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A hybrid technique for residual stress analysis through stress function fitting

Abstract

This work presents a hybrid experimental theoretical technique for residual stresses analysis of mechanical components. It was used the blind hole technique along with the Electronic Speckle Pattern Interferometry (ESPI) method and Airy stress functions for the determination of the principal stress components. Displacement information is fitted in order to extract Airy stress function coefficients. The results demonstrate that this technique is of great utility to determine the average principal stress and local stress gradients.

Keywords

residual stress, experimental mechanics, stress function, blind hole technique

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1 INTRODUCTION

The structural analysis of mechanical components is of vital importance for the industry [2, 4]. In general, Residual stresses are a type of mechanical stress that could compromise the normal life of mechanical systems. This kind of stress is even worse when combined with the local stresses inside the structure. This work studies a different form to quantify residual stresses in structural components.

Residual stresses are, by definition, internal stresses self-equilibrated presented in a free body. Residual stress is normally originated from the manufacturing and improvement processes and can be tensile or compressive varying from point to point.

This work has the objective to determinate the behavior of the residual stresses components in a previously welded region of piping system [5].

It was used the blind hole technique associated to the Electronic Speckle Pattern Interferometry (ESPI) [5] method for the determination of the stress field. Displacement data obtained experimentally through ESPI technique are combined with specific mathematical models from the theory of elasticity for local stress determination. Experimental data are acquired using ESPI technique, with radial sensitivity. Displacement information is collected from a predefined mesh around the blind hole. Stress functions coefficients are determined from the experimental data and displacement information is fitted to extract stress function coefficients.

This hybrid technique demonstrates to be very useful to determine the average principal stress and directions along with clean information about the displacement field around the drilled hole.

2 STRESS FUNCTIONS

For plane elastic problems, a function of r, and θ can be developed which automatically satisfies the equilibrium and compatibility.

If we choose a combination of analytic functions [3, 6, 10] such that

$$\Phi = \Phi^{(0)} + \Phi^{(1)} + \Phi^{(n)} + \Phi^{(*)} \tag{1}$$

where,

$$\Phi^{(0)} = a_0 + b_0 \ln(r) + c_0 r^2 + d_0 r^2 \ln(r) \tag{2}$$

$$\Phi^{(1)} = \left(a_1 r + \frac{b_1}{r} + c_1 r^3 + d_1 \ln(r)\right) \cdot \left\{ \begin{array}{c} \sin\theta \\ \cos\theta \end{array} \right\} \tag{3}$$

$$\Phi^{(n)} = \left(a_n r^n + b_n r^{-n} + c_n r^{2+n} + d_n r^{2-n}\right) \cdot \left\{ \begin{array}{c} \sin\theta \\ \cos\theta \end{array} \right\} \quad \text{for } n \geq 2 \tag{4}$$

$$\Phi^{(*)} = a_* \theta + b_* r^2 \theta + c_* r\theta \cdot sin\theta + d_* r\theta \cdot \cos\theta \tag{5}$$

One can assume that equation (6) well represents the displacement field around the drilled hole,

$$U_r(r,\theta) = \xi r + \delta r \cdot \left\{ \begin{array}{c} \sin\theta \\ \cos\theta \end{array} \right. + \left(\alpha r + \omega r^{-1} + \gamma r^{-3}\right) \cdot \left\{ \begin{array}{c} \sin 2\theta \\ \cos 2\theta \end{array} \right. \tag{6}$$

where ξ,δ, α, ω and γ, are constants.

If we look at the Kirsh solution [6] for a plate with a circular hole subjected to bi-directional tensile loads.

3 DETERMINATION OF COEFFICIENTS

The displacement data is collected using a circular mesh around the blind hole. Points are distributed in concentric circles using polar coordinates. Data are collected in chunks of 1024 points, located in 8 circles representing a diameter of 8 mm.

The calculation of the principal stresses follows the relationships between the stresses and displacement for biaxial uniform residual stress field around the drilled hole. To obtain the principal stress and direction, it is used the relation [1, 7, 8]

$$u_r(r,\theta) = A(r) \cdot \left(\sigma_1^R - \sigma_2^R\right) \cos(2\theta - 2\beta) + B(r) \cdot \left(\sigma_1^R + \sigma_2^R\right) \tag{7}$$

where,

$$A(r) = -\frac{1+\upsilon}{2E} r \cdot \left[\left(\frac{r_o}{r} \right)^4 - \frac{4}{1+\upsilon} \left(\frac{r_o}{r} \right)^2 \right]$$
$$B(r) = \frac{1+\upsilon}{2E} r \cdot \left(\frac{r_o}{r} \right)^2 \tag{8}$$

u_r is the radial component of the deformation;
σ_1^R, σ_2^R are the principal stresses;
β, is the principal direction;
r, θ are the coordinates of the point that it is being analyzed.
Observing equation (6) and (7) one can extract that:
α, δ, are constants;

$$\xi = (\sigma_1 + \sigma_2) r B(r)$$
$$\omega = (\sigma_1 - \sigma_2) \cdot [\cos(2\beta) + \sin(2\beta)] \cdot r A(r)$$
$$\gamma = (\sigma_1 - \sigma_2) \cdot [\cos(2\beta) + \sin(2\beta)] \cdot r^{-3} A(r) \tag{9}$$

In order to determine the coefficients of the stress function (6) a non-linear system is created:

$$G = -[U_r(r,\theta) - U_{re}] \tag{10}$$

where,
$U_r(r,\theta)$ is the equation (6);
U_{re} is the vector of the radial displacement data from holography.
The solution of the system is found in a least square sense for the coefficients X:

$$dX = (J`J)^{-1} \cdot J`G \tag{11}$$

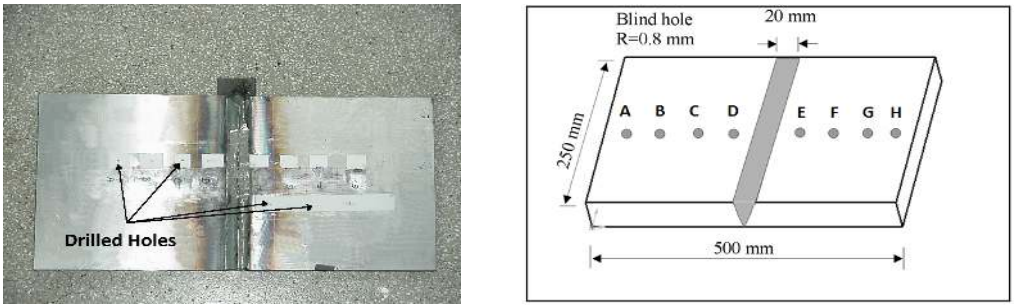
where,
J is the Jacobian matrix;
dX is the vector of increments.

4 APPLICATION

For the demonstration of the method a welded specimen made from 1020 steel was used. A series of 8 holes were located in the middle of the plate, perpendicular to the welding, with a depth of approximately 0.8 mm. Fig. 1 shows a sketch and an actual view of the welded specimen.

After the hole is drilled the displacements are acquired using the ESPI technique [9, 11], typically recording 1024 points from 4 circles centered with the hole, with radius ranging from

1.5mm to 4mm, Fig. 2. The displacements field is obtained after processing the interference images, Fig. 3. The collected data of the radial component of the displacement and its respective polar coordinates are disposed in a matrix and stored in a text file.



(a) (b)

Figure 1 Actual view (a) and sketch (b) of the specimen used for the ESPI blind-hole technique.

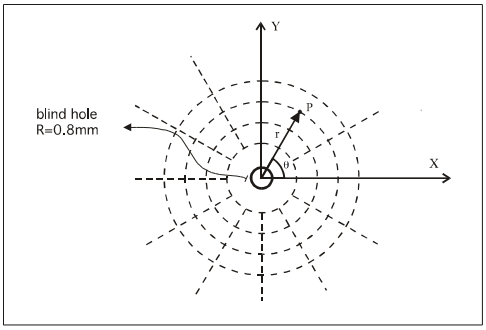


Figure 2 Used mesh for collecting data around the drilled hole.

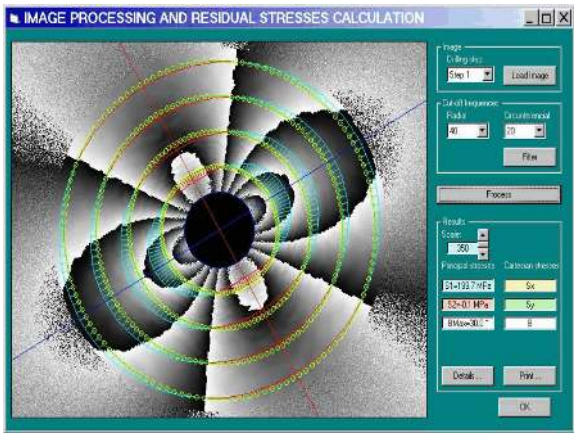


Figure 3 Displacement analysis screen.

Data from the experimental displacement field is fitted by the stress functions resulting in a equivalent system. The Airy's Stress Functions coefficients from (1) and (9) are individualized and a smooth displacement field is obtained.

Below, are represented two images that correspond to the displacement field around the drilled hole. The first one was obtained directly by the ESPI technique and the other is calculated by the method of the Airy's Stress functions. It is clear that the noise presented in the image of Fig. 4 is removed by the application of the stress coefficients in Fig. 5.

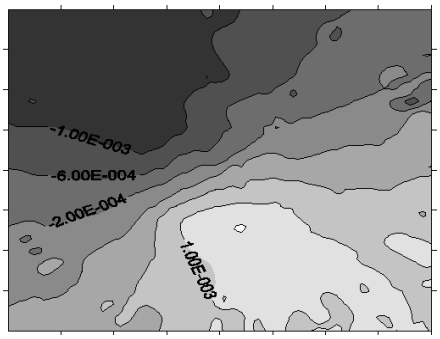


Figure 4 Displacement data obtained from the ESPI technique.

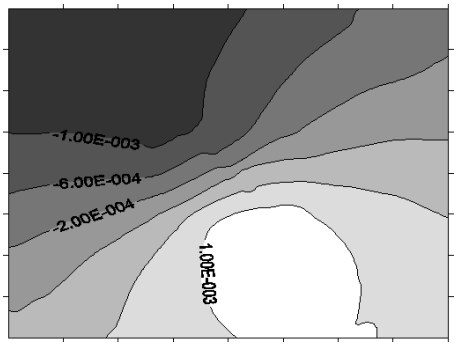


Figure 5 Displacement data from the *Airy's* stress functions coefficients.

Figure 6 below compares the values of the principal stresses from the ESPI technique and the strain-gage analysis using the RS-200 from Micro-Measurements Group (MM) to the one obtained using the Airy's Stress Functions coefficients.

The harmonics technique uses the direct determination of $A(r)$ and $B(r)$ from equation (7) [11]. The results showed in Fig. (5) for the Stress Functions method have taken into account the blind hole corrections from ASTM E 837-95.

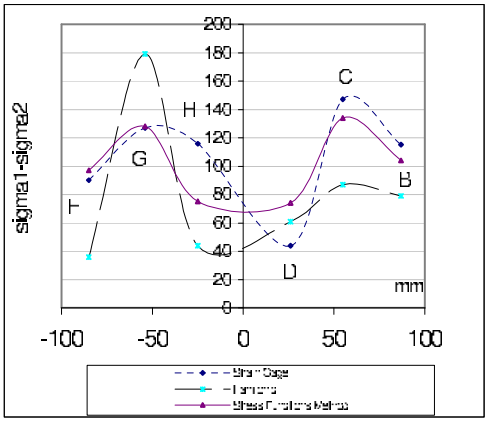


Figure 6 Comparison of difference of principal stress.

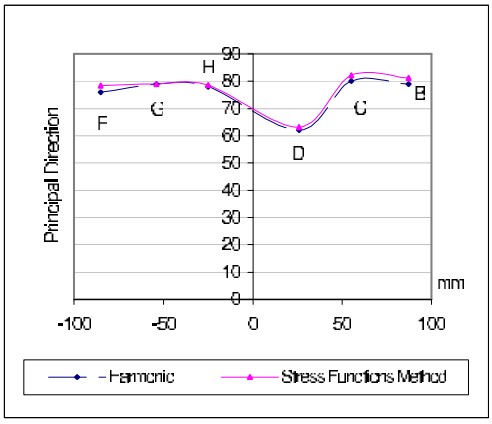


Figure 7 Principal stress directions using harmonics [11] and Stress Functions Method.

5 CONCLUSIONS

A simple but powerful method is presented for the determination of the residual stresses in mechanical components. This hybrid technique uses a modern method for experimental analysis of stresses and strain using numerical procedures, experimental data and the classical Theory of Elasticity. A practical example is presented where a welded specimen is evaluated by the method of the coefficients. A good agreement was found between classical strain gage analysis and the proposed technique.

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