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Resonance characteristics of two-span continuous beam under moving high speed trains

Abstract

The resonance characteristics of a two-span continuous beam traversed by moving high speed trains at a constant velocity is investigated, in which the continuous beam has uniform span length. Each span of the continuous beam is modeled as a Bernoulli-Euler beam and the moving trains are represented as a series of two degrees-of-freedom mass-spring-damper systems at the axle locations. A method of modal analysis is proposed in this paper to investigate the vibration of two-span continuous beam. The effects of different influencing parameters, such as the velocities of moving trains, the damping ratios and the span lengths of the beam, on the dynamic response of the continuous beam are examined. The two-span continuous beam has two critical velocities causing two resonance responses, which is different from simple supported beam. The resonance condition of the two-span continuous beam is put forward which depends on the first and second natural frequency of the beam and the moving velocity.

Keywords

critical velocity, moving trains, two-span continuous beam, resonance

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1 INTRODUCTION

With the development of high speed railway, the railway induced vibration has long been an interesting topic in the field of civil engineering, such as railway bridge vibrations. This has a huge amount of researches on the dynamic behaviors of railway bridges under the passage of the high speed trains [3–6, 8, 13–17]. Especially a comprehensive study in this topic can be found in references [4, 13, 16]. It is well known that the resonant vibrations occur when the loading frequencies of the moving trains coincide with the natural frequencies of the bridges.

In studying the fundamental problem of train-induced vibrations on bridges, a bridge was often modeled as a simply supported beam. Yang et al. obtained the condition of resonance and cancellation for a simple beam due to continuously moving loads and proposed the optimal design criteria that are effective for suppressing the resonant response [15]. Xia et al.

investigated the resonance mechanism and conditions of a train-bridge system using theoretical derivations, numerical simulations, and experimental data analyses [14]. Li and Su researched the resonant vibration for a simply supported girder bridge under high-speed trains, using an idealized vehicle model with a rigid body and four wheelsets [8]. Ju and Lin established a three dimensional finite element model to investigate the resonant characteristics of the simply supported bridges with high piers under high-speed trains [5]. But all of the above researches were based on the simple supported beam. For continuous bridges, Cheung et al. investigated the dynamic response of multi-span non-uniform bridges under moving vehicles and trains with the modified beam vibration functions [3]. Kwark et al. studied dynamic responses of two-span continuous concrete bridges under the Korean high-speed train (KHST) with experimental and theoretical methods [6]. Yau researched the effect of the number of spans on the impact response of the continuous beams with finite element method [17]. Based on these investigations, many useful results have been brought out. Unfortunately, no further details were proposed for the resonance characteristics, especially for the two-span continuous beam.

Because of the continuous action of moving trains, the certain frequencies of excitation will be imposed on the two-span continuous beam to result in beam resonance. In the present study, the dynamic response of two-span continuous beam under moving high speed trains were investigated with modal superposition method and the reasons for beam resonance were revealed also. Taking the max displacement and the dynamic impact factor of the midpoint of the beam as indices, several parameters such as moving trains velocities, damping ratios and span lengths of the beam, are chosen to investigate the effect of moving trains on the dynamic responses of the beam. Different from simple supported beam, the two-span continuous beam has two critical velocities causing two resonance responses, which depends on the first and second natural frequency of the beam and the moving velocity. From resonance view, the results are useful for suppressing the vibration of the two-span continuous beam under moving high speed trains.

2 FORMULATION

2.1 Basic considerations

Generally, in real situation the moving trains can not keep in contact with the beam all the time. Lee discussed the onset of separation between a moving mass and a beam [7], and Cheng et al. did further investigations to study the onset of separation and reestablishment of contact between the moving vehicle and the bridge [2]. However, when the mass of the wheel is much less than the beam, it is can be assumed that the moving trains always keep in contact with the beam. Cheng et al. also used bridge-track-vehicle element to study the vibration of railway bridges under moving trains with the contact model [1]. M. Ziyaeifar used the train and bridge (track) contact model to investigate the vibration of the railway bridge and its control under a Maxwell (three-element type) vehicle model [20, 21]. In this paper, it is supposed that the moving trains always keep in contact with the two-span continuous beam. The following assumptions are also made for the formulation of the vibration problem of the

two-span continuous beam subjected to moving trains as in Figure 1:

1. The moving trains are modeled as a series of two degrees-of-freedom mass-spring-damper systems at the axle locations, and each mass-spring-damper system consists of a sprung mass and an unsprung mass interconnected by a spring and a dashpot.
2. The two-span continuous beam is modeled as an elastic Bernoulli-Euler beam with uniform span length.
3. The high speed trains travel at a constant speed v. And for the initial conditions, the first mass-spring-damper system is located at the left-hand end of the continuous beam.

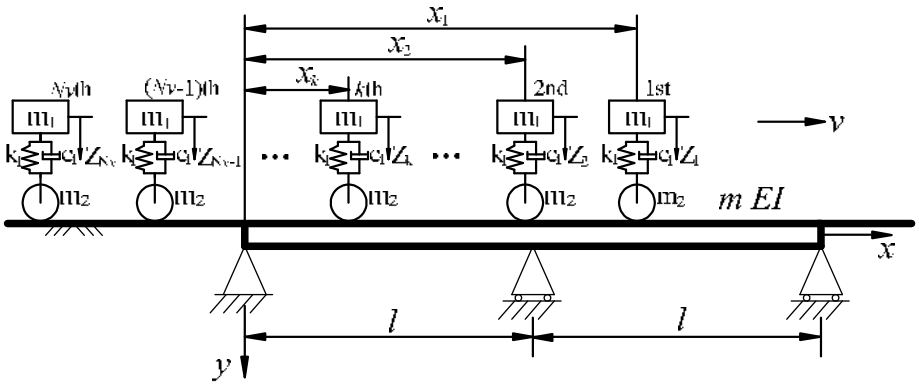


Figure 1 Two-span continuous beam under moving trains.

2.2 Vehicle

As shown in Figure 1, it is assumed that there are N_v moving mass-spring-damper systems in direct contact with the beam. For the typical kth moving mass-spring-damper system, the sprung mass m_1 and the unsprung mass m_2 are interconnected by a spring of stiffness k_1 and a dashpot of damping coefficient c_1. The motion of unsprung mass is constrained by the displacement of the beam, so its vertical displacement is also the beam vertical displacement where the kth system is located. When the kth mass-spring-damper system runs on the two-span continuous beam, the motion equation of the sprung mass m_1 can be written as

$$m_1\ddot{Z}_k(t) + c_1\left[\dot{Z}_k(t) - \dot{y}(x_k,t)\right] + k_1\left[Z_k(t) - y(x_k,t)\right] = 0 \tag{1}$$

where m_1 is the mass of sprung mass, Z_k and $y(x_k,t)$ are vertical displacements of sprung mass and unsprung mass, dots represent differentiation with respect to time t.

2.3 Two-span continuous beam

As shown in Figure 1, a two-span continuous beam with each span length l subjected to a series of mass-spring-damper systems is analyzed. Suppose the distance of the kth mass-

spring-damper system from the left-hand end of the continuous beam is x_k. Then the motion equation for moving trains on the two-span continuous beam can be written as

$$EIy''''(x,t) + m\ddot{y}(x,t) + c\dot{y}(x,t) = F(x,t) \tag{2}$$

where m is the constant mass per unit length, EI is the bending stiffness, c the damping of the beam, $y(x, t)$ the displacement of the beam, and dots and primes represent differentiation with respect to time t and coordinate x.

$$\begin{aligned} F(x,t) = \sum_{k=1}^{N_v} &\left\{(m_1+m_2)g - m_2\ddot{y}(x_k,t) + c_1\left[\dot{Z}_k(t) - \dot{y}(x_k,t)\right] + k_1\left[Z_k(t) - y(x_k,t)\right]\right\} \\ &\cdot \delta(x - x_k)\cdot\left[H(t-t_k) - H(t-t_k-\Delta t)\right] \end{aligned} \tag{3}$$

Here, t_k the arriving time of the kth mass-spring-damper system at the beam, $t_k = x_k/v$. Δt the time of the load passing the beam, $\Delta t = 2l/v$, $H(\cdot)$ is a unit step function, and $\delta(\cdot)$ is the Dirac delta function.

Based on the modal superposition method, the solution of Eq. (2) can be expressed as

$$y(x,t) = \sum_{n=1}^{\infty} q_n(t)\phi_n(x) \quad (n = 1, 2\cdots\infty) \tag{4}$$

where $q_n(t)$ is the nth modal amplitude and $\phi_n(x)$ is the nth mode shape function of the beam.

Substituting Eq. (4) and Eq. (3) into Eq. (2), and multiplying by ϕ_n and integrating the resultant equation with respect to x between 0 and $2l$; and then applying the orthogonality conditions, the equation of motion in terms of the modal displacement $q_n(t)$ is given as:

$$\ddot{q}_n(t) + 2\xi_n\omega_n\dot{q}_n(t) + \omega_n^2 q_n(t) = \frac{F_n(t)}{M_n} \tag{5}$$

where ω_n, ξ_n, $F_n(t)$ and M_n are the modal frequency, the damping ratio, the generalized force and the modal mass of the nth mode, respectively, and

$$\begin{aligned} F_n(t) = \sum_{k=1}^{N_v} &\left\{(m_1+m_2)g\phi_n(x_k) - m_2\sum_{i=1}^{\infty}\ddot{q}_i(t)\phi_i(x_k)\phi_n(x_k) + \left[k_1 Z_k(t) + c_1\dot{Z}_k(t)\right]\phi_n(x_k)\right. \\ &\left. - \sum_{i=1}^{\infty}\left[k_1 q_i(t) + c_1\dot{q}_i(t)\right]\phi_i(x_k)\phi_n(x_k)\right\}\cdot\left[H(t-t_k) - H(t-t_k-\Delta t)\right] \end{aligned} \tag{6}$$

$$M_n = \int_0^{2l} m\phi_n^2(x)dx \tag{7}$$

Subsequently, substituting Eq. (6) into Eq. (5), the motion equation of the two-span continuous beam can be written as

$$\ddot{q}_n(t) + \frac{m_2}{M_n}\sum_{k=1}^{N_v}\sum_{i=1}^{\infty}\ddot{q}_i(t)\,\Phi_{ink}\tilde{H}_k + 2\xi_n\omega_n\dot{q}_n(t) + \frac{c_1}{M_n}\sum_{k=1}^{N_v}\sum_{i=1}^{\infty}\dot{q}_i(t)\,\Phi_{ink}\tilde{H}_k$$
$$+\omega_n^2 q_n(t) + \frac{k_1}{M_n}\sum_{k=1}^{N_v}\sum_{i=1}^{\infty}q_i(t)\,\Phi_{ink}\tilde{H}_k - \frac{k_1}{M_n}\sum_{k=1}^{N_v}Z_k(t)\Phi_{nk}\tilde{H}_k - \frac{c_1}{M_n}\sum_{k=1}^{N_v}\dot{Z}_k(t)\Phi_{nk}\tilde{H}_k \qquad (8)$$
$$= \frac{(m_1+m_2)\,g}{M_n}\sum_{k=1}^{N_v}\Phi_{nk}\tilde{H}_k$$

where $\Phi_{ink} = \phi_i(x_k)\,\phi_n(x_k)$, $\Phi_{nk} = \phi_n(x_k)$ and $\tilde{H}_k = [H(t-t_k) - H(t-t_k-\Delta t)]$.

Then substituting Eq. (4) into Eq. (1), the motion equation of the sprung mass m_1 can be written as

$$m_1\ddot{Z}_k(t) + c_1\dot{Z}_k(t) + k_1 Z_k(t) - \sum_{i=1}^{\infty}\left[c_1\dot{q}_i(t) + k_1 q_i(t)\right]\phi_i(x_k) = 0 \qquad (9)$$

2.4 Natural frequency and mode shape functions

The mode shape of the two-span continuous beam can be divided into two groups, which are symmetrical and asymmetrical, respectively [12]. The circular frequency can be identified as

$$\omega_i = \lambda_i^2\sqrt{\frac{EI}{m}} = \begin{cases} \left(\frac{i+1}{2l}\pi\right)^2\sqrt{\frac{EI}{m}} & i = 1,\ 3,\ 5,\ 7,\cdots \\ \left(\frac{0.25+i/2}{l}\pi\right)^2\sqrt{\frac{EI}{m}} & i = 2,\ 4,\ 6,\ 8,\cdots \end{cases} \qquad (10)$$

When i=1,3,5... and $x \in [0, 2l]$, the mode shape is expressed as

$$\phi_i(x) = \sin(\lambda_i x) \qquad (11)$$

When i=2,4,6... and $x \in [0, l]$,

$$\phi_i(x) = \frac{\sin(\lambda_i x)}{\sin(\lambda_i l)} - \frac{\sinh(\lambda_i x)}{\sinh(\lambda_i l)} \qquad (12)$$

When i=2,4,6... and $x \in [l, 2l]$,

$$\phi_i(x) = \cos(\lambda_i x - \lambda_i l) - \cosh(\lambda_i x - \lambda_i l) - [\sin(\lambda_i x - \lambda_i l) - \sinh(\lambda_i x - \lambda_i l)]\cdot\cot(\lambda_i l) \qquad (13)$$

3 SOLUTION

After obtaining the natural frequencies and mode shape functions of the two-span continuous beam, suppose that N_b modes are used in this paper. Combining Eq. (8) and (9), the equations of motion in modal space are given in matrix form as

$$M\ddot{U} + C\dot{U} + KU = F \tag{14}$$

where M, C, K are the mass, damping and stiffness matrices; $\ddot{U}$, $\dot{U}$, U the vectors of accelerations, velocities and displacement respectively; and F the vector of external forces. So U and F are the $(N_b + N_v) \times 1$ dimensional vectors,

$$U = \begin{bmatrix} q_1 & q_2 & \cdots & q_{N_b} & Z_1 & Z_2 & \cdots & Z_{N_v} \end{bmatrix}' \tag{15a}$$

$$F = \begin{bmatrix} \rho_1\psi_1 & \rho_2\psi_2 & \cdots & \rho_{N_b}\psi_{N_b} & 0 & 0 & \cdots & 0 \end{bmatrix}' \tag{15b}$$

where $\rho_n = \frac{(m_1+m_2)g}{M_n}$ and $\psi_n = \sum_{k=1}^{N_v} \Phi_{nk}\tilde{H}_k$.

Especially, M, C, K can be assembled with four matrices.

$$M = \begin{bmatrix} M_{bb} & M_{bv} \\ M_{vb} & M_{vv} \end{bmatrix}, \tag{16a}$$

$$C = \begin{bmatrix} C_{bb} & C_{bv} \\ C_{vb} & C_{vv} \end{bmatrix}, \text{ and} \tag{16b}$$

$$K = \begin{bmatrix} K_{bb} & K_{bv} \\ K_{vb} & K_{vv} \end{bmatrix}. \tag{16c}$$

For mass matrix M,

$$M_{bb} = \begin{bmatrix} 1 + \Delta_1\Theta_{11} & \Delta_1\Theta_{21} & \cdots & \Delta_1\Theta_{N_b1} \\ \Delta_2\Theta_{12} & 1 + \Delta_2\Theta_{22} & \cdots & \Delta_2\Theta_{N_b2} \\ \cdots & \cdots & \ddots & \cdots \\ \Delta_{N_b}\Theta_{1N_b} & \Delta_{N_b}\Theta_{2N_b} & \cdots & 1 + \Delta_{N_b}\Theta_{N_bN_b} \end{bmatrix}_{N_b \times N_b}, \tag{17a}$$

$$M_{vv} = \begin{bmatrix} m_1 & & & \\ & m_1 & & \\ & & \ddots & \\ & & & m_1 \end{bmatrix}_{N_v \times N_v}, \tag{17b}$$

$$M_{bv} = 0, \tag{17c}$$

$$M_{vb} = 0. \tag{17d}$$

For damping matrix C,

$$C_{bb} = \begin{bmatrix} 2\xi_1\omega_1 + \Gamma_1\Theta_{11} & \Gamma_1\Theta_{21} & \cdots & \Gamma_1\Theta_{N_b1} \\ \Gamma_2\Theta_{12} & 2\xi_2\omega_2 + \Gamma_2\Theta_{22} & \cdots & \Gamma_2\Theta_{N_b2} \\ \cdots & \cdots & \ddots & \cdots \\ \Gamma_{N_b}\Theta_{1N_b} & \Gamma_{N_b}\Theta_{2N_b} & \cdots & 2\xi_{N_b}\omega_{N_b} + \Gamma_{N_b}\Theta_{N_bN_b} \end{bmatrix}_{N_b \times N_b}, \tag{18a}$$

$$C_{vv} = \begin{bmatrix} c_1 & & & \\ & c_1 & & \\ & & \ddots & \\ & & & c_1 \end{bmatrix}_{N_v \times N_v}, \tag{18b}$$

$$C_{bv} = \begin{bmatrix} -\Gamma_1\Psi_{11} & -\Gamma_1\Psi_{12} & \cdots & -\Gamma_1\Psi_{1N_v} \\ -\Gamma_2\Psi_{21} & -\Gamma_2\Psi_{22} & \cdots & -\Gamma_2\Psi_{2N_v} \\ \cdots & \cdots & \ddots & \cdots \\ -\Gamma_{N_b}\Psi_{N_b1} & -\Gamma_{N_b}\Psi_{N_b2} & \cdots & -\Gamma_{N_b}\Psi_{N_bN_v} \end{bmatrix}_{N_b \times N_v}, \tag{18c}$$

$$C_{vb} = \begin{bmatrix} -c_1\Phi_{11} & -c_1\Phi_{21} & \cdots & -c_1\Phi_{N_b1} \\ -c_1\Phi_{12} & -c_1\Phi_{22} & \cdots & -c_1\Phi_{N_b2} \\ \cdots & \cdots & \ddots & \cdots \\ -c_1\Phi_{1N_v} & -c_1\Phi_{2N_v} & \cdots & -c_1\Phi_{N_bN_v} \end{bmatrix}_{N_v \times N_b}. \tag{18d}$$

For stiffness matrix K,

$$K_{bb} = \begin{bmatrix} \omega_1^2 + \Lambda_1\Theta_{11} & \Lambda_1\Theta_{21} & \cdots & \Lambda_1\Theta_{N_b1} \\ \Lambda_2\Theta_{12} & \omega_2^2 + \Lambda_2\Theta_{22} & \cdots & \Lambda_2\Theta_{N_b2} \\ \cdots & \cdots & \ddots & \cdots \\ \Lambda_{N_b}\Theta_{1N_b} & \Lambda_{N_b}\Theta_{2N_b} & \cdots & \omega_{N_b}^2 + \Lambda_{N_b}\Theta_{N_bN_b} \end{bmatrix}_{N_b \times N_b}, \tag{19a}$$

$$K_{vv} = \begin{bmatrix} k_1 & & & \\ & k_1 & & \\ & & \ddots & \\ & & & k_1 \end{bmatrix}_{N_v \times N_v}, \tag{19b}$$

$$K_{bv} = \begin{bmatrix} -\Lambda_1\Psi_{11} & -\Lambda_1\Psi_{12} & \cdots & -\Lambda_1\Psi_{1N_v} \\ -\Lambda_2\Psi_{21} & -\Lambda_2\Psi_{22} & \cdots & -\Lambda_2\Psi_{2N_v} \\ \cdots & \cdots & \ddots & \cdots \\ -\Lambda_{N_b}\Psi_{N_b1} & -\Lambda_{N_b}\Psi_{N_b2} & \cdots & -\Lambda_{N_b}\Psi_{N_bN_v} \end{bmatrix}_{N_b \times N_v}, \tag{19c}$$

$$K_{vb} = \begin{bmatrix} -k_1\Phi_{11} & -k_1\Phi_{21} & \cdots & -k_1\Phi_{N_b1} \\ -k_1\Phi_{12} & -k_1\Phi_{22} & \cdots & -k_1\Phi_{N_b2} \\ \cdots & \cdots & \ddots & \cdots \\ -k_1\Phi_{1N_v} & -k_1\Phi_{2N_v} & \cdots & -k_1\Phi_{N_bN_v} \end{bmatrix}_{N_v \times N_b}. \tag{19d}$$

where $\Delta_n = \frac{m_2}{M_n}$, $\Gamma_n = \frac{c_1}{M_n}$, $\Lambda_n = \frac{k_1}{M_n}$, $\Theta_{in} = \sum_{k=1}^{N_v} \Phi_{ink}\tilde{H}_k$, $\Psi_{nk} = \Phi_{nk}\tilde{H}_k$.

To obtain simultaneously the dynamic responses of the two-span continuous beam, the equations of motion as given in Eq. (14) will be solved in a step-by-step integration method using the Newmark-β method [11, 13]. The integration scheme of Newmark-β method consists of the following equations:

$$\left\{\ddot{U}\right\}_{t+\Delta t} = a_0\left(\{U\}_{t+\Delta t} - \{U\}_t\right) - a_2\left\{\dot{U}\right\}_t - a_3\left\{\ddot{U}\right\}_t \tag{20}$$

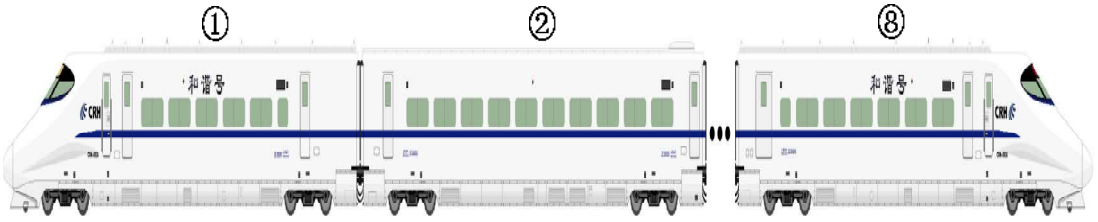
$$\left\{\dot{U}\right\}_{t+\Delta t} = \left\{\dot{U}\right\}_t + a_6\left\{\ddot{U}\right\}_t + a_7\left\{\ddot{U}\right\}_{t+\Delta t} \tag{21}$$

where the coefficients are

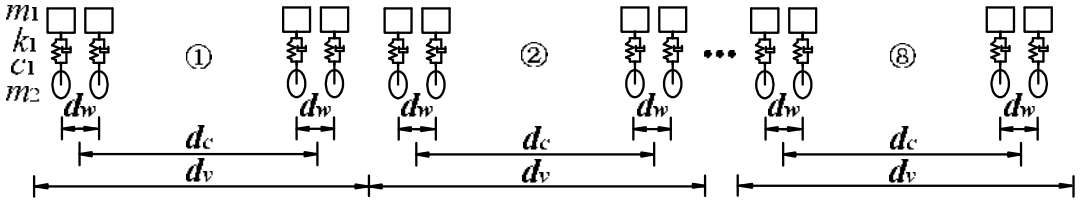
$$\alpha_0 = \frac{1}{\beta\Delta t^2},\ \alpha_1 = \frac{\gamma}{\beta\Delta t},\ \alpha_2 = \frac{1}{\beta\Delta t},\ \alpha_3 = \frac{1}{2\beta} - 1,$$
$$\alpha_4 = \frac{\gamma}{\beta} - 1,\ \alpha_5 = \frac{\Delta t}{2}\left(\frac{\gamma}{\beta} - 2\right),\ \alpha_6 = \Delta t(1-\gamma),\ \alpha_7 = \gamma\Delta t. \tag{22}$$

In this study, β=1/4 and γ=1/2 are selected, which implies a constant acceleration with unconditional numerical stability.

4 NUMERICAL INVESTIGATION



(a) CRH high speed train



(b) Axle arrangements of CRH high speed train model

Figure 2 CRH high speed train and axle arrangements.

Consider a two-span continuous beam with l=20m, EI=2.5×10^{10} Nm^2, m=3.4088×10^4 kg/m, ξ=0.025. CRH (China Railway High-speed) high speed train consisting of eight cars, as shown in Figure 2(a), is used as the external moving trains acting on the beam. Figure 2(b) shows the axle arrangements of CRH high speed train model, with the full length of each car d_v=25m, the rated distance between the two bogies of a car d_c=17.5m, and the fixed distance between the two wheel-axles of a bogie d_w=2.5m. Each wheel assembly is modelled as an equivalent 2DOF system with a sprung mass m_1=9500kg, an unsprung mass m_2=3300kg, a spring with stiffness k_1=2.5×10^5 N/m and a dashpot with damping coefficient c_1=4.5×10^4 Ns/m. The properties of the two-span continuous beam and the moving CRH high speed train model are also listed in Table 1 and 2.

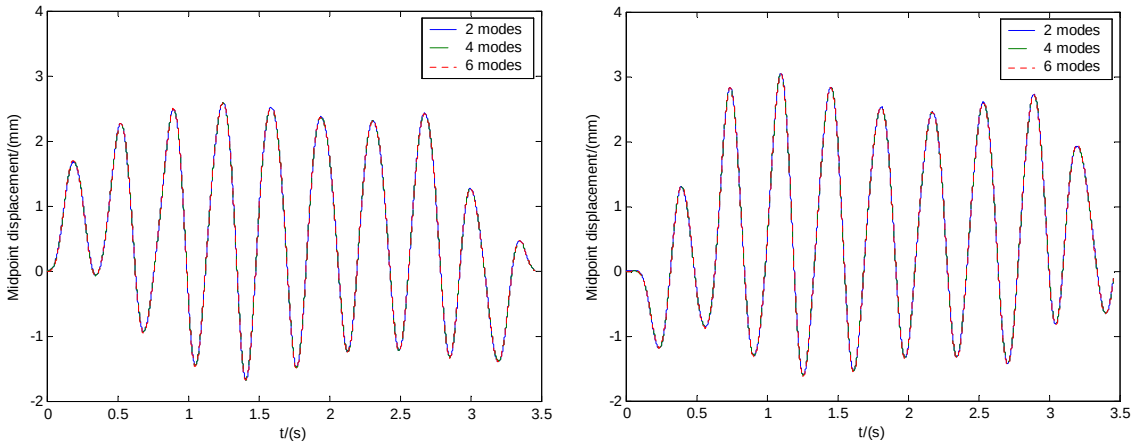
Table 1 Properties of the two-span continuous beam.

l(m)	EI(Nm^2)	m(kg/m)	ξ	ω_1 (rad/s)	ω_2 (rad/s)
20	2.5×10^{10}	3.4088×10^4	0.025	21.13	33.02

Table 2 Properties of the moving CRH high speed train model.

N_v	d_v(m)	d_c(m)	d_w(m)	m_1(kg)	m_2(kg)	k_1(N/m)	c_1(Ns/m)
4×8	25	17.5	2.5	9500	3300	2.5×10^5	4.5×10^4

Figure 1 shows a two-span continuous beam with each span length l subjected to a series of mass-spring-damper systems at constant speed v and Figure 2 shows the moving CRH high speed train model. To compute the dynamic response of the two-span continuous beam under moving trains, the number of modes of the beam should be considered first. In order to verify that a sufficient number of modes has been used in the analysis, we first compute the displacement response at the midpoint of each span of the two-span continuous beam under the CRH high speed train at 250km/h using either 2, 4, or 6 modes. From the convergent verification of computed results in Figure 3, the first 6 modes are sufficient to compute the displacement response of the two-span continuous beam under moving trains. For this reason, the same number of modes will be considered in the following examples.



(a) 1st span (b) 2nd span

Figure 3 Test of convergence.

4.1 Effect of moving trains velocities

To compare the resonance characteristics of simple supported beam with the two-span continuous beam, a simple supported beam was chosen with span length l=20m, EI=2.5×10^{10} Nm^2,

m=3.4088×10^4 kg/m, ξ=0.025. Then the displacement responses of the two beams under moving CRH high speed trains were computed. For different moving trains velocities from 5km/h to 600km/h, the max displacement solved for the midpoint of each span of the continuous beam and the simple supported beam has been plotted against the moving trains velocities in Figure 4.

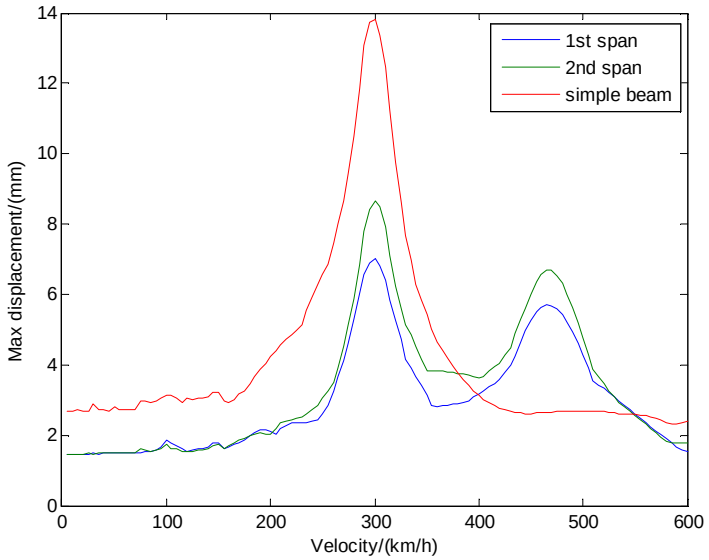
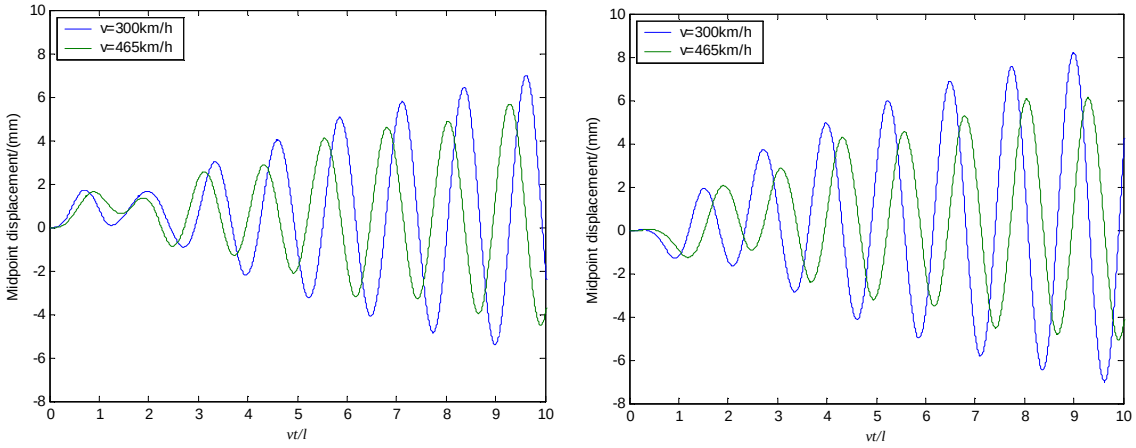


Figure 4 Max displacement for the midpoint of the two beams vs moving trains velocities.

As one can see, for the two-span continuous beam, there exist two resonant peaks for the max displacement response curves of the 1st and 2nd span of the continuous beam. As shown in Figure 4, the two critical velocities of the two-span continuous beam are about 300km/h and 465km/h respectively. To illustrate the resonant phenomena of the two-span continuous beam subjected to the trains traveling at velocities of 300km/h and 465km/h, the time history responses of the midpoint displacement of the beam have been plotted in Figure 5. Both the displacement responses of the 1st and 2nd span are built up continuously as there are more moving loads passing through the beam. This is so called resonance phenomenon for train-induced response of railway bridges [15].

As one can see, for simple-supported beam, there exists only one resonant peak for the max displacement response curves. As shown in Figure 4, the critical velocity of the simple supported beam is about 300km/h. So when the moving high speed trains is certain, the first critical velocity of the two-span continuous beam is the same with the simple supported beam. But at that resonance condition the max displacement of the simple supported beam is much larger than the two-span continuous beam.

Besides, for the two-span continuous beam, it is observed that the max displacement at both resonant speeds for the second span is larger than that of the first span. These conclusions are consistent with the previous literature [18]. This can be attributed to the uplift action by the last moving vehicle from the first span and entance into the second span. For this reason,



(a) 1st span (b) 2nd span

Figure 5 Resonant response of midpoint displacement of 1st and 2nd span.

only the dynamic response of the second span will be considered in the subsequent research.

Resonance is produced by the periodic repetition of moving high speed trains circulating along the bridge. It can be deduced from Figure 2 that the most important characteristic length related to the appearance of resonance must be the length of the vehicle, which has been validated by Museros P. [9]. For moving trains traveling on the beam, the resonant condition of the beam can be derived as follows [13, 14]:

$$v_{brn} = \frac{3.6 \cdot f_{bn} \cdot d}{i}, (n = 1, 2, 3\cdots; i = 1, 2, 3, \cdots) \tag{23}$$

where v_{brn} is the resonant moving trains velocity (km/h); f_{bn} is the nth natural frequency of the beam (Hz); d is the characteristic length of the moving high speed trains (m); i represents the number of complete oscillation cycles for the nth mode of the beam to vibrate during the passage of two adjacent loads [10, 19].

For the two-span continuous beam, the exact natural frequencies corresponding to the first and second modes are 21.13rad/s and 33.02rad/s respectively listed in Table 1, and the characteristic length is taken as the load interval which is approximately 25m. So the first and second resonant moving trains speeds for resonant conditions estimated by Eq. (23) are

$$v_{br1} = \frac{3.6 \cdot f_{b1} \cdot d}{i} = \frac{3.6 \times 21.13 \times 25}{1 \times 2 \times \pi} = 302.67 km/h$$

$$v_{br2} = \frac{3.6 \cdot f_{b2} \cdot d}{i} = \frac{3.6 \times 33.02 \times 25}{1 \times 2 \times \pi} = 472.98 km/h$$

For the simple supported beam, the first natural frequency is 21.13rad/s also. So the moving trains speeds for resonant conditions estimated by Eq. (23) is

$$v_{br1} = \frac{3.6 \cdot f_{b1} \cdot d}{i} = \frac{3.6 \times 21.13 \times 25}{1 \times 2 \times \pi} = 302.67 km/h$$

The resonant moving loads speeds estimated by Eq. (23) are in good accordance with the critical moving loads speeds from the simulated results, as shown in Figure 4. So for the simple supported beam it is evident that the resonant peak is mainly due to coincidence of the excitation frequencies implied by the moving trains at different speeds with the first frequency of the beam. But for the two-span continuous beam, the two resonant peaks are mainly with the first and second frequencies of the beam.

4.2 Effect of damping ratios

Theoretically, the structural damping is difficult to determine. Generally, the range of damping ratios is between 0.01 and 0.06. The damping ratio of the two-span continuous beam has been chosen as 0.01, 0.02, 0.04 and 0.06 to study the effect of damping on the dynamic response of the beam. The moving trains velocity is also from 5km/h to 600km/h, the max displacement solved for the midpoint of the beam has been plotted against the moving trains velocities in Figure 6.

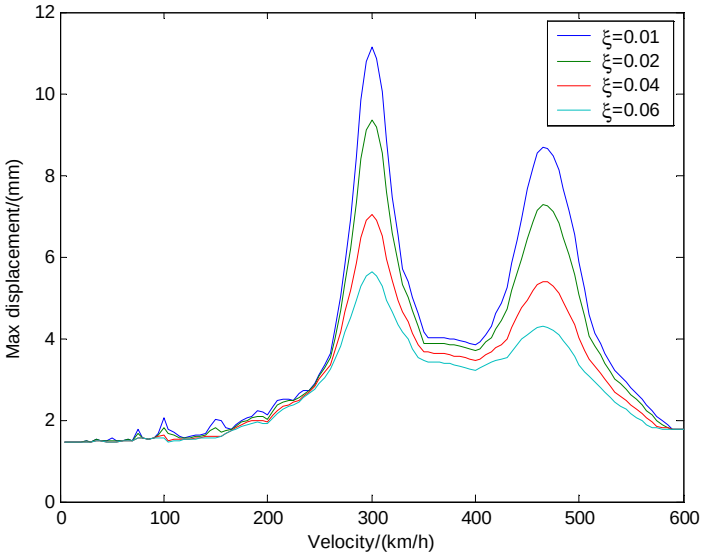


Figure 6 Max displacement of the beam with different damping ratios vs moving trains velocities.

As shown in Figure 6, it is proved that the damping ratio of the two-span continuous beam has little impact on the maximum deflection of the midpoint of the beam when moving trains travel except at the critical velocities. But in resonance condition, the damping of the beam can suppress the vibration of the beam obviously, and with the increase of the damping ratio the max deflection of the beam decreases rapidly. When the damping ratio ξ is 0.01, 0.02, 0.04 and 0.06 respectively, the critical velocities of the two-span continuous beam are approximate, as 300km/h and 465km/h. So damping ratio does not affect the critical velocity

of the continuous beam, and it only affects the max deflection of the beam. Taking into the effect of the damping ratios on the continuous beam resonance is little, in the following research the damping ratio ξ=0.025 is still used.

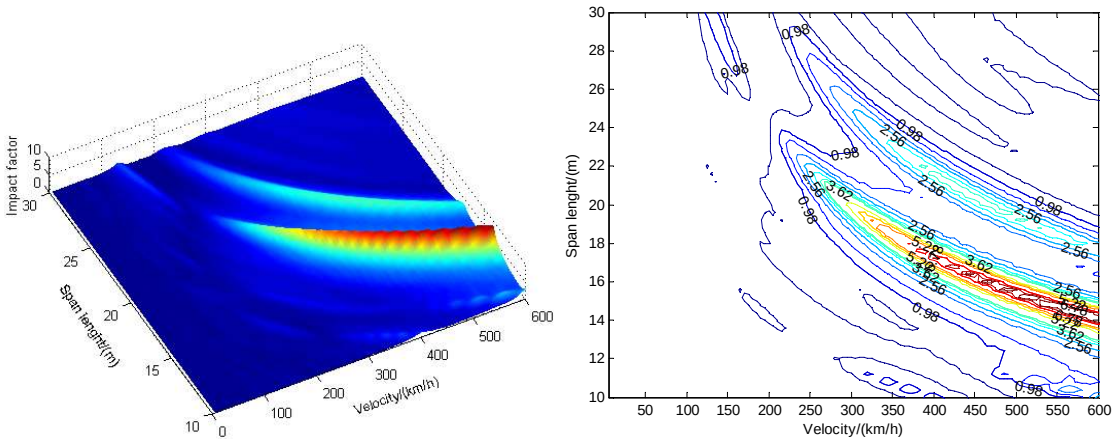
4.3 Effect of span length

In order to study the effect of different span length on the dynamic responses of the two-span continuous beam, the dynamic impact factor of the midpoint of the beam is taken as index. The dynamic impact factor for the displacement of the two-span continuous beam subjected to the moving trains is defined as [15]

$$I = \frac{y_d(x) - y_s(x)}{y_s(x)} \tag{24}$$

where $y_d(x)$ and $y_s(x)$ denote the max dynamic and static displacement of the two-span continuous beam at position x due to the action of moving trains respectively. In this paper, when the high speed trains move at a much lower velocity, such as 0.36km/h, then the displacement of the beam was taken as the static displacement.

As we all know, with different span lengths, the section stiffness and the unit-length mass should be changed. But in this paper, to study the effect of span length to the resonance conditions in theoretically only, the other parameter values of the beam are supposed to be the same with section 4.1. The span length of the continuous beam is chosen from 10m to 30m and the moving trains velocity is from 5km/h to 600km/h also. The dynamic impact factor solved for the midpoint of the beam has been plotted against the span lengths and the velocities in Figure 7(a), with its contour lines given in Figure 7(b).



(a) I-L-v (b) Contour lines

Figure 7 Effects of span lengths and velocities on the dynamic impact factor of the midpoint of the beam.

As one can see from Figure 7(a) and 7(b), two resonance bands were excited for the two-span continuous beam under moving trains. With the decrease of span length, the resonance

bands of the two-span continuous beam moves shift to higher moving trains velocities areas, and at the same time the vibration intensified strongly. When the span length is certain, there are two critical velocities, under which the dynamic impact factor of the beam is large, and the resonance occurred. So to suppress the vibration of the two-span continuous beam under moving high speed trains, the two resonances should be considered together.

5 CONCLUSIONS

In this paper according to the vibration differential equation of two-span continuous beam under moving trains, the resonance characteristics of the beam is investigated. Then the influences of moving load velocities, damping ratios and span lengths of the beam to the vibration of the two-span continuous beam were studied in detail. On the base of the study, the following conclusions can be drawn:

1. The two-span continuous beam has two critical velocities causing the resonance response, which is different from simple supported beam. These two resonances are mainly due to the coincidence of the excitation frequencies implied by the moving trains at different speeds with the first and second natural frequency of the continuous beam.

2. The damping ratios of the two-span continuous beam has great impact on the maximum deflection merely on the conditions of moving trains at the critical velocity, otherwise, it has little effect on suppressing the vibration of the beam. On the resonance condition, the bigger damping ratios lead to the smaller deflection.

3. The two-span continuous beam has two resonance bands. So to control the vibration of the two-span continuous beam, the two resonance should be considered together. From resonance view, the results are useful for suppressing the vibration of the two-span continuous beam under moving high speed trains.

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