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Structural reliability model considering mixed probabilistic and interval variables

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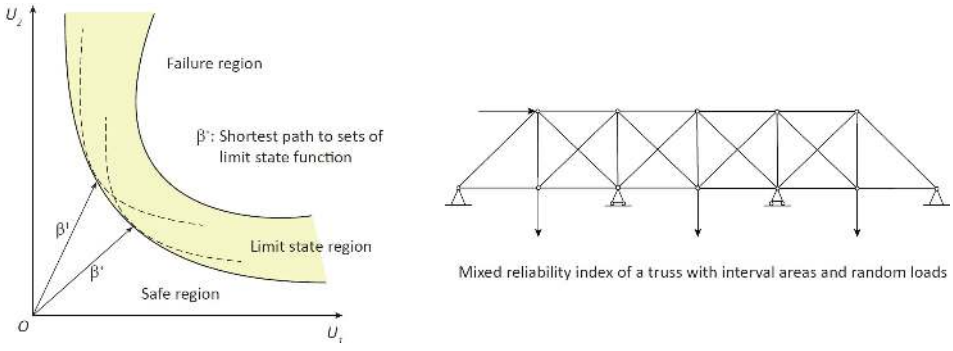
Abstract

Traditional probability-based structural reliability analysis method can only consider random uncertainties described by random distribution functions, which required sufficient experimental samples. On comparison, interval uncertainties can be more appropriate when lacking information. In realistic situations, several structural reliability models considering mixed probabilistic and interval variables are proposed. Besides choosing reliability model according to the information available of the uncertainties, this paper presents a perspective in structural design that for a specific design, the controllable structural uncertain parameters should be better described as interval variables while other uncontrollable uncertain parameters such as external loads described as random variables if sufficient information is available. The corresponding reliability analysis model is proposed. Further, for simple truss structures, this paper demonstrates that the extreme hybrid reliability index can be attained when the interval variables reach their upper or lower bounds. To solve the hybrid reliability index, a sequential single-loop strategy combined with an intermediate-variable based response surface and the method of moving asymptotes optimization solver is proposed. Finally, numerical examples are given to demonstrate the applicability of the proposed method and concluding remarks are made.

Keywords

Hybrid reliability analysis, Random variable, Interval variable, Truss structures

Graphical Abstract



Mixed reliability index of a truss with interval areas and random loads

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1 INTRODUCTION

Uncertainty exists widely in practical engineering structures, and sometimes these uncertainties may lead to the unexpected failure of structures. With the very powerful tools of probabilistic theory and the ever increasing computational method, probability-based reliability analysis method has been a successful method to consider uncertainties, and many techniques such as first order reliability method (FORM) (Hasofer, Lind, 1974; Rackwitz, Flessler, 1978), second order reliability method (SORM) (Breitung, 1984; Polidori et al. 1999a) and Monte-Carlo simulation method (MCS) (Hurtado, Barbat, 1998) are well established for this method. Though well developed, probability-based reliability analysis method can only consider uncertainties which are described by random distributions. But to construct such random distributions, large experimental samples, which are not always available, are required. Further, Elishakoff (1995a) pointed out that in particular cases a small error in the random distribution function may cause large errors in the reliability analysis results.

Due to the disadvantages of the probability-based reliability method mentioned above, in 1990s, Ben-Haim and Elishakoff (1990) proposed the convex sets such as ellipsoid and interval to consider uncertainties. Compared with the probabilistic variables, convex model needs only bounds of the uncertainties which are more convenient to obtain. After that, Ben-Haim (1999b; 2004) proposed the info-gap theory and used robustness function to assess the robustness of the structures. Guo et al. (2001) presented non-probabilistic reliability index, which is evaluated as the ratio between the nominal value to the dispersion of performance function, to evaluate the structural reliability considering interval uncertainties. Cao and Duan (2005a) used the shortest distance from the origin point to the limit-state function in the normalized space to define the hybrid reliability index on the basis of convex set. Wang et al. (2007a) considered the stress-strength interference situation and took the ratio of the volume of safe region to the total volume of the region constructed by the basic interval variables as the measure of structural non-probability reliability. Jiang et al. (2007b) proposed an interval satisfaction degree method.

In practical engineering, a more realistic situation is that some uncertain parameters have sufficient samples which can be used to construct the random distribution functions while others are lacking information that can only be treated as intervals. Currently, an increasing number of studies have been contributed to the hybrid reliability models. Ben-Haim and Elishakoff (1995b) pointed out that due to the existence of interval variables, the structural reliability is no longer a single value, but an interval. Guo and Lu (2002) proposed a two-stage performance function method to deal with the hybrid reliability analysis considering probabilistic and interval uncertainties. Qiu et al. (2008a) utilized interval analysis and employed parameter estimation methods to evaluate the reliability interval. Du et al. (2005b) treated probabilistic and interval uncertainties as independent variables and transformed the double level optimization procedure into a sequential single level one, short for SSL, which greatly improved the computational efficiency. Jiang et al. (2012a; 2011a) divided the hybrid probabilistic and convex reliability model into two types, named “type I hybrid model” and “type II hybrid model”, in which the former referred to that the probabilistic and convex variables are included separately while the latter meant that only probabilistic variables are adopted to describe the uncertainties but the characteristic values of the random distribution function were intervals. Kang and Luo (2010a) combined the probabilistic and convex set variables to perform the reliability-based optimization and developed efficient methods to improve the computational efficiency. Gao et al. (2010b; 2011b) presented a mixed perturbation MC method for static and reliability analysis of structural systems with a mixture of random and interval parameters. Yoo and Lee (2013) proposed sampling-based method and employed MCS method to evaluate the hybrid reliability.

It is worth noting that in the studies on hybrid reliability models mentioned above, the probabilistic and interval uncertainties are selected according to the known information of the uncertainties. However, for a specific structural design, those controllable parameters such as structural geometry and material properties are better described as interval variables. Because it is a more acceptable way if the controllable parameters of structural components are satisfied within the interval requirement, the structures will absolutely be safe. While for other parameters such as external loads, if there have sufficient information, it is better described as random variables. Take a building structure in civil engineering for example, the geometric parameters of the columns and beams should better be satisfied the interval requirement in order to make the building absolutely work as expected under given loads. The designers have the right to refuse the unsatisfied components since it may bring dangerous disasters. For external loads, because there have lots of historical records for the loads such as winds and earthquakes in a specific place, it can be modeled as random variables.

This paper aims to present the idea of structural reliability considering hybrid probabilistic and interval variables, and proposes the corresponding reliability analysis model. Further, it demonstrates that for truss structures under the proposed reliability model, the reliability can attain its extreme value when the interval variables take either their upper or lower bound value. The remainder of the paper is organized as follows. Section 2 proposes the new reliability analysis

model considering mixed probabilistic and interval variables. Section 3 demonstrates that for truss structures, the reliability can attain its extreme value when interval variables reach their interval bounds. Section 4 provides several numerical examples and finally, concluding remarks are given in Section 5.

2 RELIABILITY MODEL CONSIDERING PROBABILISTIC AND INTERVAL VARIABLES

2.1 Probability-based reliability model

In the traditional probability-based reliability analysis method, the uncertainties are modeled as random variables which are described by random distribution functions. The structural failure probability can be evaluated by

$$P_f = \Pr[G(\mathbf{X}) \le 0] = \int \cdots \int_{G(\mathbf{X}) \le 0} f_X(\mathbf{x}) dx_1 \ldots dx_n \tag{1}$$

where $\Pr[\cdot]$ denotes the probability, $\mathbf{X} = \{X_1, X_2, \ldots, X_n\}^T$ denotes the vector of random variables, $f_X(\mathbf{x})$ is the joint probability density function of $\mathbf{X}$, $G(\mathbf{X})$ denotes a limit-state function and the structural failure state is denoted by $G(\mathbf{X}) < 0$. Usually the above integration is unable to evaluate, and many approximated techniques such as FORM, SORM and MCS method are well developed to approximate the integration.

2.2 Probabilistic and interval hybrid reliability model

With the idea that those controllable parameters be modeled as interval variables while other parameters, if there have sufficient samples, be modeled as random variables, the structural hybrid reliability model can be defined as the structural minimum probabilistic reliability or maximum structural failure probability under all the combinations of the interval variables, mathematically it can be expressed as

$$P_f = \max_{\mathbf{Y}} \Pr[G(\mathbf{X}, \mathbf{Y}) \le 0] \tag{2}$$

where $\mathbf{X} = \{X_1, X_2, \ldots, X_n\}^T$ denotes the vector of random variables, $\mathbf{Y} = \{Y_1, Y_2, \ldots, Y_m\}$ denotes the vector of interval variables, $\Pr[\cdot]$ denotes the probability, $G(\mathbf{X}, \mathbf{Y})$ denotes a limit-state function and $G(\mathbf{X}, \mathbf{Y}) \le 0$ denotes the failure state of structures.

Compared with the probability-based reliability model, due to the existence of the interval variables, the failure probability is no longer a value, but an interval. So the hybrid reliability model mentioned above is to find the maximum failure probability in the interval. If reliability index is adopted to represent the probability, recalled that reliability index is the shortest distance from the origin point to the limit-state function in the normalized random variable space, the hybrid reliability index can be defined as the shortest distance from the origin point to the limit-state function cluster in the normalized random variable space. Let $\mathbf{U}$ and $\mathbf{V}$ are the normalized vector of random and interval variables respectively, then hybrid reliability index can be mathematically expressed as Eqn. (3), schematically seen as Figure 1.

$$\begin{aligned} &\beta = \min_{\mathbf{V}} \min_{\mathbf{U}} \|\mathbf{U}(\mathbf{V})\| \\ &s.t.\ G(\mathbf{U}, \mathbf{V}) = 0 \end{aligned} \tag{3}$$

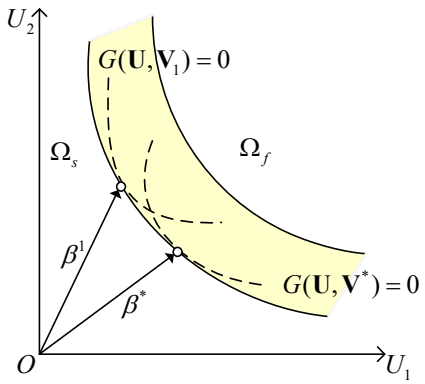


Figure 1: Representation of hybrid reliability index

3 EXTREME RELIABILITY VALUES FOR TRUSS STRUCTURES

The probabilistic and interval hybrid reliability model proposed in Sec. 2 has the mathematical form as Eqn. (3). If hybrid reliability index is adopted to approximate the hybrid reliability probability, it can be obviously found that the evaluation of hybrid reliability index is in fact a double level optimization procedure. To solve this problem, Du et al. (2005b) proposed a sequential single level (SSL) procedure to transform the double level procedure into single level procedure. Kang and Luo (2010a) employed performance measure approach (PMA) and developed an efficient iteration scheme for the hybrid reliability based optimization. Jiang et al. (2012b) simply took interval variables as uniformly distribution random variables within the interval and transform the hybrid problem into random problem, and demonstrated that the two problems will converge to the same solution.

Recently, Guo et al. (2008b) found that the structural responses of truss structures taking Young's moduli of the bars as interval uncertainties will attain their extreme values when interval variables take either their upper or lower bound values. This conclusion has significant meaning in the worst case analysis since it can provide with the global solution under the double level optimization problem in which global solution of the inner optimization problem must be satisfied. Based on this conclusion, it can be further demonstrated that the following theorem is tenable.

Theorem:

For the hybrid reliability model presented in Section 2, the extreme hybrid reliability index of truss structures can be attained at a combination of interval variables' bounds.

Proof:

Assume that hybrid reliability index β of the truss structures taking Young's moduli or cross section areas of bars as interval uncertainties has its minimum value at $(\mathbf{U}_p, \mathbf{V}_p)$. According to the definition of hybrid reliability index, β is the shortest distance from the origin of coordinates to the limit-state function cluster $G(\mathbf{U}, \mathbf{V}) = 0$ in the normalized random variable space, where $\mathbf{U}$ and $\mathbf{V}$ are normalized random variable and interval variable vectors, respectively. Then for the given $\mathbf{U}_p$, $G(\mathbf{U}_p, \mathbf{V})$ should satisfied

$$G(\mathbf{U}_p, \mathbf{V}) \geq 0 \tag{4}$$

Consider the following two situations:

(1) If $\mathbf{V}_p$ is already the combination of the bounds of interval variables, then the theorem is demonstrated.

(2) If $\mathbf{V}_p$ is not the combination of the interval bounds, according to the definition of the hybrid reliability index, we have

$$G(\mathbf{U}_p, \mathbf{V}_p) = R - S(\mathbf{U}_p, \mathbf{V}_p) = 0 \tag{5}$$

where R is the structural resistance, $S(\mathbf{U}_p, \mathbf{V}_p)$ is the structural responses. Since $\mathbf{U}_p$ is fixed, based on the conclusion of Guo et al. (2008b), there will always be a combination of interval bounds $\mathbf{V}^*$ that make the structural responses $G(\mathbf{U}_p, \mathbf{V}^*)$ attain its extreme value, that is

$$S(\mathbf{U}_p, \mathbf{V}^*) > S(\mathbf{U}_p, \mathbf{V}_p) \tag{6}$$

Substituting Eqn. (6) into Eqn. (5), one obtained

$$G(\mathbf{U}_p, \mathbf{V}^*) = R - S(\mathbf{U}_p, \mathbf{V}^*) < R - S(\mathbf{U}_p, \mathbf{V}_p) = 0 \tag{7}$$

Obvious Eqn. (7) is contradicted with Eqn. (4), that is to say there always exists another point $(\mathbf{U}_p, \mathbf{V}^*)$ making β smaller to that evaluated at $(\mathbf{U}_p, \mathbf{V}_p)$, thus $(\mathbf{U}_p, \mathbf{V}^*)$ will be the new design point. Therefore, if $\mathbf{V}_p$ is not the combination of the interval bounds, the hybrid reliability index of truss structures will not reach its extreme value, and the theorem is demonstrated.

With this property, the evaluation of the hybrid reliability index can be restricted to those combinations of the bounds of interval variables.

4 SOLUTION ALGORITHM OF HYBRID RELIABILITY INDEX

Mathematical model of hybrid reliability index for truss structures is shown in Eqn. (3), obviously it is a double-layer nested optimization problem. In order to solve the hybrid reliability index, Du et al. (2005b) proposed a sequential single-loop (SSL) method to transform the double-layer nested optimization problem into single layer optimization problem, which improves the computational efficiency. Jiang et al. (2012b) treat interval variables as random variables with uniform distributions, and solve the model by probabilistic reliability theories. Han et al. (2014) proposed a method to construct a response surface with both interval variables and random variables, then Monte Carlo simulation method is used to solve the failure probability based on the response surface.

In this paper, the basic idea of sequential single-loop method proposed by Du et al. (2005b) is adopted to solve the double-layer nested optimization problem. The basic calculation steps are as follows:

(1) Setup the initial design points for both random variables and interval variables;

(2) Fix random variables, find the combination of interval bounds which minimize the limit-state function;

(3) Fix interval variables, calculate the most probable point (MPP) and the corresponding reliability index;

(4) Check the convergences of both MPP and interval variables. If not convergent, then go to step 2, otherwise, quit the procedure with convergent MPP and the combination of interval variables.

Different with the method proposed by Du et al. (2005b), the method of moving asymptotes (MMA) (Svanberg, 1987) is adopted as the optimization solver in step 2, and an intermediate-variable based response surface method are constructed to find out the MPP and the corresponding reliability index in step 3. With these two strategies, the accuracy and the computational efficiency will be further improved.

5 NUMERICAL EXAMPLES

With the idea that structural controllable parameters should be described as interval variables while other parameters as probabilistic variables, in this section, two numerical examples are presented to illustrate the property that the minimum reliability index of truss structures, which taking Young's moduli or cross sectional areas as interval uncertainties, can be attained at a combination of the interval bounds.

5.1 Ten-bar truss

As shown in Figure 2, the hybrid reliability index of a ten-bar truss structure suffering from cross sectional areas as interval uncertainties and external loads as random uncertainties is considered in this example. Three random concentrated loads are applied to nodes 5 and 6. The cross sectional areas of ten bars are treated as interval variables. Characteristic values of the random distribution function and interval variables are listed in Table 1. Young's modulus of each bar is 1.0×10^7 psi. The truss structure should satisfy the requirement that the vertical displacement of node 3 be less than an allowable value $d_{ymax}=20$ in.

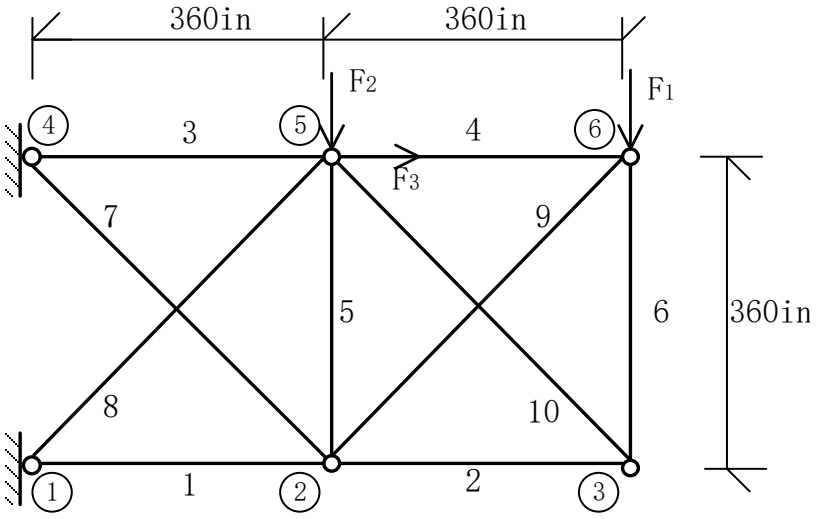


Figure 2: Ten-bar truss with mixed uncertainties

Table 1 Distributions of the uncertain parameters in ten-bar truss structure

Uncertainties	Parameter 1	Parameter 2	Distribution
$A_1 \sim A_{10}$	$2.5\ in^2$	0.1	Interval
$F_1 \sim F_2$	$-10^5\ lbs$	0.1	Normal
F_3	$5\times10^4\ lbs$	0.1	Normal

For interval variables, parameter 1 is the nominal value and parameter 2 is the ratio between dispersion and the nominal value; for random normal distribution, parameter 1 is the mean value and parameter 2 is the coefficient of variance.

To solve this problem, a sequential single-loop procedure is employed to improve the efficiency, an intermediate-variable based response surface method is used to approximate the limit-state functions and the MMA solver is adopted as the optimization solver. The analysis results are listed in Table 2. As can be seen from Table 2, all the interval variables reach their upper or lower bounds, which is coincident with the conclusion in section 3. And this combination is also a worst case solution in Guo et al. (2008b).

Table 2 Analysis results of ten-bar truss structure

Interval variables	A_1	2.25
	A_2	2.25
	A_3	2.25
	A_4	2.25
	A_5	2.25
	A_6	2.75
	A_7	2.25
	A_8	2.25
	A_9	2.25
	A_{10}	2.25
Random variables	F_1	-105089.07
	F_2	-113256.83
	F_3	50652.73
β	1.43	

5.2 25-bar truss structure

Consider a 25-bar truss structure, as shown in Figure 3. The external loads are treated as random variables and the cross sectional areas are treated as interval variables, their characteristic parameters are listed in Table 3. Young's modulus and density of each bar are 29000ksi and 0.1lb/in^3, respectively. There is a vertical displacement constraint $d_{ymax} = 10$ in on node 9.

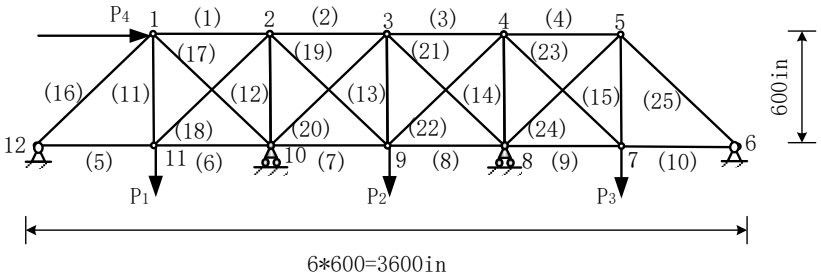


Figure 3: 25 -bar truss with mixed uncertainties

With the same solution strategies as example 1, the sequential single-loop procedure combined with the intermediate-variable based response surface method and the MMA optimization solver is used, and the analysis results are listed in Table 4. It can also be seen from Table 4 that all the interval variables lie on their upper or lower bounds, which coincidences with the theorem proposed in section 3.

Table 3 Distributions of the uncertain parameters in 25-bar truss structure

Uncertainties	Parameter 1	Parameter 2	Distribution
$A_1 \sim A_{10}$	$10\ in^2$	0.1	Interval
$A_{11} \sim A_{25}$	$5\ in^2$	0.2	Interval
P_1, P_3	$-400\ kip$	0.1	Normal
P_2	$-500\ kip$	0.1	Normal
P_4	$300\ kip$	0.2	Normal

For interval variables, parameter 1 is the nominal value and parameter 2 is the ratio between dispersion and the nominal value; for random normal distribution, parameter 1 is the mean value and parameter 2 is the coefficient of variance.

Table 4 Analysis results of 25-bar truss structure

Interval variables	A_1	11.00	A_{14}	4.00
	A_2	9.00	A_{15}	4.00
	A_3	9.00	A_{16}	6.00
	A_4	11.00	A_{17}	4.00
	A_5	11.00	A_{18}	6.00
	A_6	11.00	A_{19}	4.00
	A_7	11.00	A_{20}	4.00
	A_8	11.00	A_{21}	4.00
	A_9	9.00	A_{22}	4.00
	A_{10}	11.00	A_{23}	6.00
	A_{11}	4.00	A_{24}	4.00
	A_{12}	4.00	A_{25}	6.00
	A_{13}	4.00		
Random variables	F_1	-404.17	F_2	-621.65
	F_3	-404.15	F_4	311.64
β	2.45			

6 CONCLUSION

This paper studies the structural reliability model considering both random and interval variables from a new perspective, which is to describe the structural controllable uncertain parameters such as structural geometry and material strength as interval variables while other uncontrollable uncertain parameters such as external loads as random variables. Corresponding reliability model are constructed which is defined as the minimum failure probability under all combinations of intervals. If hybrid reliability index is used to approximate this probability, the hybrid reliability index can be represented as the shortest distance from the origin point to the limit-state function clusters in the normalized random variable space. Further, based on the conclusion that structural responses of truss structures taking Young's moduli or cross sectional areas of bars as interval variables will attain their extreme values while interval variables are at their bounds, the hybrid reliability index for truss structures will also attain its minimum value with the combination of the interval bounds when Young's moduli or cross sectional areas of bars are selected as interval variables. A sequential single-loop strategy with response surface and MMA optimization solver is adopted to solve the hybrid reliability index. Finally, some numerical examples are given to illustrate this property and the applicability of the proposed method.

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References

Ben-Haim, Y. (1999b). Set-models of information-gap uncertainty: axioms and an inference scheme. Journal of the Franklin Institute 336, 1093-1117.

Ben-Haim, Y. (2004). Uncertainty, probability and information-gaps. Reliability Engineering and System Safety 85, 249-266.

Ben-Haim, Y., Elishakoff, I. (1990). Convex models of uncertainty in applied mechanics, Elsevier Amsterdam.

Ben-Haim, Y., Elishakoff, I. (1995b). Discussion on: A non-probabilistic concept of reliability. Structural Safety 17, 195-199.

Breitung, K. (1984). Asymptotic approximations for multinormal integrals. Journal of Engineering Mechanics 110, 357-366.

Cao, H. J., Duan, B. Y. (2005a). An approach on the non-probabilistic reliability of structures based on uncertainty convex models. Chinese Journal of Computational Mechanics 22, 546-549.

Du, X., Sudjianto, A., Huang, B. (2005b). Reliability-based design with the mixture of random and interval variables. Journal of Mechanical Design 127, 1068-1076.

Elishakoff, I. (1995a). Essay on uncertainties in elastic and viscoelastic structures: from AM Freudenthal's criticisms to modern convex modeling. Computers and Structures 56, 871-895.

Gao, W., Song, C., Tin-Loi, F. (2010b). Probabilistic interval analysis for structures with uncertainty. Structural Safety 32, 191-199.

Gao, W., Wu, D., Song, C., et al. (2011b). Hybrid probabilistic interval analysis of bar structures with uncertainty using a mixed perturbation Monte-Carlo method. Finite Elements in Analysis and Design 47, 643-652.

Guo, S. X., Lu, Z. Z. (2002). Hybrid probabilistic and non-probabilistic model of structural reliability. Journal of Mechanical Strength 24, 524-526.

Guo, S. X., Lu, Z. Z., Feng, Y. S. (2001). A non-probabilistic model of structural reliability based on interval analysis. Chinese Journal of Computational Mechanics 18, 56-60.

Guo, X., Bai, W., Zhang, W. (2008b). Extreme structural response analysis of truss structures under material uncertainty via linear mixed 0–1 programming. International Journal for Numerical Methods in Engineering 76, 253-277.

Han, X., Jiang, C., Liu, L. X., et al. (2014). Response-surface-based structural reliability analysis with random and interval mixed uncertainties. Science China 57, 1322-1334.

Hasofer, A. M., Lind, N. C. (1974). Exact and invariant second-moment code format. Journal of the Engineering Mechanics Division 100, 111-121.

Hurtado, J. E., Barbat, A. H. (1998). Monte Carlo techniques in computational stochastic mechanics. Archives of Computational Methods in Engineering 5, 3-29.

Jiang, C., Han, X., Li, W. X., et al. (2012a). A hybrid reliability approach based on probability and interval for uncertain structures. Journal of Mechanical Design 134, 031001.

Jiang, C., Han, X., Liu, G. R. (2007b). Optimization of structures with uncertain constraints based on convex model and satisfaction degree of interval. Computer Methods in Applied Mechanics and Engineering 196, 4791-4800.

Jiang, C., Li, W. X., Han, X., et al. (2011a). Structural reliability analysis based on random distributions with interval parameters. Computers & Structures 89, 2292-2302.

Jiang, C., Lu, G. Y., Han, X., et al. (2012b). A new reliability analysis method for uncertain structures with random and interval variables. International Journal of Mechanics and Materials in Design 8, 169-182.

Kang, Z., Luo, Y. (2010a). Reliability-based structural optimization with probability and convex set hybrid models. Structural and Multidisciplinary Optimization 42, 89-102.

Polidori, D. C., Beck, J. L., Papadimitriou, C. (1999a). New approximations for reliability integrals. Journal of engineering mechanics 125, 466-475.

Qiu, Z., Yang, D., Elishakoff, I. (2008a). Combination of structural reliability and interval analysis. Acta Mechanica Sinica 24, 61-67.

Rackwitz, R., Flessler, B. (1978). Structural reliability under combined random load sequences. Computers & Structures 9, 489-494.

Svanberg, K. (1987). The method of moving asymptotes—a new method for structural optimization. International journal for numerical methods in engineering 24, 359-373.

Wang, X. J., Qiu, Z. P., Wu, Z. (2007a). Non-probabilistic set-based model for structural reliability. ACTA MECHANICA SINICA-CHINESE EDITION- 39, 641.

Yoo, D., Lee, I. (2013). Sampling-based approach for design optimization in the presence of interval variables. Structural and Multidisciplinary Optimization, 1-14.