homogeneous linear algebraic equations for the eight unknown constants A_i, $i = 1,2,...,8$ as follows:

$$\begin{bmatrix} [P] & [Q] \\ [R] & [T] \end{bmatrix} [A_1 \quad A_2 \quad A_3 \quad A_4 \quad A_5 \quad A_6 \quad A_7 \quad A_8]^T = 0 \tag{32}$$

where

$$[P] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$[Q]=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ e^{\lambda_1} & e^{\lambda_2} & e^{\lambda_3} & e^{\lambda_4} \\ \lambda_1 e^{\lambda_1} & \lambda_2 e^{\lambda_2} & \lambda_3 e^{\lambda_3} & \lambda_4 e^{\lambda_4} \end{bmatrix}, \tag{33}$$

$$[R]=\begin{bmatrix} e^{\lambda_1\zeta_c} & e^{\lambda_2\zeta_c} & e^{\lambda_3\zeta_c} & e^{\lambda_4\zeta_c} \\ \lambda_1^2 e^{\lambda_1\zeta_c} & \lambda_2^2 e^{\lambda_2\zeta_c} & \lambda_3^2 e^{\lambda_3\zeta_c} & \lambda_4^2 e^{\lambda_4\zeta_c} \\ \lambda_1^3 e^{\lambda_1\zeta_c} & \lambda_2^3 e^{\lambda_2\zeta_c} & \lambda_3^3 e^{\lambda_3\zeta_c} & \lambda_4^3 e^{\lambda_4\zeta_c} \\ (\lambda_1-\varphi\lambda_1^2)e^{\lambda_1\zeta_c} & (\lambda_2-\varphi\lambda_2^2)e^{\lambda_2\zeta_c} & (\lambda_3-\varphi\lambda_3^2)e^{\lambda_3\zeta_c} & (\lambda_4-\varphi\lambda_4^2)e^{\lambda_4\zeta_c} \end{bmatrix}$$

$$[T]=\begin{bmatrix} -e^{\lambda_1\zeta_c} & -e^{\lambda_2\zeta_c} & -e^{\lambda_3\zeta_c} & -e^{\lambda_4\zeta_c} \\ -\lambda_1^2 e^{\lambda_1\zeta_c} & -\lambda_2^2 e^{\lambda_2\zeta_c} & -\lambda_3^2 e^{\lambda_3\zeta_c} & -\lambda_4^2 e^{\lambda_4\zeta_c} \\ -\lambda_1^3 e^{\lambda_1\zeta_c} & -\lambda_2^3 e^{\lambda_2\zeta_c} & -\lambda_3^3 e^{\lambda_3\zeta_c} & -\lambda_4^3 e^{\lambda_4\zeta_c} \\ -\lambda_1 e^{\lambda_1\zeta_c} & -\lambda_2 e^{\lambda_2\zeta_c} & -\lambda_3 e^{\lambda_3\zeta_c} & -\lambda_4 e^{\lambda_4\zeta_c} \end{bmatrix}$$

The system of equations (32) has the nontrivial solution if and only if the determinant of the coefficients becomes zero. So the characteristic equation of the system will be:

$$|\Delta| = f(\alpha_c, \zeta_c, k_m, k_s, c, \eta, u, \alpha, \beta) = 0 \tag{34}$$

Now, by solving the Eq. (34), the dimensionless natural frequency, β, can be obtained.

5 VALIDATION

To the best knowledge of the authors, there is no analytical or experimental result in the literature for the vibration analysis of a problem that is completely the same of one stated here. So in order to validate the aforementioned theoretical model and analytical procedure, we use available data for a similar problem. For instance, comparison made between the present work results and the experimental data of Mahjoob and Shahsavari (2007) for the case of a clamped-clamped cracked pipe conveying fluid without foundation (i.e. $k_m = k_s = c = 0$). The implemented mechanical and geometrical properties required to calculate the results are listed in Table 2. The maximum difference between the experimental data of Mahjoob and Shahsavari (2007) and the present model results are 0.29%, 0.47% and 0.45% for the first, second and third natural frequencies, respectively (Table 3). Therefore, this comparison indicates an excellent agreement between present study and the experimental data.

	Pipe					Fluid	
Parameter	E	ρ_p	Ri	R_o	l	m_f	ρ_f
Dimension	GPa	kg m^{-3}	mm	mm	m	kg m^{-1}	kg m^{-3}
Value	210	7800	8	10.5	1	0.02	980

Table 2: Mechanical properties and geometrical dimensions of the investigated pipe, fluid and foundation.

Flow velocity, m s^{-1}	Crack depth, mm	Natural frequencies (Hz)								
		Present analytical method			Experimental Data (Mahjoob and Shahsavari 2007)			Error (%)		
		f_1	f_2	f_3	f_1	f_2	f_3	f_1	f_2	f_3
1	Intact pipe	118.94	327.87	642.75	118.97	327.98	643.00	0.10	0.03	0.06
	1	118.92	327.83	642.48	118.72	326.46	639.60	0.16	0.42	0.45
	2	118.82	327.68	641.00	118.47	326.22	639.35	0.30	0.45	0.29
5	Intact pipe	118.93	327.81	642.74	118.94	327.92	642.9	0.01	0.03	0.02
	1	118.91	327.82	641.36	118.72	326.39	639.49	0.16	0.44	0.29
	2	118.81	327.67	640.79	118.47	326.15	639.24	0.29	0.47	0.24

Table 3: Natural frequencies of the clamped-clamped fluid-conveying cracked pipe for $\zeta_c = 0.4$.
Comparison between results of present analytical method and the experimental data of Mahjoob and Shahsavari (2007)for various flow velocity.

6 RESULTS AND DISCUSSION

Having insured the accuracy of the present model, it is used to examine the vibration behavior of the cracked pipe conveying fluid resting on a visco-elastic foundation. The elastic, shear and damping constants used for the foundation as a Kelvin–Voigt model are respectively K_m=3.08 MPa, K_s=12550 kN and C=3110 Pa.s as given in (Gerolymos and Gazetas 2006, Younesian et al. 2012). In all numerical calculations, the flow-profile-modification factor, α, has been taken 1.33 except where it has been clearly specified.

6.1 Effect of the Crack Parameters

Figure 4, illustrates first natural frequency of the cracked pipe conveying fluid as a function of the dimensionless crack location, for three different values of $\alpha_c = 0.025$, $\alpha_c = 0.25$ and $\alpha_c = 0.4$, where the foundation parameters were $k_s = k_m = 0.01$ and $c = 0$. It is found that the crack parameters affect the vibration behavior of the cracked pipe. It is obvious that the frequencies of a system generally decrease due to the crack. Results show that when the crack location approaches to the fixed ends of the pipe, the effect of the crack on the frequency reduction is increased. This tendency is intensified with an increase in the crack depth. Also, as seen in Figure 4, for the crack located at $\zeta_c = 0.225$ and $\zeta_c = 0.775$, there is no reduction in the first natural frequency. Indeed, the mentioned locations are the inflection points for the first vibration mode of a clamped-clamped pipe. In these locations, the second derivation of the deflection function is equal to zero ($d^2W(X)/dX^2 = 0$) causing the bending moment to be completely vanished. Furthermore, since the major factor that decreases the natural frequencies caused by the crack is the bending moment, thus for the first vibration mode,

the crack located at $\zeta_c = 0.225$ and $\zeta_c = 0.775$ has no effect on the first natural frequency. The same behavior is observed for the alteration of the second natural frequency shown in Figure 5.

Figure 6 presents the natural frequency of the cracked pipe for the first mode as a function of the dimensionless crack depth for different values of ζ_c. When the crack depth, a_c, increases, the local flexibility of the pipe reduces according to Eq. (7) causing reduction in the natural frequency.

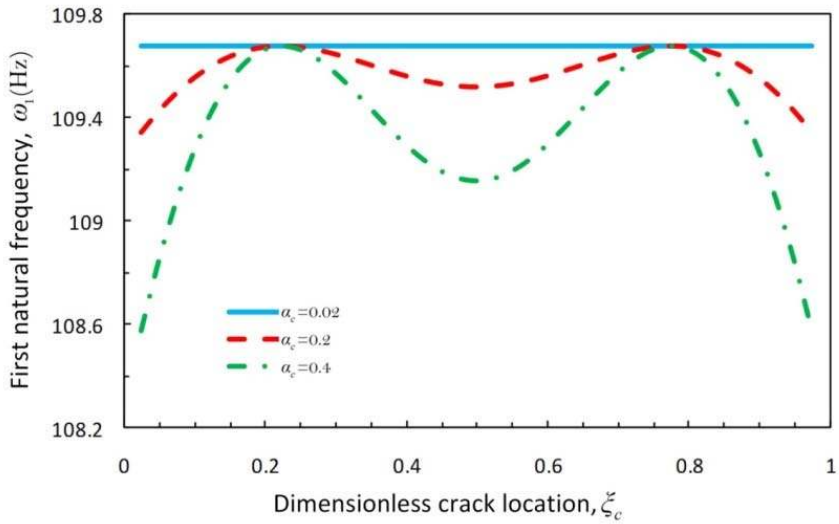


Figure 4: First dimensionless natural frequency of the fluid-conveying cracked pipe as a function of the dimensionless crack location.

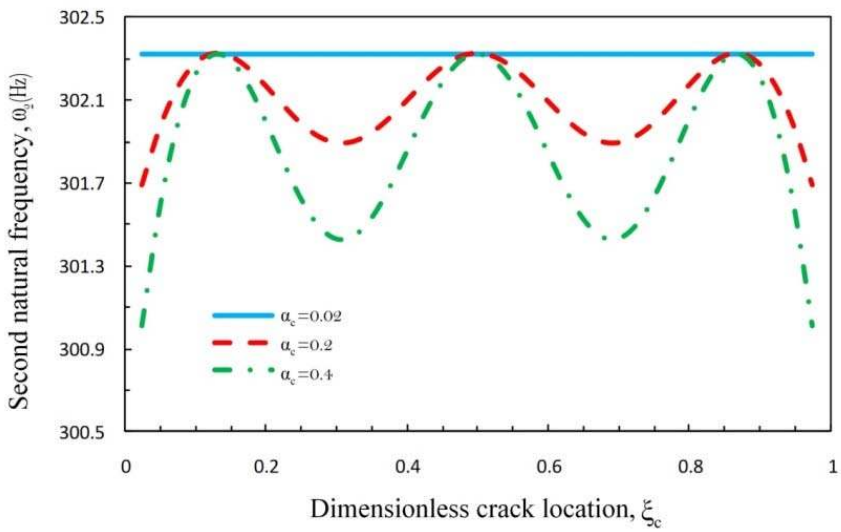


Figure 5: Second dimensionless natural frequency of the fluid-conveying cracked pipe as a function of the dimensionless crack location.

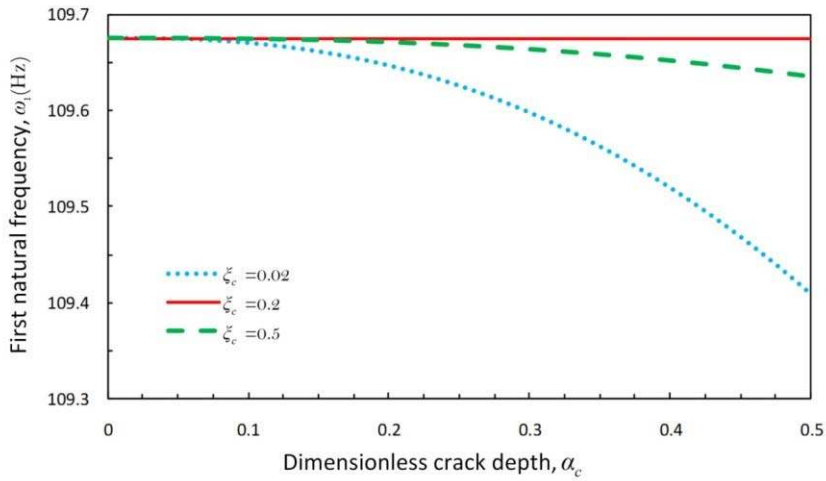


Figure 6: First natural frequency of the fluid-conveying cracked pipe.

6.2 Effect of the Flow Velocity

As seen from frequency equation (Eq. 34), the natural frequency of the cracked pipe conveying fluid in visco-elastic medium depends on the fluid flow velocity. Figure 7 shows the natural frequency of the intact pipe versus the dimensionless flow velocity for the first and second vibration modes. For the clamped-clamped intact pipe, the dimensionless critical flow velocity is $u^{cr}_{\text{Intact}} = 2\pi$ (Doare and Langer 2002). As it is obvious from the Figure 7, when the flow velocity increases, the pipe natural frequency decreases smoothly until it completely vanishes for u=2π. This equals to the lowest critical flow velocity that causes instability in the system. It is worth to note that u=2π corresponding to the mean flow velocity of U=498.5 m/s. This high velocity for a typical compressible fluid corresponds to Mach numbers greater than unit (Ma>1). Since for Ma>0.3, the compressibility should be taken into account, the present results for the critical flow velocity are not valid for the compressible fluids.

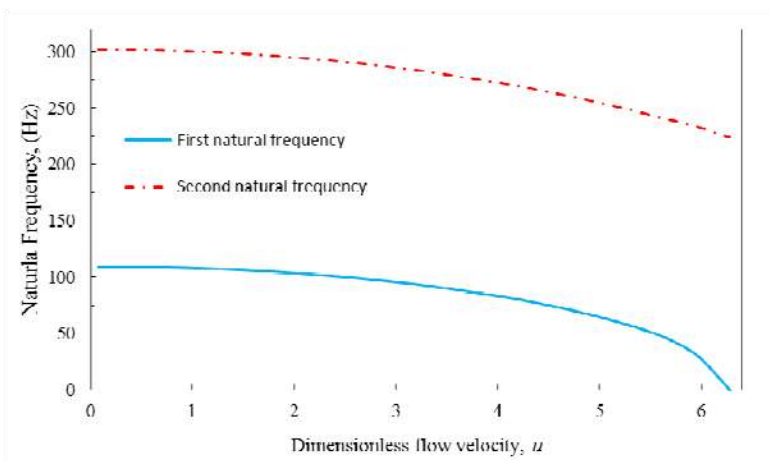


Figure 7: Variation of the natural frequency of the clamped-clamped fluid-conveying cracked pipe against the dimensionless flow velocity.

Experiments of Nikuradse revealed that fluid velocity profile in the circular pipes changes based on the quality of wall roughness. In order to take into account the effect of the mentioned change in the velocity profile, on vibration behavior of the fluid-conveying pipes, Guoa et al. (2010) derived a modified equation of motion for the pipe. The expression, a, appears in the centrifugal term of the modified equation (Eq. 17). As depicted in Figure 8, the flow-profile-modification factor would influence the critical flow velocity of both intact and cracked fluid-conveying pipe. As Guoa et al. (2010) indicated, when a decreases from 1.33 to 1.015, the velocity profile in the pipe cross section tends to a typical turbulent flow profile. This trendency corresponds to increase in critical flow velocity according to Figure 8. Thus it can be deduced that the instability in a typical fluid-conveying pipe would be delayed in turbulence regime in comparison with the laminar one. For an instance, the dimensionless critical flow velocity of intact pipe increases from 5.45 to 6.24 as the flow-profile-modification factor decreases from 1.33 to 1.015.

In the case of a cracked fluid-conveying pipe, the crack depth influences significantly the impact of flow-profile-modification factor on critical flow velocity. An interesting behavior observed in Figure 8 is about the effect of crack and its depth on the system instability. As it is seen, existence of a crack on the pipe causes the instability in a lower fluid flow velocity and this effect is intensified proportionally with crack depth for a given crack location ($\zeta_c = 0.01$ in Figure 8). This behavior shows that the crack and the instability would practically reinforce each other in a pipe system causing its destruction.

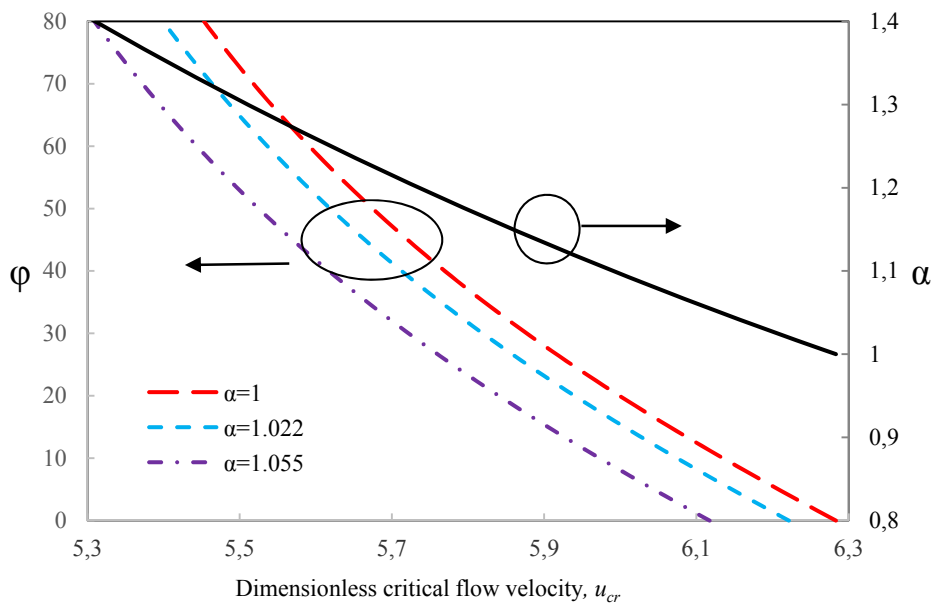


Figure 8: Flow-profile-modification factor influences the impact of crack depth, φ, on the critical flow velocity, u_{cr} for $\zeta_c = 0.01$ (left vertical axis). Also, it affects the critical flow velocity of an intact clamped-clamped pipe (right vertical axis).

6.3 Effect of the Visco-Elastic Foundation

Figure 9 shows the effect of the foundation parameters on the critical velocity as a function of the dimensionless damping factor, c, for the different foundation parameters. The comparison is made for the crack parameters $\alpha_c = 0.2$ and $\zeta_c = 0.1$. As seen in Figure 9, divergence instability for the first mode occurs when $u_{cr} \simeq 1.01$ but by increasing the dimensionless damping parameter of the visco-elastic foundation, c and/or decreasing the dimensionless stiffness parameter, k_s, the instability of the pipe takes place at a lower flow velocity.

Figure 10 shows the first three dimensionless frequencies of the cracked pipe as a function of the dimensionless parameters k_m where $c = 0.01$, $u = 0$. The results indicate that any augmentation in the foundation stiffness constant is followed by increasing the natural frequencies. This means that increasing the elastic constant makes the pipe stiffer. Also, effect of the foundation stiffness is more pronounced in the lower natural frequencies than the upper ones.

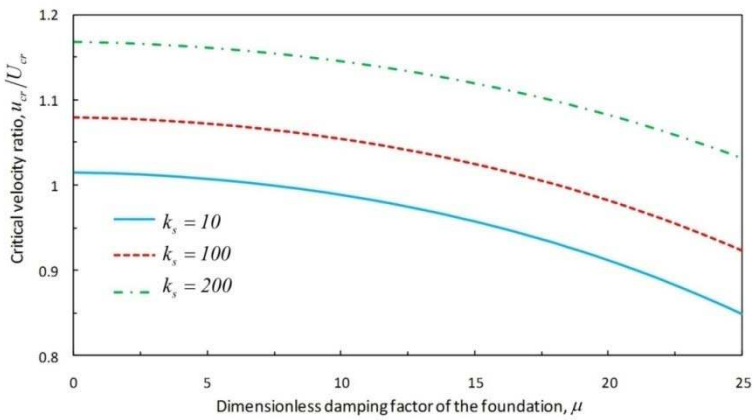


Figure 9: Variation of the dimensionless critical flow velocity of the clamped-clamped cracked pipe against the dimensionless damping factor for different values of the dimensionless stiffness, k_s.

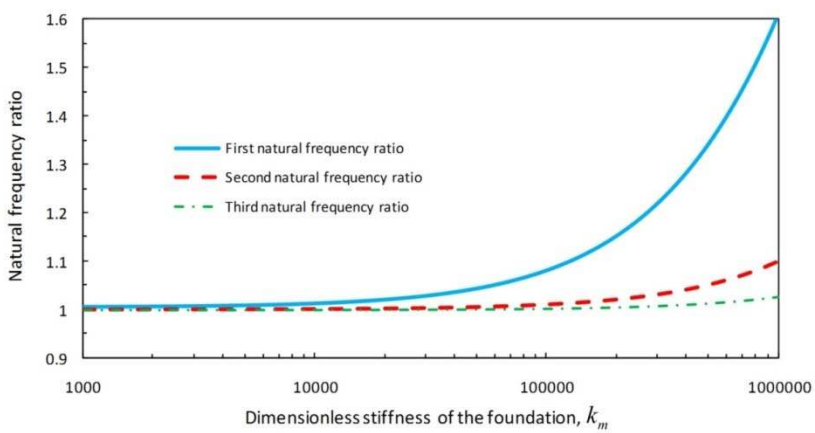


Figure 10: Effect of the foundation stiffness, k_m, on the first three natural frequencies of the fluid-conveying cracked pipe.

7 CONCLUSION

In the current study, the vibration behavior of a cracked pipe conveying fluid resting on a visco-elastic foundation was investigated analytically. A new approach for calculating the local flexibility caused by a crack was developed. The mentioned approach is applicable for various pipes with different mechanical properties. Comparison between the analytical results and available experimental data in literature shows good agreement for a wide range of the crack parameters.

The results indicate that increasing the crack depth improves the flexibility, and therefore the local stiffness reduces at the crack location. This leads to reduction in the both natural frequency and critical flow velocity of the cracked pipe system. This feature can be utilized for crack detection based on the vibration analysis. Also, when the crack location approaches to the fixed ends of the pipe, reduction in the natural frequency becomes more considerable. Furthermore, the results show that a decrease in the natural frequency reduces when the crack gets closer to the inflection points until it completely vanishes in the inflection points.

The fluid flow inside the pipe and its profile shape influences the natural frequency of the cracked pipe significantly. Moreover, the crack decreases the critical flow velocity so that the system instability onsets at a lower flow velocity.

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