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Analysis of flexibility of the support and its influence on dynamics of the grab crane

Abstract

Dynamic analysis of the model of the grab crane with the flexibly supported base is presented in the paper. The analyzed grab crane is the structure of the open-loop kinematic chain with rigid links. Joint coordinates and homogenous transformations are used in order to describe dynamic behavior of the system. Equations of the motion of the systems are derived using Lagrange's equations of the second order and integrated Newmark's method with iterative procedure. The commercial package MSC.ADAMS is used in order to verify own program. The range of analysis concerns influence of different values of vertical stiffness coefficients in the supports and means of fixing of the load on its motion.

Keywords

grab crane, dynamic analysis, flexibility of the support, joint coordinates, homogenous transformation

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1 INTRODUCTION

During the design process of lifting machines flexibility of the support should be taken into special consideration [1,2,3,4,5]. It has mainly influence on stability of the system and the same efficiency of reloading work and safety of machine operations. Therefore it has been formulated the mathematical models which already in the first phase of the project include flexibility of the support and carried out numerical simulations illustrate phenomenon sometimes undesirable (loss of stability, uncontrolled balances of the load, unrealization of desired trajectory).

The mathematical model of the grab crane with flexibly supported base and numerical simulations are presented in the paper. The formulated model includes small and large motions of the flexible platform. In description of dynamics of the system formalism of joint coordinates and homogenous transformations is used. Equations of the motion are derived using Lagrange's equations of the second order. Own computer program has been elaborated in which equations are integrated using Newmark's method with iterative procedure and constant step size. Correctness of the formulated model has been verified using the commercial program MSC.ADAMS.

2 MATHEMATICAL MODEL OF THE GRAB CRANE

The model of the analyzed grab crane is shown in Fig.1. This system consists of seven rigid bodies constituting the structure of the open-loop kinematic chain.

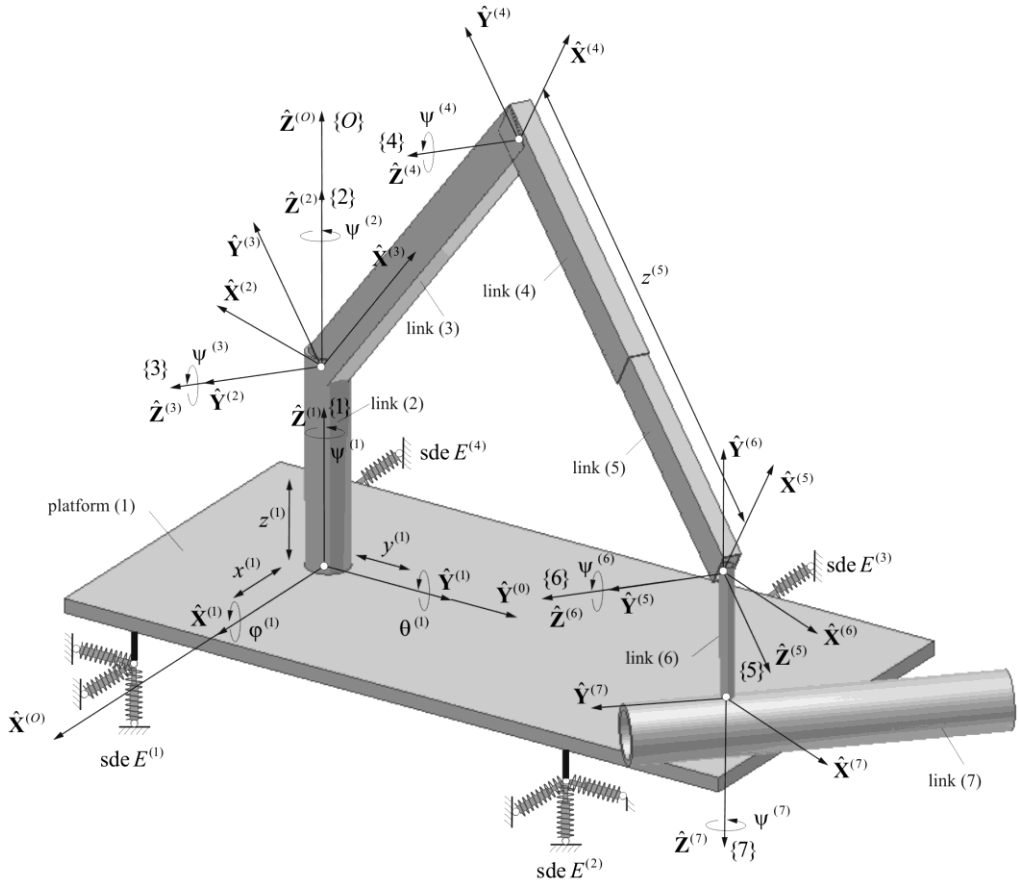


Figure 1 Model of grab crane

In description of geometry of the system Denavit-Hartenberg's notation is used (tab.1).

Table 1 Denavit-Hartenberg's parameters

p link	$a^{(p-1)}$	$\alpha^{(p-1)}$	$d^{(p)}$	$\theta^{(p)}$
2	0	0	$L^{(2)}$	$\psi^{(2)}$
3	0	270°	0	$\psi^{(3)}$
4	$L^{(3)}$	0	0	$\psi^{(4)}$
5	0	90°	$z^{(5)}$	0
6	0	270°	0	$\psi^{(6)}$
7	0	90°	$L^{(6)}$	$\psi^{(7)}$
where $L^{(p)}$ is lenght of p link				

The motion of each link with respect to the previous one is described by vectors of generalized coordinates:

$$\tilde{\mathbf{q}}^{(1)} = \begin{bmatrix} x^{(1)} & y^{(1)} & z^{(1)} & \psi^{(1)} & \theta^{(1)} & \varphi^{(1)} \end{bmatrix}^T, \tag{1.1}$$

$$\tilde{\mathbf{q}}^{(2)} = \begin{bmatrix} \psi^{(2)} \end{bmatrix}, \tag{1.2}$$

$$\tilde{\mathbf{q}}^{(3)} = \begin{bmatrix} \psi^{(3)} \end{bmatrix}, \tag{1.3}$$

$$\tilde{\mathbf{q}}^{(4)} = \begin{bmatrix} \psi^{(4)} \end{bmatrix}, \tag{1.4}$$

$$\tilde{\mathbf{q}}^{(5)} = \begin{bmatrix} z^{(5)} \end{bmatrix}, \tag{1.5}$$

$$\tilde{\mathbf{q}}^{(6)} = \begin{bmatrix} \psi^{(6)} \end{bmatrix}, \tag{1.6}$$

$$\tilde{\mathbf{q}}^{(7)} = \begin{bmatrix} \psi^{(7)} \end{bmatrix}. \tag{1.7}$$

The motion of p link with respect to the reference system $\{O\}$ is defined by vector of generalized coordinates:

$$\mathbf{q}^{(p)} = \begin{bmatrix} \mathbf{q}^{(p\ \ 1)} \\ \tilde{\mathbf{q}}^{(p)} \end{bmatrix}, \quad p = 2,..,7 \tag{2}$$

where $\mathbf{q}^{(1)} = \tilde{\mathbf{q}}^{(1)}$.

Therefore the motion of the analyzed system is described by vector of generalized coordinates:

$$\mathbf{q} = \mathbf{q}^{(7)} = \mathrm{col}\{q_j\}_{j=1,\ldots,12} = \begin{bmatrix} x^{(1)} & y^{(1)} & z^{(1)} & \psi^{(1)} & \theta^{(1)} & \varphi^{(1)} & \psi^{(2)} & \psi^{(3)} & \psi^{(4)} & z^{(5)} & \psi^{(6)} & \psi^{(7)} \end{bmatrix}^T \tag{3}$$

It is assumed kinematic inputs:

$$q_{5\ \ p} = \tau^{(p)}, \ \ j = 2,..,5 \tag{4}$$

where $\tau^{(p)}$ are functions of time.

The local transformation matrices can be written in form:

$$\tilde{\mathbf{B}}^{(1)} = \begin{bmatrix} c\psi^{(1)}c\theta^{(1)} & c\psi^{(1)}s\theta^{(1)}s\varphi^{(1)} - s\psi^{(1)}c\varphi^{(1)} & c\psi^{(1)}s\theta^{(1)}c\varphi^{(1)} + s\psi^{(1)}s\varphi^{(1)} & x^{(1)} \\ s\psi^{(1)}c\theta^{(1)} & s\psi^{(1)}s\theta^{(1)}s\varphi^{(1)} + c\psi^{(1)}c\varphi^{(1)} & s\psi^{(1)}s\theta^{(1)}c\varphi^{(1)} - c\psi^{(1)}s\varphi^{(1)} & y^{(1)} \\ -s\theta^{(1)} & c\theta^{(1)}s\varphi^{(1)} & c\theta^{(1)}c\varphi^{(1)} & z^{(1)} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5.1}$$

for large angles $\psi^{(1)}, \theta^{(1)}, \varphi^{(1)}$,

$$\tilde{\mathbf{B}}^{(1)} = \begin{bmatrix} 1 & -\psi^{(1)} & \theta^{(1)} & x^{(1)} \\ \psi^{(1)} & 1 & -\varphi^{(1)} & y^{(1)} \\ -\theta^{(1)} & \varphi^{(1)} & 1 & z^{(1)} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{for small angles } \psi^{(1)}, \theta^{(1)}, \varphi^{(1)}, \tag{5.2}$$

$$\tilde{\mathbf{B}}^{(p)} = \begin{bmatrix} c\theta^{(p)} & -s\theta^{(p)} & 0 & a^{(p-1)} \\ s\theta^{(p)} c\alpha^{(p-1)} & c\theta^{(p)} c\alpha^{(p-1)} & -s\alpha^{(p-1)} & -s\alpha^{(p-1)} d^{(p)} \\ s\theta^{(p)} s\alpha^{(p-1)} & c\theta^{(p)} s\alpha^{(p-1)} & c\alpha^{(p-1)} & c\alpha^{(p-1)} d^{(p)} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad p = 2,\ldots,7. \tag{5.3}$$

where: $s = \sin$, $c = \cos$.

The transformation matrices of coordinates from the local systems to the reference system $\{O\}$ are written as follows:

$$\mathbf{B}^{(p)} = \mathbf{B}^{(p-1)} \tilde{\mathbf{B}}^{(p)}, \quad p = 1,..,7 \tag{6}$$

The equations of the motion of the system are formulated using Lagrange's equation of the second order:

$$\frac{d}{dt}\frac{\partial E_k}{\partial \dot{q}_j} - \frac{\partial E_k}{\partial q_j} + \left(\frac{\partial E_p}{\partial q_j} + \frac{\partial E_{sde}}{\partial q_j} \right) + \frac{\partial D_{sde}}{\partial \dot{q}_j} = Q_j \tag{7}$$

where: E_k is the kinetic energy of the links,

E_p is the potential energy of gravity forces of the links,

E_{sde} is the potential energy of spring deformation of the spring-damping elements,

D_{sde} is function of the dissipation of energy of the spring-damping elements,

Q_j are non-potential generalized forces,

$q_j, \dot{q}_j$ are generalized coordinates and velocities.

2.1 Kinetic energy and potential energy of gravity forces

The kinetic energy and the potential energy of gravity forces can be written as follows:

$$E_k = \sum_{p-1}^{7} \tilde{E}_k^{(p)} \tag{8.1}$$

$$E_p = \sum_{p-1}^{7} \tilde{E}_p^{(p)}, \tag{8.2}$$

where: $\tilde{E}_k^{(p)} = \frac{1}{2}\mathrm{tr}\ \ \dot{\mathbf{B}}^{(p)}\mathbf{H}^{(p)}\dot{\mathbf{B}}^{(p)T}$,

$\tilde{E}_p^{(p)} = m^{(p)} g \boldsymbol{\theta}_3 \mathbf{B}^{(p)} \tilde{\mathbf{r}}_{C^{(p)}}^{(p)}$,

$m^{(p)}$ is mass of the links,

g is the acceleration of gravity,

$\boldsymbol{\theta}_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$,

$\tilde{\mathbf{r}}_{C^{(p)}}^{(p)}$ is vector of coordinates of the center of mass in local coordinate system of p link,

$\mathbf{H}^{(p)}$ is pseudo-inertial matrix [6].

2.2 Potential energy of spring deformation and function of dissipation energy

It is assumed that the analyzed crane is flexibly supported on four supports which are modelled by means of spring-damping elements (sdes) $E^{(k)}\,(k = 1,2,3,4)$ - Fig.2.

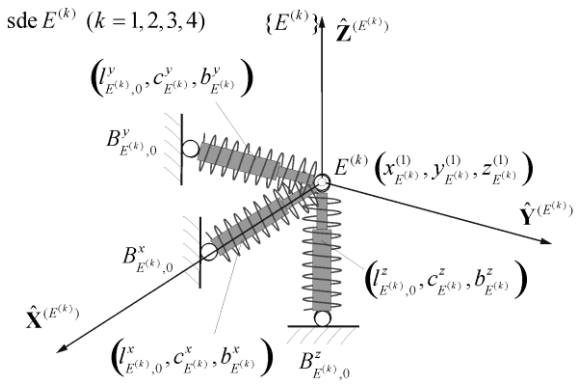


Figure 2 Flexible connections of the crane

The potential energy of spring deformation and function of the dissipation energy can be expressed by following forms:

$$E_{sde} = \sum_{k-1}^{4} \sum_{\alpha\in\{x,y,z\}} E_{E^{(k)}}^{\alpha} \tag{9.1}$$

$$D_{sde} = \sum_{k-1}^{4} \sum_{\alpha\in\{x,y,z\}} D_{E^{(k)}}^{\alpha} \tag{9.2}$$

where: $E^{\alpha}_{E^{(k)}} = \frac{1}{2} c^{\alpha}_{E^{(k)}} \left(d^{\alpha}_{E^{(k)}} \right)^2$,

$D^{\alpha}_{E^{(k)}} = \frac{1}{2} b^{\alpha}_{E^{(k)}} \left(\dot{d}^{\alpha}_{E^{(k)}} \right)^2$,

$d^{\alpha}_{E^{(k)}} = l^{\alpha}_{E^{(k)}} - l^{\alpha}_{E^{(k)},0}$ is deformation of the spring in sde $E^{(k)}$ in α direction,

$l^{\alpha}_{E^{(k)}} = \left| \mathbf{U}^{(E^{(k)})}_{B^{\beta}_{E^{(k)},0}} \mathbf{q}^{(1)} \right|$ is length of strained spring,

$l^{\alpha}_{E^{(k)},0}$ is length of unstrained spring,

$c^{\alpha}_{E^{(k)}}$, $b^{\alpha}_{E^{(k)}}$ are stiffness and damping coefficients,

$$\mathbf{U}^{(E^{(k)})}_{B^{\beta}_{E^{(k)},0}} = \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 0 & z^{(E^{(k)})}_{B^{\beta}_{E^{(k)},0}} & -y^{(E^{(k)})}_{B^{\alpha}_{E^{(k)},0}} \\ 0 & 1 & 0 & -z^{(E^{(k)})}_{B^{\beta}_{E^{(k)},0}} & 0 & x^{(E^{(k)})}_{B^{\alpha}_{E^{(k)},0}} \\ 0 & 0 & 1 & y^{(E^{(k)})}_{B^{\beta}_{E^{(k)},0}} & -x^{(E^{(k)})}_{B^{\beta}_{E^{(k)},0}} & 0 \end{array}\right].$$

The equations of the motion of the system can be written in matrix form:

$$\mathbf{A}\ddot{\mathbf{q}} + \mathbf{B}\dot{\mathbf{q}} + \mathbf{C}\mathbf{q} - \mathbf{D}\mathbf{P} = \mathbf{f} \tag{10.1}$$

$$\mathbf{D}^T\ddot{\mathbf{q}} = \mathbf{\Gamma} \tag{10.2}$$

where: $\mathbf{A} = \mathbf{A}(\mathbf{q})$,

$\mathbf{B} = \mathbf{B}(\mathbf{q}, \dot{\mathbf{q}})$,

C is stiffness matrix,

D is matrix with constant coefficients,

$\mathbf{P} = \left[P^{(2)} \quad P^{(3)} \quad P^{(4)} \quad P^{(5)} \right]^T$ is vector of driving forces and moments,

$\mathbf{\Gamma} = \left[\ddot{\tau}^{(2)} \quad \ddot{\tau}^{(3)} \quad \ddot{\tau}^{(4)} \quad \ddot{\tau}^{(5)} \right]^T$,

f is vector of generalized force without driving forces and moments.

Newmark's method with iterative procedure [7] is used to integrate the equations of the motion. This iterative procedure is necessary because elements of **A** and **B** matrices depend on the generalized coordinates and velocities.

3 NUMERICAL RESULTS

The geometric parameters of the system are presented in Fig. 3.

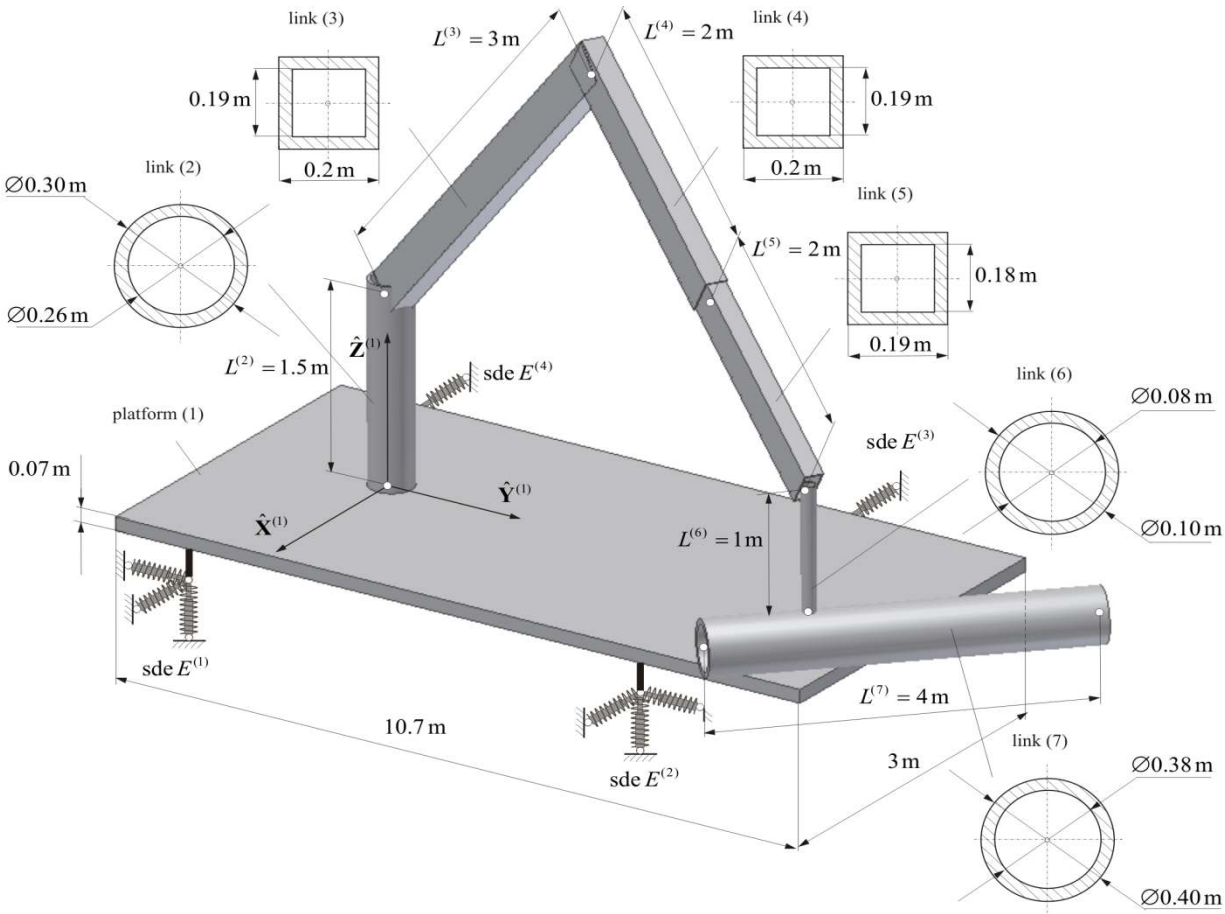
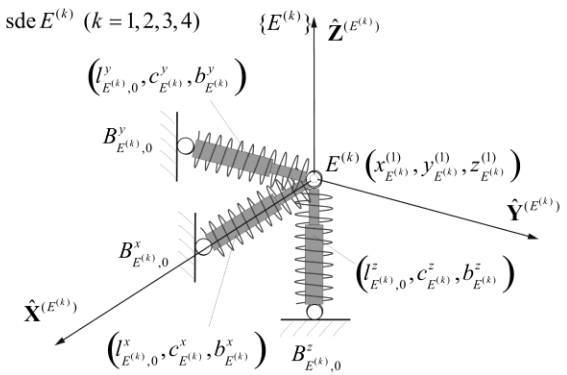


Figure 3 Parameters of the grab crane

Parameters of sdes are presented in tab. 2.

Table2 Parameters of sdes



No. sde $E^{(k)}$	Coordinates of sdes $E^{(k)}$ in $\{1\}$, [m]			Coordinates of points $B^{x}_{E^{(k)},0}$ in $\{E^{(k)}\}$ [m]			Coordinates of points $B^{y}_{E^{(k)},0}$ in $\{E^{(k)}\}$ [m]			Coordinates of points $B^{z}_{E^{(k)},0}$ in $\{E^{(k)}\}$ [m]			Stiffness coefficients of sde $\{E^{(k)}\}$ [Nm^{-1}]			Damping coefficients of sde $\{E^{(k)}\}$ [Nsm^{-1}]		
	$x^{(1)}_{E^{(k)}}$	$y^{(1)}_{E^{(k)}}$	$z^{(1)}_{E^{(k)}}$	$x^{(E^{(k)})}_{B^{x}_{E^{(k)},0}}$	$y^{(E^{(k)})}_{B^{x}_{E^{(k)},0}}$	$z^{(E^{(k)})}_{B^{x}_{E^{(k)},0}}$	$x^{(E^{(k)})}_{B^{y}_{E^{(k)},0}}$	$y^{(E^{(k)})}_{B^{y}_{E^{(k)},0}}$	$z^{(E^{(k)})}_{B^{y}_{E^{(k)},0}}$	$x^{(E^{(k)})}_{B^{z}_{E^{(k)},0}}$	$y^{(E^{(k)})}_{B^{z}_{E^{(k)},0}}$	$z^{(E^{(k)})}_{B^{z}_{E^{(k)},0}}$	$c^{x}_{E^{(k)}}$	$c^{y}_{E^{(k)}}$	$c^{z}_{E^{(k)}}$	$b^{x}_{E^{(k)}}$	$b^{y}_{E^{(k)}}$	$b^{z}_{E^{(k)}}$
1	1.5	-2.0	-0.35	0.3	0	0	0	-0.3	0	0	0	-0.35	10^5	10^5	$0.5\cdot10^6$	0	0	0
2	1.5	3.8	-0.35	0.3	0	0	0	0.3	0	0	0	-0.35			10^6			
3	-1.5	3.8	-0.35	-0.3	0	0	0	0.3	0	0	0	-0.35			∞			
4	-1.5	-2.0	-0.35	-0.3	0	0	0	-0.3	0	0	0	-0.35						

Kinematic inputs $\ddot{\tau}^{(p)}$ (for $p = 2,\ldots,5$) assumed are shown in Fig 4.

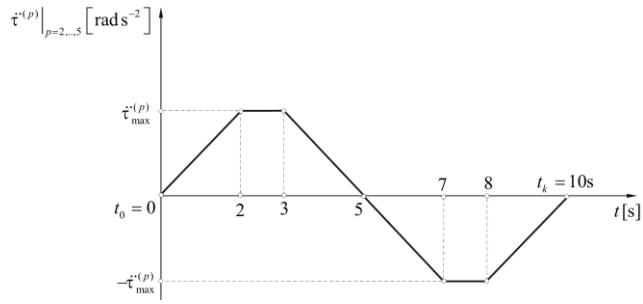


Figure 4 Kinematic input

Maximal values of accelerations $\ddot{\tau}^{(p)}_{\max}$ are calculated by assuming that:

$$\tau^{(2)}\Big|_{t_0=0} = 180^{\mathrm{o}}\,,\ \tau^{(2)}\Big|_{t_k=10\mathrm{s}} = 270^{\mathrm{o}}\,,$$
$$\tau^{(3)}\Big|_{t_0=0} = 180^{\mathrm{o}}\,,\ \tau^{(3)}\Big|_{t_k=10\mathrm{s}} = 250^{\mathrm{o}}\,,$$
$$\tau^{(4)}\Big|_{t_0=0} = 90^{\mathrm{o}}\,,\ \tau^{(4)}\Big|_{t_k=10\mathrm{s}} = -50^{\mathrm{o}}\,,$$
$$\tau^{(5)}\Big|_{t_0=0} = 4\,\mathrm{m}\,,\ \tau^{(5)}\Big|_{t_k=10\mathrm{s}} = -0.5\,\mathrm{m}\,.$$

The initial and final positions of the system for following parameters are presented in Fig 5.

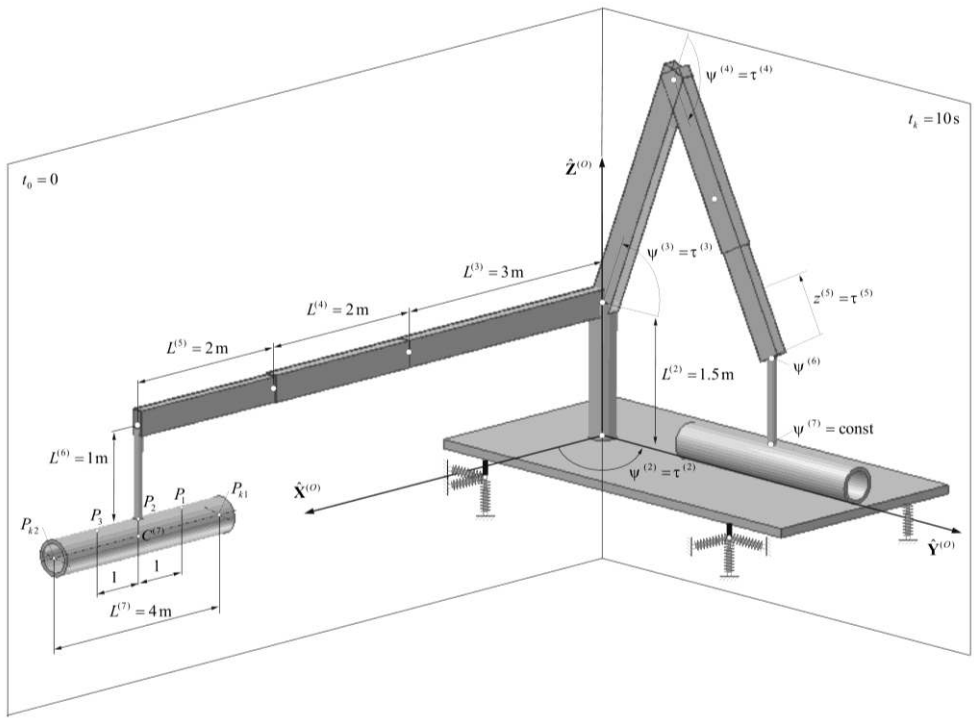
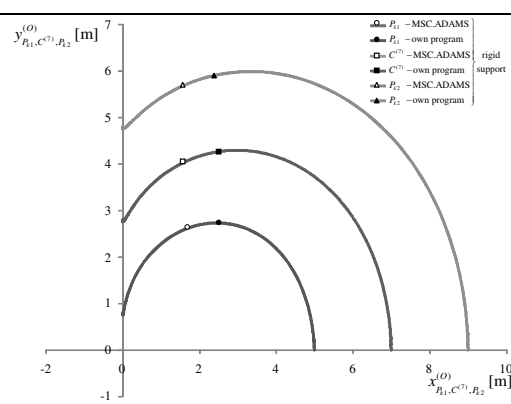
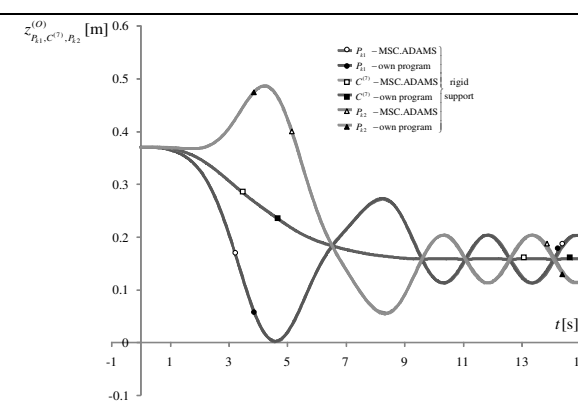


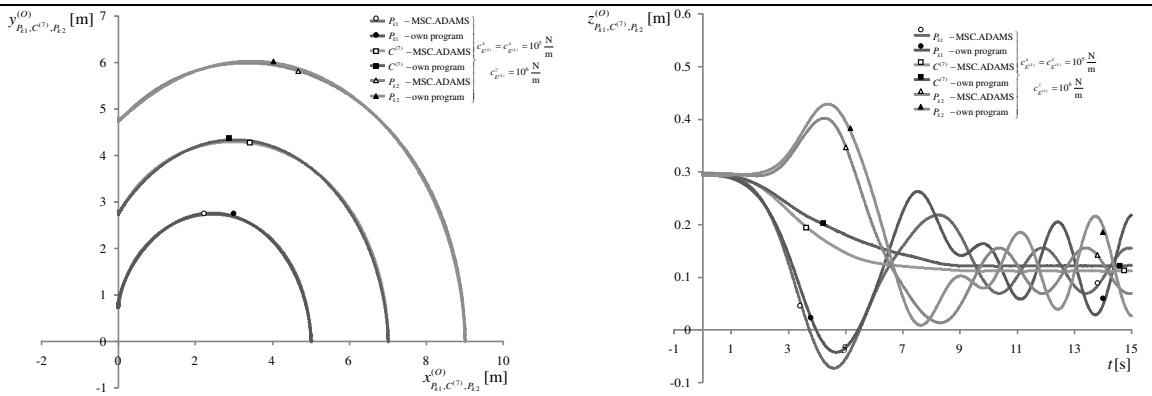
Figure 5 Configuration of the system – initial and final

The comparison of the obtained results using own program with those obtained from MSC.ADAMS is presented in tab.3. The results were compared for trajectory and z coordinate of $P_{k_1}, C^{(7)}, P_{k_2}$ points in the case of rigid and flexible support. It was also assumed that the load is fixed in P_2 point.

Table 3 Trajectory and z coordinate of P_{k_1}, $C^{(7)}, P_{k_2}$ points

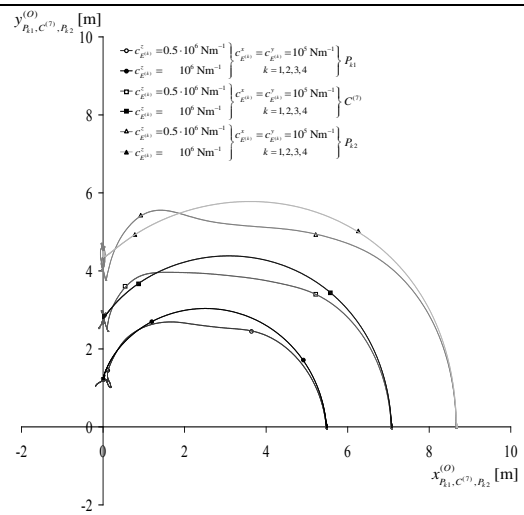
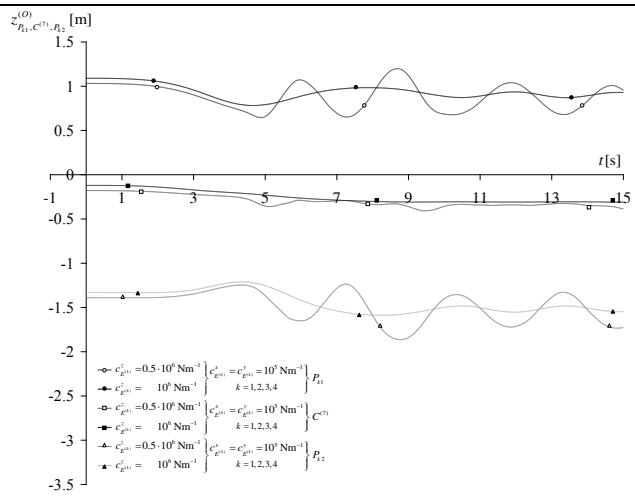
P_2	trajectory of P_{k_1}, $C^{(7)}, P_{k_2}$ points	$z^{(O)}_{P_{k1},C^{(7)},P_{k2}}$ coordinate
rigid		

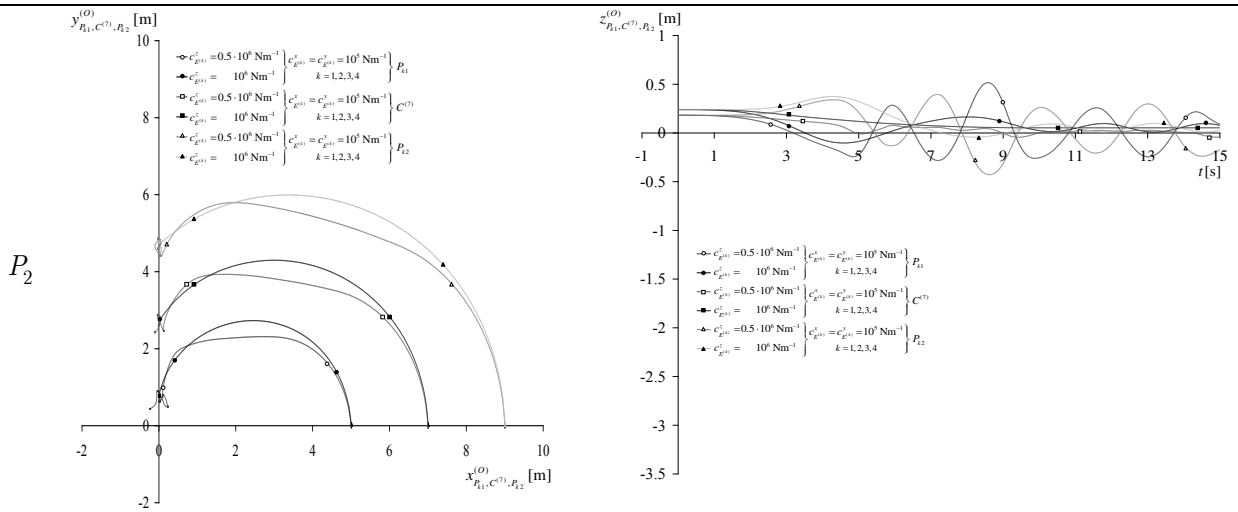
flexible

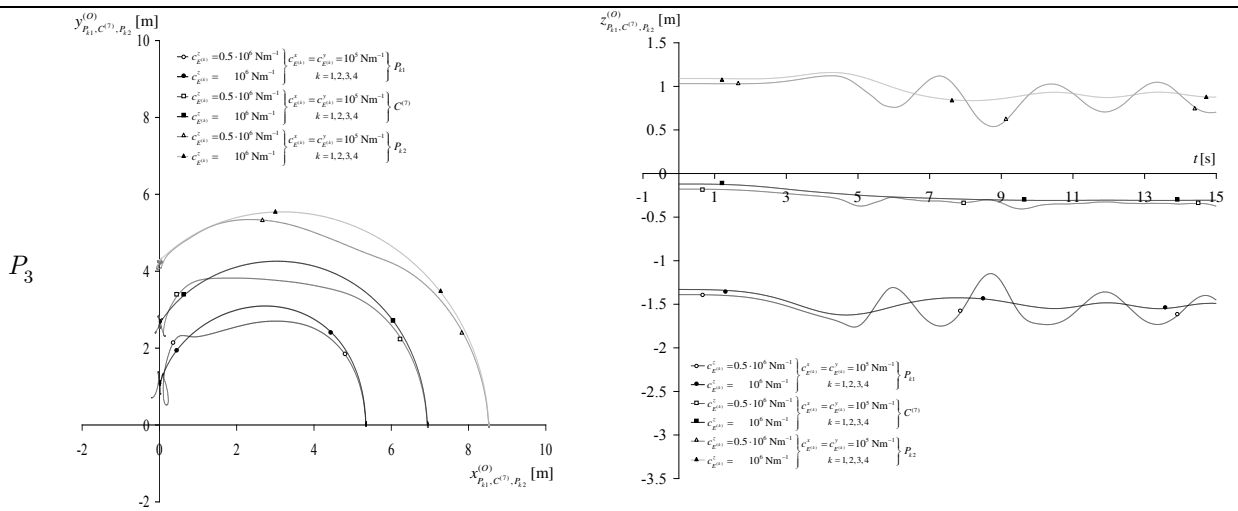


Influence of different values of stiffness coefficients of sdes on trajectory and z coordinate of $P_{k_1}, C^{(7)}, P_{k_2}$ points for different fixing points of the load is shown in tab.4. The results are obtained using own program.

Table 4 Trajectory and z coordinate of P_{k_1}, $C^{(7)}, P_{k_2}$ points

	trajectory of P_{k_1}, $C^{(7)}, P_{k_2}$ points	$z^{(O)}_{P_{k1},C^{(7)},P_{k2}}$ coordinate
P_1		





4 CONCLUSIONS

In the paper the model of the grab crane with flexibly supported base has been presented. The model of the system has been obtained by means of joint coordinates and homogenous transformations. The influence of flexibility of the support on dynamic behavior of the system has been analyzed. Good compatibility of the results of own program with those obtained from MSC.ADAMS confirms correctness of the formulated mathematical model of the grab crane and the same any structure with open-loop kinematic chain with flexibly supported base.

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