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Modal Parameter Extraction of a Huge Four Stage Centrifugal Compressor Using Operational Modal Analysis Method

Abstract
In recent years, modal analysis has become one of the essential methods for modification and optimization of dynamic characteristics of engineering structures. This is the first published study to identify modal parameters of a complex four-stage centrifugal compressor using Operational Modal Analysis (OMA). Vibrational response was measured continuously with sampling frequency of 44100(Hz) by four noncontact eddy current sensors. Applied loads in actual working condition during compressor's operation were considered as excitation forces. In this study, modal parameters were extracted and compared using various OMA methods, including Frequency Domain Decomposition (FDD), Enhanced Frequency Domain Decomposition (EFDD) and Stochastic Subspace Identification (SSI). PULSE™ commercial software as well as an in-house MATLAB code employed to data analysis. The results show that SSI method has a higher accuracy compared to FDD and EFDD methods. However, FDD shows better results when system damping is low in one of the modes.

Keywords
operational modal analysis, harmonic component, modal parameters, centrifugal compressor.

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1 INTRODUCTION

Experimental Modal Analysis (EMA) is an empirical technique used to determine the modal model of a linear time invariant vibrational system. Modal model is determined using Frequency Response Functions (FRF) or by measuring free vibrations of the system. The collection of all measured frequency responses is structural response model. Extracting modal parameters is the same as going from structural response model to modal model. In experimental modal analysis, excitation force and structural responses are measured. In many real structures, it is not possible to apply EMA to determine modal parameters. These structures are effectively excited by forces created by their own operation which are impossible to measure. Therefore, other methods were created which do not need to measure excitation forces in order to determine modal parameters. These methods are Operational Modal Analysis (OMA) or Output-only modal analysis and are carried out by measuring structure's response during normal working conditions. Among identification methods in OMA are methods based on random space and frequency domain analysis. The basis of all identification methods in OMA is the assumption of randomness of excitation forces. This can limit the application of OMA in rotating mechanical structures. In these types of equipment, other than random background stimulation, rotational harmonics of the equipment are also present in exciting forces and invalidate the assumption of randomness. In 1990s, a novel method for identification of state space of systems based on subspaces was proposed in control engineering. Based on this method, in 1993, Van Overschee et al. proposed stochastic subspace method using structure's response to random excitation ((Overschee and Moor, 1996); (Overschee and Moor, 1993)). In 2008, Reynders and Roeck used a combination of random and determined methods for experimental and operational modal analysis and evaluated the performance of this method using data gathered from a bridge. Along with speed and accuracy, this method had managed to surpass some of the limitations of operational modal analysis method. Another work by the same authors proposed a bias removal method based on stability diagram and variance error estimation using first order sensitivity of modal parameters determined by stochastic subspace method. They verified the accuracy and performance of this

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method by simulated and experimental data ((Reynders and Roeck, 2008); (Reynders et al., 2008)). Cara et al. (2013) proposed a novel method for identification of system modes using modal participation calculated by Kalman filter. Frequency Domain Decomposition (FDD) is one of the applied methods in frequency domain. It is based on relation between output and input power spectrum density matrices of a random process (Bendat and Piersol, 2010). Simplest method in the frequency domain is peak selection method which was proposed first in 1970 by Mclamore et al. for environmental vibrations of suspension bridges (Mclamore et al., 1971). This method was then organized by Felber in 1993 which led to development of modal identification applications (Cunha et al., 2005). The main advantage of this method compared to other similar methods is its speed and ease of use. In 2000, Brincker et al proposed a criterion for identification and separation of structure's real modes from its harmonic modes for the first time. This criterion is based on fundamental difference between statistical characteristics of harmonic and random responses in a narrow band near structural modes (Brincker et al., 2000a). Pintelon et al. (2010) proposed a single-output continuous time operational modal analysis algorithm in order to reduce disturbances caused by harmonic stimulations with varying frequencies. Agneni et al. (2012) proposed a method based on entropy. Weijtjens et al. (2014) proposed multi-reference transmissibility method based on operational modal analysis.

This study aims to investigate the performance of common operational modal analysis algorithms for structural modal identification of a complex four-stage centrifugal compressor in presence of harmonic excitations. In order to extract modal parameters, first these parameters were extracted through a commercial operational modal analysis software. Subsequently, the data was fed to some developed codes in MATLAB and the results were compared to each other.

2 MODAL IDENTIFICATION ALGORITHMS IN OPERATIONAL CONDITIONS

Frequency Domain Decomposition (FDD), Enhanced Frequency Domain Decomposition (EFDD) and Stochastic Subspace Identification (SSI) methods are the most common modal identification algorithms. These methods are introduced in the following section.

2.1 Frequency Domain Decomposition

Frequency decomposition method uses Singular Value Decomposition (SVD) on power spectrum density matrix to decompose it to a set of density functions of the spectrum. Each function is equal to one degree of freedom. Assumptions of this method include white noise in excitation signal, low decay and orthogonality of mode shapes for close modes. Deviation from these assumptions reduces the accuracy of results (Brincker et al., 2000b). In this method, the first step is estimation of power spectrum density matrix. Estimation of known output power spectrum density $\hat{G}_{yy}\left(j\omega\right)$ in different frequencies $\omega=\omega_i$ is carried out by singular value decomposition:

$$\widehat{\boldsymbol{G}_{yy}}\left(\boldsymbol{j\omega_i}\right)=\boldsymbol{U_i}\boldsymbol{S_i}\boldsymbol{U_i^H} \tag{1}$$

In which $U_i=\lfloor u_{i1},u_{i2},\ldots,u_{im}\rfloor$ is the unity matrix made up from u_{ij} singular vectors and S_i is a diagonal matrix made from singular scalar values of S_{ij}.

2.2 Enhanced Frequency Domain Decomposition (EFDD)

Enhanced frequency domain decomposition method is the improved version of frequency domain decomposition method which offers the possibility of estimating damping (Brincker et al, 2001). In EFDD method, the power spectrum density function of an identified degree of freedom will be transferred to time domain using Inverse Fourier Transform. Natural frequency is determined using the number of zero crossing over time and damping by logarithmic decay of normalized correlation function of each degree of freedom. EFDD method is based on decomposition of measured response spectrum. Therefore, in enhanced frequency domain decomposition method, decomposition of singular values is carried out for each measured response in every frequency of $\omega=\omega_i$. This means that power spectrum density estimation of output is as follows:

$$\hat{\boldsymbol{G}}_{\boldsymbol{yy}}\left(\boldsymbol{j\omega}\right)=\boldsymbol{U_i}\boldsymbol{S_i}\boldsymbol{U_i^H}=\sum_{\boldsymbol{k}}^{\boldsymbol{n_y}}\boldsymbol{u_{ki}}\boldsymbol{u_{ki}^H}\boldsymbol{s_{ki}} \tag{2}$$

In the resonant frequency, the rank of $G_{yy}(j\omega)$ will be close to one. Using this method, natural frequencies can be calculated. Damping ratio is also calculated using logarithmic decrement of correlation function (Jacobsen and Andersen, 2008). Mode shapes are determined using weighted sum of singular vectors of different frequencies:

$$\varphi_i = \sum_{\omega=\Omega_i} U_1(\omega) S_i(\omega) \tag{3}$$

In which Ω_i is the set of selected frequencies in the vicinity of mode *i*.

2.3 Time domain stochastic subspace identification (SSI) method

In this method, time domain responses are gathered in a matrix called Hankel matrix which is then divided into two past and future submatrices. In order to create a relation between responses, future matrix is projected into past matrix and projection matrix is created. By decomposition of projection matrix, observation matrix and Kalman states are calculated. Then observation matrix is used to calculate system matrix and then, system poles are determined. Stability diagram is used to calculate the natural frequency, mode shapes and damping coefficients (Brincker and Andersen, 2006). The first step in determination of modal parameters is eigenvalue decomposition of system matrix of $\hat{A}_d$. By calculating eigenvalues and eigenvectors of system matrix, it is possible to calculate modal parameters:

$$\hat{\boldsymbol{A}}_{\boldsymbol{d}} = \boldsymbol{\Psi}\boldsymbol{\mu}_{\boldsymbol{i}}\boldsymbol{\Psi}^{-1} \tag{4}$$

Continuous time poles λ_i can be calculated using discrete time poles of μ_i:

$$\boldsymbol{\mu}_{\boldsymbol{i}} = \boldsymbol{exp}(\boldsymbol{\lambda}_{\boldsymbol{i}}) \tag{5}$$

Which leads to the following equations:

$$\lambda_i = \frac{Ln(\mu_i)}{\Delta T} \qquad \omega_t = |\lambda_i|$$
$$f_i = \frac{\omega_i}{2\pi} \qquad \zeta_i = \frac{\mathrm{Re}(\lambda_i)}{|\lambda_i|} \tag{6}$$

Mode shape matrixes are calculated using the following equation:

$$\boldsymbol{\Phi} = \boldsymbol{C}\boldsymbol{\Psi} \tag{7}$$

3 EMPIRICAL EXPERIMENTS

In equipments such as a complex four-stage centrifugal compressor, the harmonic components of excitation are dominant. Therefore, using operational modal analysis methods requires proper identification and elimination of these harmonics. However, due to the complex nature of the equipment which results in numerous harmonic components and due to importance of the participating natural frequencies in compressor's working conditions, only the working frequency range of the compressor was investigated. Vibrational response in four accessible locations gathered using noncontact sensors placed directly inside the bearings to reconstruct shaft vibrations (figure 1). These responses were measured continuously with the sampling frequency of 44100 Hz. Excitation was exerted using actual working loads of the equipment. It was assumed that the excitation stimulates all modes. Using the assumption of white noise excitation for the equipment under working condition which is close to reality, the stimulation possibility of all modes increases.

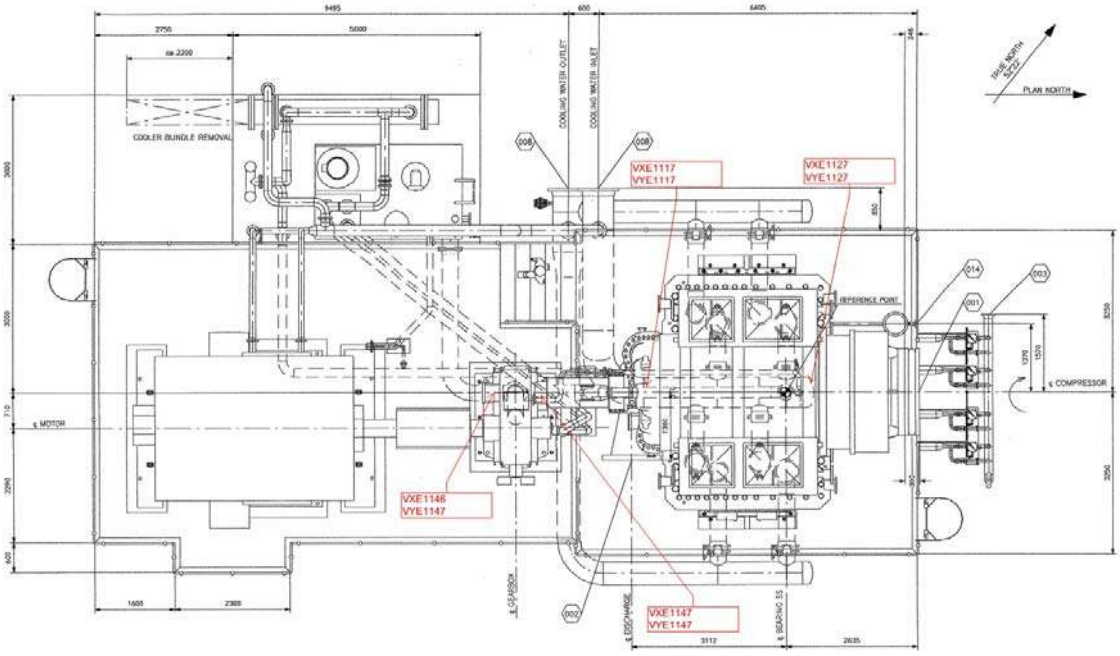


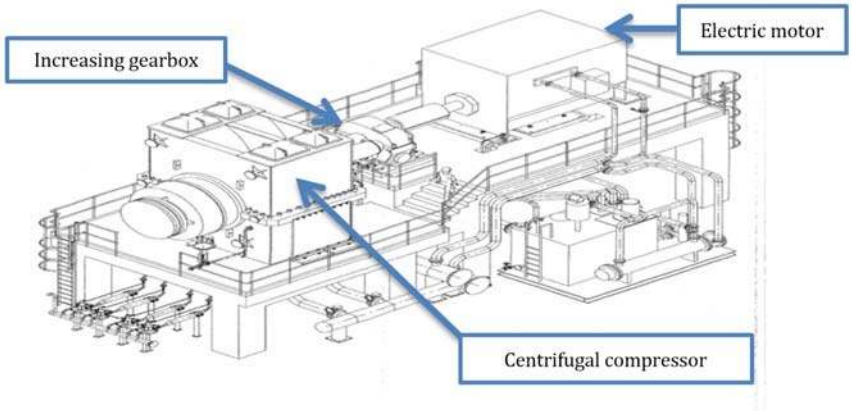
Figure 1: *Exact position of sensors inside the bearings is marked with red.*

3.1 Complex four-stage centrifugal compressor

The investigated four-stage centrifugal compressor is composed of a giant 30 Megawatt electrical engine with the speed of 1500 RPM, an increasing gearbox with transmission ratio of 3.31 and a huge four-stage centrifugal compressor with rotational speed of 4960 RPM (figure 2). The three-dimensional model of the compressor is shown in figure 3. Table 1 shows some characteristics of the compressor.



(a)



(b)

Figure 2 (a) & (b): *Four-stage centrifugal compressor*

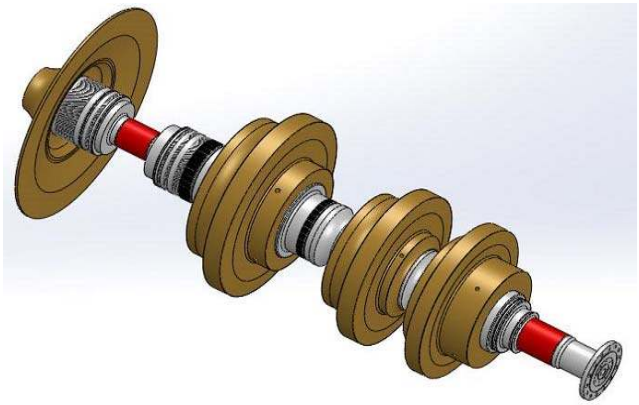


Figure 3: *Three-dimensional model of rotor in four-stage centrifugal compressor*

Table 1: *Properties of centrifugal compressor, gearbox and electromotor*

MAIN AIR COMPRESSOR	
RADIAL-ISOTHERM COMPRESSOR FOR AIR SEPARATION UNIT 10-1-C1161	
MANUFACTURE	MAN TURBOMASCHINEM AG
TYPE OF MACHINE	RIKT 125 1 + 1 + 2
CAPACITY (NM3/HR)	293400 303750
MOMENT OF INERTIA (kgm2)	8200
MOTOR / COMPRESSOR SPEED (1/min)	1500 / 4950
1ST PINION SPEED (1/min)	4950
NO. OF STAGES	4
DIMENSION OF COMPRESSOR	17 x 10.5 x 8
WEIGHT (TON)	150
HIGH SPEED, PARALLEL SHAFT GEAR UNIT	
Manufacturer	BHS
No. of Teeth	29 / 96
Power Rating	31000 kW
Gear Ratio	3.31
Total Weight	12000 kg
MEDIUM VOLTAGE SYNCHRONOUS MOTOR	
Manufacturer	SIEMENS AG
Rated Frequency	50HZ
Total Weight	49500 kg
Speed	1500 rpm
Rated Shaft Power	30400 kW
Rated Voltage	11 kV

3.2 Identifying harmonic components using different methods

As mentioned before, due to dominance of harmonic stimulations, it is necessary to identify and eliminate these harmonics before operational modal analysis. The data gathered by instruments showed high effect of harmonic components on time data as shown in figure 4. Extended kurtosis test, singular value decomposition and stability diagram were used for identification of harmonic components. Kurtosis is defined in terms of the fourth central moment of the stochastic variable normalized with respect to the standard deviation. Random data corresponds to kurtosis value of 3 while the kurtosis for pure harmonic data is 3/2. This point is the basis of harmonic

component identification in the extended kurtosis method. In figure 5, the result of extended kurtosis test with threshold of 0.333 on singular value decomposition graph is shown. Vertical green lines show the limits of identified harmonics and line thickness shows the harmonic intensity. By changing the threshold value, these harmonics are more clearly determined (figure 6). Figure 7 shows the stability diagram of data. A red "+" sign shows stable modes while a green "x" sign shows unstable modes and possibly harmonics.

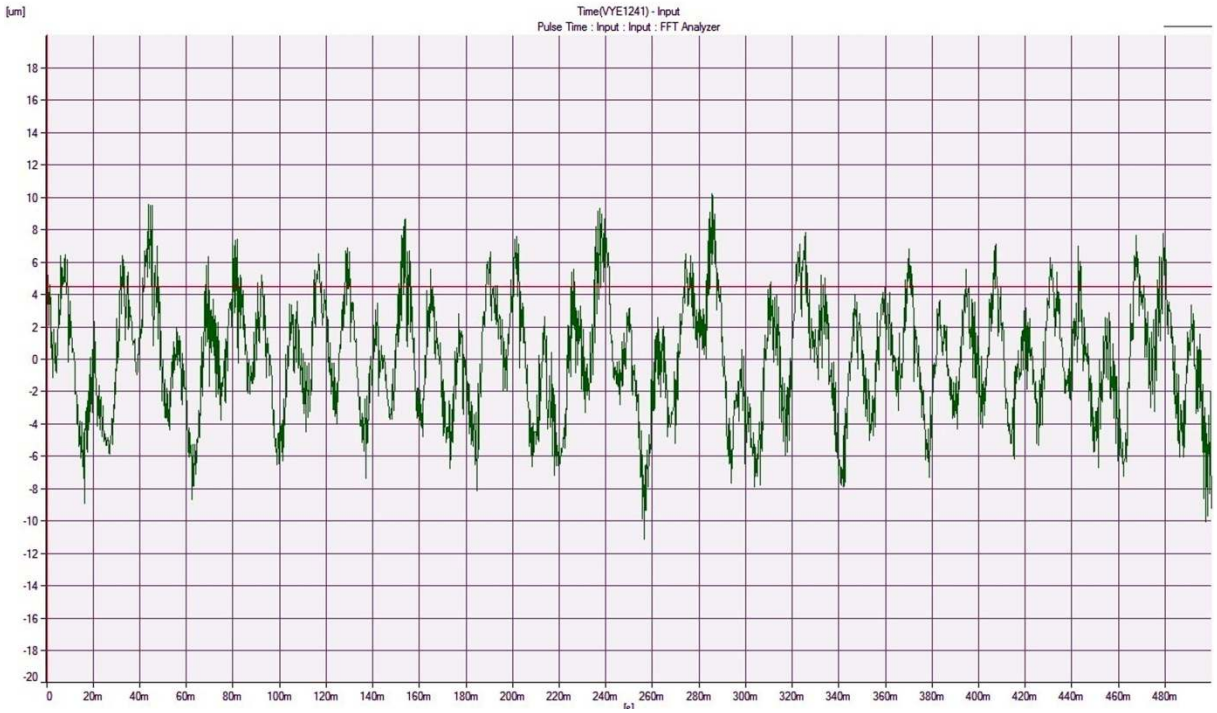


Figure 4: *Effect of harmonic components on time data gathered from the compressor*

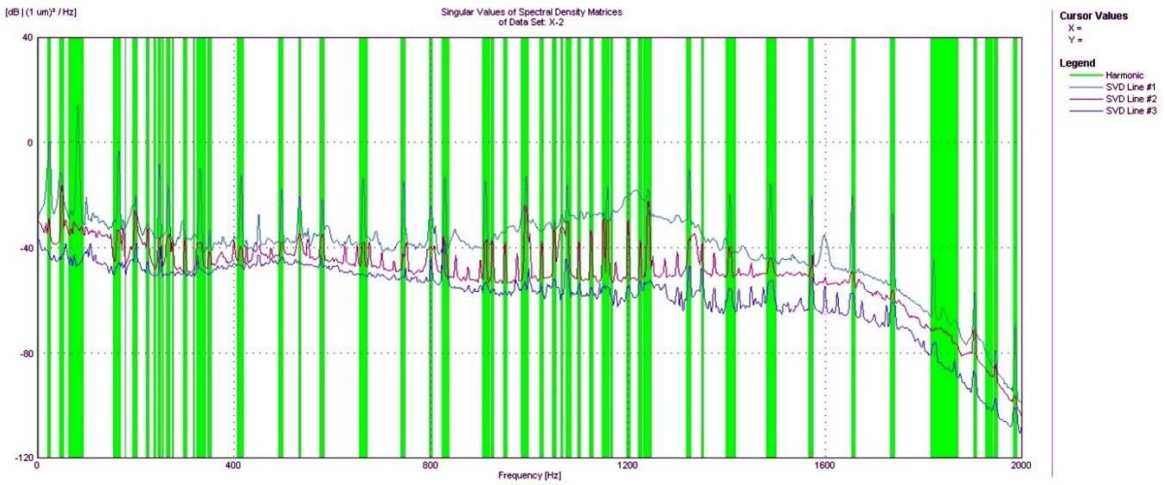


Figure 5: *The intensity of harmonic components by changing threshold value in SVD diagram*

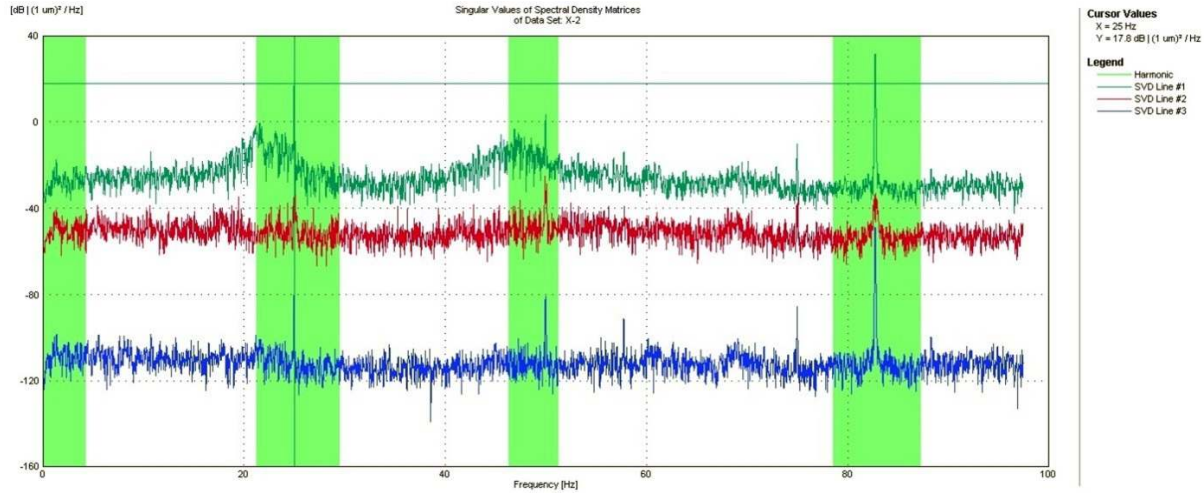


Figure 6: *Identification of harmonic components using extended kurtosis test of singular value decomposition*

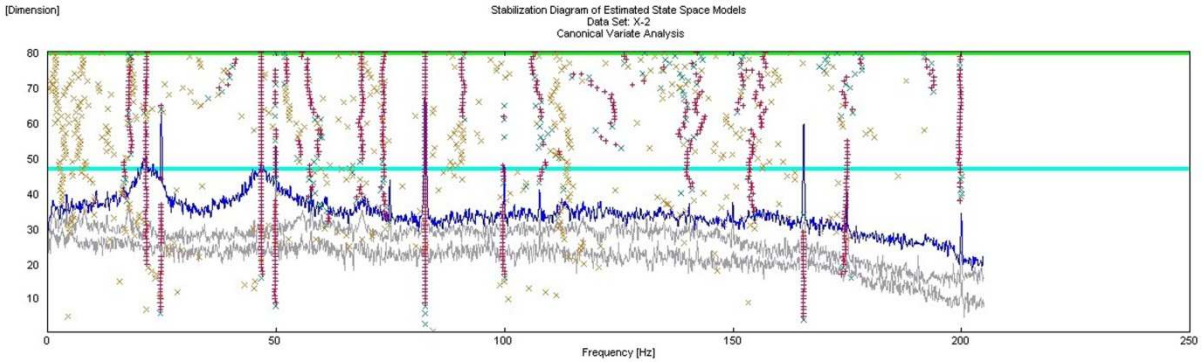


Figure 7: *Identification of harmonic components using stability diagram method*

3.3 Extraction of modal parameters using FDD, EFDD and SSI methods

After identification of harmonic components, natural frequencies and damping ratios were extracted in the working frequency range of equipment via FDD, EFDD and SSI methods with the help of PULSE™ type 3560-D system as well as MATLAB codes. Figure 8 shows the SVD diagram after elimination of harmonic components using linear interpolation in EFDD method. As mentioned before, EFDD seeks for frequency points in which the response spectrum gets unit rank. In the vicinity of this resonant frequency, the response spectrum is highly dominated by resonant singular vector. Collecting a set of data whos singular vectors are highly correlated to resonant singular vector, an SDOF data corresponding to the intended mode is obtained. Transfering data to time domain through inverse FFT, provides decaying SDOF time data. Exact natural frequency and damping ratio can be extracted from time data via well-known technique of logarithmic decrement. When a peak due to a harmonic component locates in the analysis range, frequency data get distorted causing considerable error in the results. In this case, SVD data in the pre-identified harmonic frequency is replaced with interpolated value of nabor points. Interpolation and replacing new SVD values is fulfilled several times till the spourious harmonic peak vanishes. In Figure 8, interpolated data indicated in red has supressed down the harmonic peaks corresponding to first and second harmonics of electromotor rotation at 24.8 and 49.6 Hz. Figure 9 shows the elimination results using EFDD Matlab code. This diagram has a frequency band of 100 Hz and resolution of 0.5 Hz. After extraction of modal parameters using frequency domain methods such as FDD and EFDD, parameters were also extracted using SSI which is one of the most efficient operational modal analysis methods in time domain. Resulting stability diagram is showed in figure 10. In order to extract natural frequencies, the optimal order was considered to be 59. Figure 11 compares the original and reconstructed spectrums. It is evident that discarding unstable poles corresponding to harmonic excitations, eliminates efficiently these harmonics in reconstructed data. After selecting the optimum model order, natural frequencies and damping ratios were extracted. Table 2 shows parameters extracted using FDD, EFDD and SSI methods with the help of operational modal analysis software and MATLAB codes. Values presented in table 2 show that stochastic subspace method has higher accuracy compared to frequency domain decomposition and enhanced frequency domain decomposition methods. However, when system damping in one of the modes is low, frequency decomposition method present a better estimation.

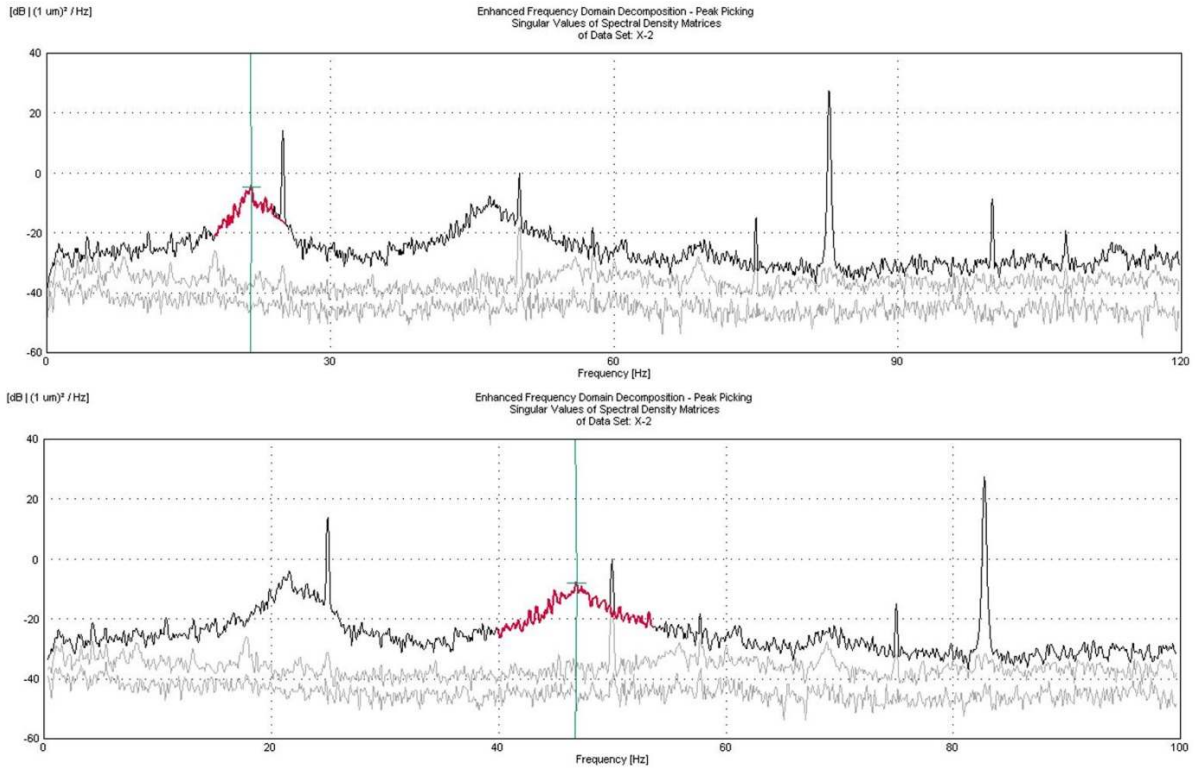


Figure 8: *Elimination of harmonic peak due to electromotor rotation in harmonic frequency of 24.8 Hz (up) and harmonic frequency of 49.6Hz.*

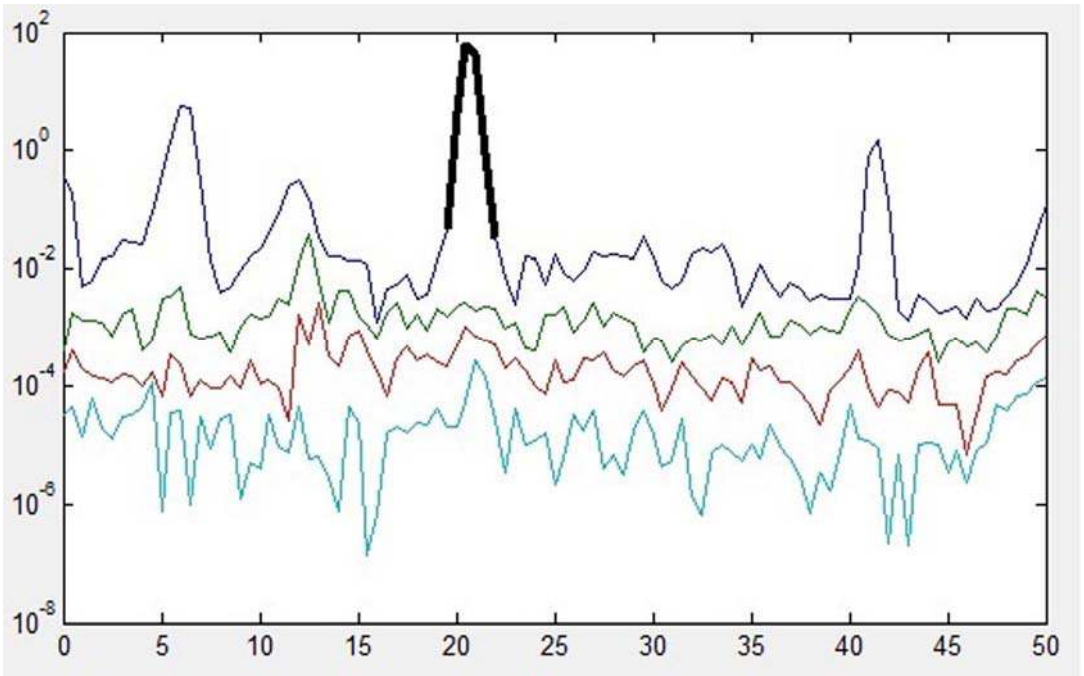


Figure 9: *Elimination of harmonic peak of electromotor in harmonic frequency of 24.8Hz*

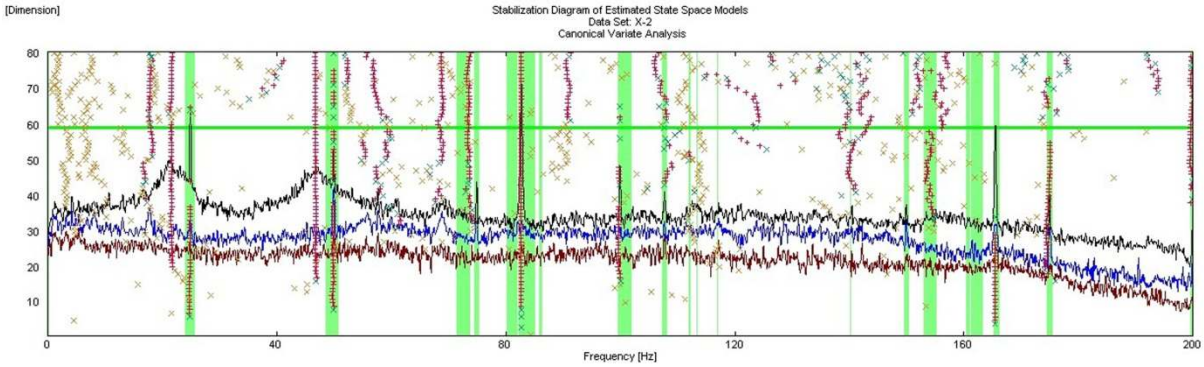


Figure 10: *Stability diagram used for extraction of modal compressor parameters with SSI method*

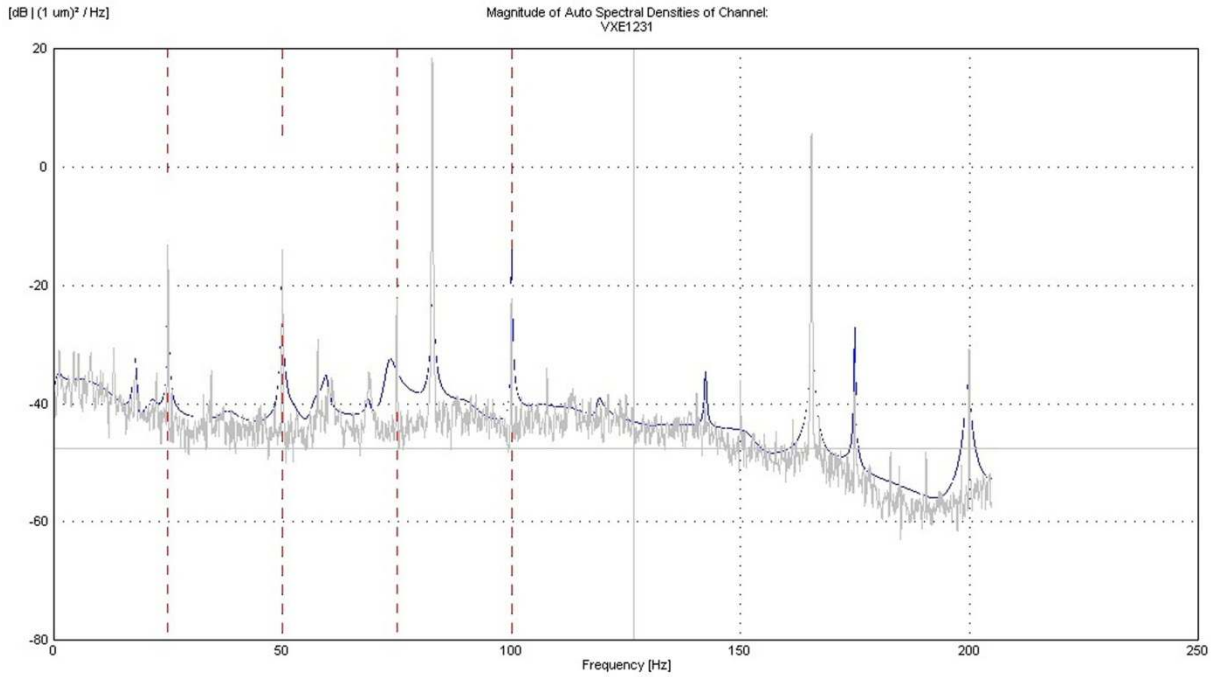


Figure 11: *Comparing measured signal in point 1 with estimated signal. Grey spectrum is measured signal and blue spectrum is the estimated signal.*

Table 2: *Comparing modal compressor parameters extracted using different operational medal analysis methods.*

Modal compressor parameters extracted using operational medal analysis methods (SSI, EFDD, FDD)

Method	Mode													
	1th	***1th***	***2nd***	***2nd***	***3rd***	***3rd***	***4th***	***4th***	***5th***	***5th***	***6th***	***6th***	***7th***	***7th***
	ω_k	ζ_k	ω_k	ζ_k	ω_k	ζ_k	ω_k	ζ_k	ω_k	ζ_k	ω_k	ζ_k	ω_k	ζ_k
FDD/matlab	16.2	-	21.1	-	44.0	-	56.2	-	61.1	-	68.2	-		
FDD/pulse	16.7	-	21.7	-	46.8	-	55.9	-	60.8	-	69.5	-		
EFDD/matlab	18.1	1.03	21.5	4.2	45.6	2.7	56.2	0.41	67.9	0.55	71.8	0.14	81.4	0
EFDD/pulse	17.83	0.87	21.62	6.37	46.75	3.7	58.28	0.09	69.36	0.82	73.05	0.07	83.2	0
SSI/matlab	17.6	1.8	21.8	2.8	46.4	1.9	58.0	3.24	69.9	0.99	72.9	1.41	82.7	0.0004
SSI/pulse	17.85	2.04	21.67	3.9	46.83	2.84	57.65	2.41	69.16	1.61	73.64	1.08	82.73	0.0004

4 CONCLUSIONS

This study investigated the performance of various methods for identification and elimination of harmonic components in common Operational Modal Analysis (OMA) methods including Frequency Domain Decomposition (FDD), Enhanced Frequency Domain Decomposition (EFDD) and Stochastic Subspace Identification (SSI) under

practical conditions. A complex four-stage centrifugal compressor under normal working conditions was investigated using operational modal analysis. Excitation was carried out by working loads. In order to extract modal parameters, first these parameters were extracted using a comercial modal analysis software. Then parameters were extracted using developed MATLAB codes and the results were compared with each other. The results of this experimental study showed that the accuracy of SSI method is higher compared to FDD and EFDD methods. However, when system damping of a specific mode is low, FDD method performs a better estimation.

References

Agneni A., Coppotelli G., Grappasonni C. (2012). A method for the harmonic removal in operational modal analysis of rotating blades, Mechanical Systems and Signal Processing, 27:604-618. DOI: 10.1016/j.ymssp.2011.09.013

Bendat J. B., Piersol A. G. (2010). Analysis and Measurement Procedures, Fourth Edition, Book Series: Wiley Series in Probability and Statistics. DOI: 10.1002/9781118032428

Brincker R., Andersen P. (2006). Understanding Stochastic Subspace Identification, Conference Proceedings: IMAC-XXIV: A Conference & Exposition on Structural Dynamics. Society for Experimental Mechanics, St. Louis, Missouri, United States, 31 Jan- 02Feb, 2006.

Brincker R., Andersen P., Möller N. (2000a). An indicator for separation of structural and harmonic modes in output-only modal testing, Proceedings of SPIE - The International Society for Optical Engineering 2:265-272.

Brincker R., Zhang L., Andersen P. (2000b). Modal Identification from Ambient Responses using Frequency Domain Decomposition, IMAC 18: Proceedings of the International Modal Analysis Conference (IMAC), San Antonio, Texas, USA, February 7-10, 2000 Pages 625-630

Brincker R., Ventura C.E., Andersen P., (2001) Damping estimation by frequency domain decomposition. In: Proceedings of 19th international modal analysis conference (IMAC-19), Kissimmee, Florida: Society for Experimental Mechanics (SEM), 5–8 February 2001, pp.698–703.

Cara F. J., Juan J., Alarcón E., Reynders E., Roeck G. D. (2013). Modal contribution and state space order selection in operational modal analysis, Mechanical Systems and Signal Processing, 38(2):276-298. DOI: 10.1016/j.ymssp.2013.03.001

Cunha Á., Caetano E., Magalhães F., Moutinho C. (2005). From input-output to output-only modal identification of civil engineering structures (2005) 1st International Operational Modal Analysis Conference, 26-27 April 2005, Copenhagen, Denmark.

Jacobsen N-J., Andersen P. (2008). Modal Analysis: Curve-Fitted Enhanced Frequency Domain Decomposition-A Robust Technique To Harmonic Excitation In Operational Modal Analysis, 15th International Congress on Sound and Vibration 6-10 July 2008, Daejeon, Korea.

Mclamore V. R., Hart G. C., Stubbs I. R. (1971). Ambient vibration of two suspension bridges, Journal of the Structural Division, 99(st7):2567-2582.

Overschee P. V., Moor L. R. D. (1996) Subspace identification for linear systems: theory, implementation, applications: Kluwer Academic Publishers. DOI: 10.1007/978-1-4613-0465-4

Overschee P.V., Moor B. D. (1993). Subspace algorithms for the stochastic identification problem, Automatica (Journal of IFAC), 29(3): 649-660. DOI: 10.1016/0005-1098(93)90061-W

Pintelon R., Peeters B., Guillaume P. (2010). Continuous-time operational modal analysis in the presence of harmonic disturbances, Mechanical Systems and Signal Processing, 24(1):90-105. DOI: 10.1016/j.ymssp.2009.03.011

Reynders E., Pintelon R., Roeck G. D. (2008). Uncertainty bounds on modal parameters obtained from stochastic subspace identification, Mechanical Systems and Signal Processing, 22(4):948-969. DOI: 10.1016/j.ymssp.2007.10.009

Reynders E., Roeck G. D. (2008). Reference-based combined deterministic–stochastic subspace identification for experimental and operational modal analysis, Mechanical Systems and Signal Processing, 22(3):617-637. DOI: 10.1016/j.ymssp.2007.09.004

Weijtjens W., Sitter G. D., Devriendt C., Guillaume P. (2014). Operational modal parameter estimation of MIMO systems using transmissibility functions, Automatica, 50(2):559-564. DOI: 10.1016/j.automatica.2013.11.021