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Propagation of waves in micropolar generalized thermoelastic materials with two temperatures bordered with layers or half-spaces of inviscid liquid

Abstract

The aim of the present paper is to study the propagation of Lamb waves in micropolar generalized thermoelastic solids with two temperatures bordered with layers or half-spaces of inviscid liquid subjected to stress-free boundary conditions in the context of Green and Lindsay (G-L) theory. The secular equations for governing the symmetric and skew-symmetric leaky and nonleaky Lamb wave modes of propagation are derived. The computer simulated results with respect to phase velocity, attenuation coefficient, amplitudes of dilatation, microrotation vector and heat flux in case of symmetric and skew-symmetric modes have been depicted graphically. Moreover, some particular cases of interest have also been discussed.

Keywords

Micropolar thermoelastic solid, Two temperatures, Leaky and nonleaky Lamb waves, Phase velocity, Symmetric and Skew-symmetric amplitudes.

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1 INTRODUCTION

Eringen (1966) developed the theory of micropolar elasticity which has aroused much interest in recent years because of its possible usefulness in investigating the deformation properties of solids for which the classical theory is inadequate. There are at least two different generalizations related to the classical theory of thermoelasticity. The first one given by Lord and Shulman (1967) admits only one relaxation time and the second one given by Green and Lindsay (1972) involves two relaxation times.

The linear theory of micropolar thermoelasticity has been developed by extending the theory of micropolar continua. Eringen (1970, 1999) and Nowacki (1986) have given detailed reviews on the subject. Boschi and Iesan (1973) extended the generalized theory of micropolar thermoelasticity which allows the transmission of heat as thermal waves of finite speed. The generalized ther-

moelasticity was presented by Dost and Taborrok (1978) by using Green and Lindsay theory. Chandrasekharaiah (1986) developed a heat flux dependent micropolar thermoelasticity.

Thermoelasticity with two temperatures is one of the non-classical theories of thermoelasticity. The thermal dependence is the main difference of this theory with respect to the classical one. A theory of heat conduction in deformable bodies depends on two distinct temperatures, the conductive temperature ϕ and thermodynamic temperature T . The thermodynamic temperature $T\,\theta$ has been introduced by Chen et al. (1968, 1969). The difference between these two temperatures is proportional to the heat supply for time independent situations. For time dependent and for wave propagation problems, the two temperatures are in general different, regardless of the presence of heat supply. Warren and Chen (1973) investigated the wave propagation in the two temperatures theory of thermoelasticity.

The study related to the interaction of elastic waves with fluid loaded solids has been recognized as a viable means for deriving the non-destructive evaluation of solid structures. The reflected acoustic field from a fluid-solid interface provides details of many characteristics of solids.

These phenomena have been investigated for the simple isotropic semi-space as well as the complicated systems of multilayered anisotropic media. A detailed review in this respect is given by Nayfeh (1995). Qi (1994) investigated the influence of viscous fluid loading on the propagation of leaky Rayleigh waves in the presence of heat conduction effects. Wu and Zhu (1995) suggested an alternative approach to the treatment of Qi (1994). They presented solutions for the dispersion relations of leaky Rayleigh waves in the absence of heat conduction effects. Zhu and Wu (1995) used this technique to study Lamb waves in submerged and fluid coated plates. Nayfeh and Nagy (1997) formulated the exact characteristic equations for leaky waves propagating along the interfaces of systems which involve isotropic elastic solids loaded with viscous fluids, including half-spaces and finite thickness fluid layers.

Youssef (2006) presented a new theory of generalized thermoelasticity by considering the interaction of heat conduction in deformable bodies. A uniqueness theorem for generalized linear thermoelasticity involving a homogeneous and isotropic body was also recorded in this study. Various authors, e.g. (Puri and Jordan (2007), Youssef and Al-Lehaibi (2007), Youssef and Al-Harby (2007), Magana and Quintanilla (2009), Mukhopadhyay and Kumar (2009), Kumar and Mukhopadhyay (2010), Kaushal et al. (2010, 2011) studied the problems of thermoelastic media with two temperatures.

Various investigators have studied the wave propagation problems in micropolar media e.g. Boschi (1973), Maugin (1974), Yerofeyev and Soldatov (1999), Erofeev (2003), Altenbach et al. (2010), Eremeyev et al. (2007), Eremeyev (2005).

The problems of wave propagation in micropolar thermoelastic plates have been investigated by Nowacki and Nowacki (1969), Kumar and Gogna (1988), Tomar (2002, 2005), Kumar and Pratap (2006, 2007a,2007b, 2008, 2009, 2010), Sharma et al. (2008), Sharma and Kumar (2009).

Qingyong Sun et al. (2011) studied propagation characteristics of longitudinal displacement wave in micropolar fluid with micropolar elastic plate, Singh (2010) discussed propagation of thermoelastic waves in micropolar mixture of porous media.

In the present paper, the propagation of waves in an infinite homogeneous micropolar generalized thermoelastic plate with two temperatures bordered with layers or half-spaces of inviscid

liquid have been investigated. The secular equations have been derived. The phase velocity, attenuation coefficient, amplitudes of dilatation, microrotation vector and heat flux for the symmetric and skew-symmetric wave modes are computed numerically and presented graphically for G-L theory.

2 VIBRATION OF PLATE ON FOUNDATION AND INTEGRAL TRANSFORM

The field equations following Eringen (1966), Ezzat and Awad (2010) and Green and Lindsay (1967) in a homogeneous, isotropic, micropolar elastic medium in the context of generalized theory of thermoelasticity with two temperatures, without body forces, body couples and heat sources, are as follows

$$(\lambda+\mu)u_{j,ij}+(\mu+K)u_{i,jj}+K\varepsilon_{ijk}\varphi_{k,j}-\nu[(\Phi-a\Phi_{,jj})_{,i}+\tau_1(\dot{\Phi}-a\dot{\Phi}_{,jj})_{,i}]=\rho\ddot{u}_i, \tag{1}$$

$$(\alpha+\beta)\phi_{j,ij}+\gamma\phi_{i.jj}+K\varepsilon_{imn}u_{n,m}-2K\phi_i=\rho j\ddot{\phi}_i\ , \tag{2}$$

$$K^*\Phi_{,rr}=\rho c^*[(\dot{\Phi}+\tau_0\ddot{\Phi})-a(\dot{\Phi}_{,jj}+\tau_0\ddot{\Phi}_{,jj})]+\nu T_0\dot{u}_{j,j}\ , \tag{3}$$

and the constitutive relations are

$$t_{ij}=\lambda u_{r,r}\delta_{ij}+\mu(u_{i,j}+u_{j,i})+K(u_{j,i}-\varepsilon_{ijr}\phi_r)-\nu\left(T+\tau_1\dot{T}\right)\delta_{ij}, \tag{4}$$

$$m_{ij}=\alpha\phi_{r,r}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i}\ , \qquad i,j,r=\ 1,2,3 \tag{5}$$

λ and μ are Lame's constants. K, α, β and γ are micropolar constants. t_{ij} and m_{ij} are the components of stress tensor and couple stress tensor respectively. u_i and φ_i are the displacement and microrotation vectors, ρ is the density , j is the microinertia, K^* is the thermal conductivity, c^* is the specific heat at constant strain, T is the thermodynamic temperature, Φ is the conductive temperature, T_0 is the reference temperature, $\nu=(3\lambda+2\mu+K)\alpha_T$, where α_T is the coefficient of linear thermal expansion, τ_0 and τ_1 are the thermal relaxation time δ_{ij} is the Kronecker delta, ε_{ijr} is the alternating tensor. The relation connecting T and Φ is given by $T=(1-a\nabla^2)\Phi$, where a is a two temperature parameter.

Following Achenbach (1976), the field equations can be expressed in terms of velocity potential for inviscid fluid as

$$p_L = -\rho_L \dot{\varphi}_L, \tag{6}$$

$$\begin{gathered}(\varphi_{L,ii} - \frac{1}{c_L^2}\ddot{\varphi}_L) = 0, \\ \boldsymbol{u}_L = \nabla \varphi_L,\end{gathered} \tag{7}$$

where $c_L^2 = \dfrac{\lambda_L}{\rho_L}$ is the velocity of acoustic fluid, λ_L is the bulk modulus, ρ_L is the density of the fluid, p_L is the acoustic pressure in the fluid, φ_L is the velocity potential of the fluid, $\boldsymbol{u}_L$ is the velocity vector, ∇ is gradient operator, ∇^2 is the Laplacian operator.

3 NUMERICAL RESULTS

An infinite homogeneous isotropic, thermally conducting micropolar thermoelastic plate of thickness $2d$ initially undisturbed and at uniform temperature T_0 is considered. We consider that two infinitely large homogeneous inviscid liquid half-spaces or layers of thickness h bordered the plate on both sides as shown in figures 1(a) and 1(b) respectively. The origin of the coordinate system (x_1, x_2, x_3) is taken on the middle of the plate and the axis normal to the solid plate along the thickness is taken as x_3-axis. We consider the propagation of plane waves in the x_1x_3 −plane with a wavefront parallel to the x_2-axis so that field components are independent of x_2-coordinates.

The displacement and microrotation vectors for two dimensional problem are taken as

$$\mathbf{u} = \left(u_1(x_1, x_3), 0, u_3(x_1, x_3)\right), \qquad \varphi = \left(0, \varphi_2(x_1, x_3), 0\right) \tag{8}$$

For inviscid fluid, we take

$$\boldsymbol{u}_L = \left(u^L(x_1, x_3), 0, w^L(x_1, x_3)\right)$$

The physical quantities can be made dimensionless by defining the following

$$x_1^{'} = \frac{\omega^* x_1}{c_1}, \quad x_3^{'} = \frac{\omega^* x_3}{c_1}, \quad u_1^{'} = \frac{\rho\omega^* c_1}{\nu T_0} u_1, \quad u_3^{'} = \frac{\rho\omega^* c_1}{\nu T_0} u_3,$$

$$\phi_2^{'} = \frac{\rho c_1^{\,2}}{\nu T_0}\phi_2, \quad t^{'} = \omega^* t, \quad \tau_1^{'} = \omega^*\tau_1, \quad T^{'} = \frac{T}{T_0}, \quad \Phi^{'} = \frac{\Phi}{T_0},$$

$$t_{ij}^{'} = \frac{1}{\nu T_0} t_{ij}, \quad m_{ij}^{'} = \frac{\omega^*}{c_1 \nu T_0} m_{ij}, \quad \tau_0^{'} = \omega^*\tau_0, \quad a^{'} = \frac{\omega^{*2}}{c_1^2} a, \tag{9}$$

$$u^{L'} = \frac{\rho c_1}{\nu T_0} u^L, \quad w^{L'} = \frac{\rho c_1}{\nu T_0} w_L, \quad \phi_L^{'} = \frac{\rho\omega^*}{\nu T_0}\phi_L, \quad c_L^2 = \frac{\lambda_L}{\rho_L}, \quad p_L^{'} = \frac{1}{\nu T_0} p_L,$$

$$h^{'} = \frac{c_1 h}{\omega^*}, \quad d^{'} = \frac{\omega^* d}{c_1}$$

where

$$\omega^* = \frac{\rho c^* c_1^2}{K^*}, \; c_1^2 = \frac{\lambda + 2\mu + K}{\rho}$$

such that ω^* is the characteristic frequency of the medium.

The displacement components u_1, u_3 and u_3 are related to the potential functions ϕ, ψ and ψ in dimensionless form as

$$u_1 = \frac{\partial\phi}{\partial x_1} - \frac{\partial\psi}{\partial x_3}, \qquad u_3 = \frac{\partial\phi}{\partial x_3} + \frac{\partial\psi}{\partial x_1}, \tag{10}$$

In the liquid layers, we have

$$u_i^L = \frac{\partial\phi_{L_i}}{\partial x_1}, \qquad w_i^L = \frac{\partial\phi_{L_i}}{\partial x_3}, \tag{11}$$

where ϕ_{L_i} are the scalar velocity potential components for the top liquid layer ($i=1$) and for the bottom liquid layer ($i=2$). u_i^L and w_i^L ($i=1,2$) are the x_1 and x_3 components of the particle velocity for the top liquid layer and the bottom liquid layer respectively.

Using eqs. (8)-(11) in eqs. (1)-(3) and (7) and after suppressing the primes, we obtain the following equations

$$\nabla^2\phi-\left(1+\tau_1\frac{\partial}{\partial t}\right)(1-a\nabla^2)\Phi-\frac{\partial^2\phi}{\partial t^2}=0, \tag{12}$$

$$\nabla^2\psi+a_1\phi_2-a_2\frac{\partial^2\psi}{\partial t^2}=0\,, \tag{13}$$

$$\nabla^2\phi_2-a_3\nabla^2\psi-a_4\phi_2-a_5\frac{\partial^2\phi_2}{\partial t^2}=0\,, \tag{14}$$

$$\nabla^2\Phi=a_6\left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}(1-a\nabla^2)\Phi+a_7\frac{\partial}{\partial t}\nabla^2\phi\,, \tag{15}$$

$$\nabla^2\phi_{L_i}-\frac{1}{\delta_L^2}\frac{\partial^2\phi_{L_i}}{\partial t^2}=0,\quad i=1,2 \tag{16}$$

where

$$a_1=\frac{K}{\mu+K}\,,\qquad a_2=\frac{\rho c_1^2}{\mu+K}\,,\qquad a_3=\frac{Kc_1^2}{\gamma\omega^{*2}}\,,\qquad a_4=2\,a_3\,,\qquad a_5=\frac{\rho jc_1^2}{\gamma}\,,$$
$$a_6=\frac{\rho c^{*}c_1^2}{K^{*}\omega^{*}},\qquad a_7=\frac{\nu^2T_0}{\rho\mathrm{K}^{*}\omega^{*}}\,,\ \nabla^2=\frac{\partial^2}{\partial x_1{}^2}+\frac{\partial^2}{\partial x_3{}^2}\,,\quad \delta_L^2=\frac{c_L^2}{c_1^2}$$

The solutions of eqs. (12)-(16) are assumed as

$$(\varphi,\psi,\varphi_2,\Phi,\varphi_{L_1},\varphi_{L_2})=\left[f_1(x_3),f_2(x_3),f_3(x_3),f_4(x_3),f_5(x_3),f_6(x_3)\right]e^{t\xi(x_1-ct)}, \tag{17}$$

such that $c=\dfrac{\omega}{\xi}$ is the phase velocity, ω is the frequency and ξ is the wave number.

Making use of eq. (17) in eqs. (12)-(16) and subsequent elimination of $f_4(x_3)$ and $f_3(x_3)$ from the resulting equations, yields the equations

$$(\nabla^{*4}+A\nabla^{*2}+B)f_1(x_3)=0, \tag{18}$$

$$(\nabla^{*4}+C\nabla^{*2}+D)f_2(x_3)=0, \tag{19}$$

such that $\nabla^{*2}=d^2/dx_3{}^2-\xi^2$ and the values of A,B,C and D are given in Appendix.

The roots of eqs. (18) and (19) are given as $n_{1,2}^2 = \frac{1}{2}\left[-A \pm \sqrt{A^2 - 4B}\right]$ and $n_{3,4}^2 = \frac{1}{2}\left[-C \pm \sqrt{C^2 - 4D}\right]$

The appropriate potentials $\varphi, \Phi, \psi, \varphi_2, \varphi_{L_1}$ and φ_{L_2} are taken as

$$\varphi = (A_1 \cos n_1 x_3 + A_2 \cos n_2 x_3 + B_1 \sin n_1 x_3 + B_2 \sin n_2 x_3) e^{\iota\xi(x_1 - ct)}, \tag{20}$$

$$\Phi = (h_1 A_1 \cos n_1 x_3 + h_2 A_2 \cos n_2 x_3 + h_1 B_1 \sin n_1 x_3 + h_2 B_2 \sin n_2 x_3) e^{\iota\xi(x_1 - ct)}, \tag{21}$$

$$\psi = (A_3 \cos n_3 x_3 + A_4 \cos n_4 x_3 + B_3 \sin n_3 x_3 + B_4 \sin n_4 x_3) e^{\iota\xi(x_1 - ct)}, \tag{22}$$

$$\phi_2 = (h_3 A_3 \cos n_3 x_3 + h_4 A_4 \cos n_4 x_3 + h_3 B_3 \sin n_3 x_3 + h_4 B_4 \sin n_4 x_3) e^{\iota\xi(x_1 - ct)}, \tag{23}$$

$$\phi_{L_1} = (E_5 e^{\gamma_L x_3} + F_5 e^{-\gamma_L x_3}) e^{\iota\xi(x_1 - ct)}, \tag{24}$$

$$\phi_{L_2} = (E_6 e^{\gamma_L x_3} + F_6 e^{-\gamma_L x_3}) e^{\iota\xi(x_1 - ct)}, \tag{25}$$

where the expressions for $h_i, h_j; i = 1, 2, \quad j = 3, 4$ are given in Appendix.

4 BOUNDARY CONDITIONS

We consider the following boundary conditions at the solid-liquid interfaces $x_3 = \pm d$:

(i) Continuity of the normal stress component of solid and liquid.

$$(t_{33})_s = -p_L \tag{26}$$

(ii) Vanishing of the tangential stress component.

$$(t_{31})_s = 0 \tag{27}$$

(iii) Vanishing of the tangential couple stress component.

$$(m_{32})_s = 0 \tag{28}$$

(iv) Continuity of the normal velocity component of solid and liquid.

$$(\dot{u}_3)_s = (w^L) \tag{29}$$

(v) The thermal boundary condition is given by

$$\frac{\partial T}{\partial x_3} + HT = 0, \tag{30}$$

where H is the surface heat transfer coefficient. Here $H \to 0$ and $H \to \infty$ corresponds to thermally insulated and isothermal boundaries, respectively.

4.1 Leaky Lamb Waves

The solutions for solid media of finite thickness $2d$ sandwiched between two liquid half-spaces is given by eqs. (20)-(23) together with

$$\phi_{L_1} = E_5 e^{\gamma_L (x_3 + d)} e^{\iota\xi(x_1 - ct)}, -\infty < x_3 < -d \tag{31}$$

$$\phi_{L_2} = F_6 e^{-\gamma_L (x_3 - d)} e^{\iota\xi(x_1 - ct)}, d < x_3 < \infty \tag{32}$$

4.2 Nonleaky Lamb Waves

The corresponding solutions for a solid media of finite thickness $2d$ sandwiched between two finite liquid layers of thickness h are given by eqs. (20)-(23) and

$$\phi_{L_1} = E_5 \sinh \gamma_L [x_3 + (d + h)] e^{\iota\xi(x_1 - ct)}, -(d + h) < x_3 < -d \tag{33}$$

$$\phi_{L_2} = F_6 \sinh \gamma_L [x_3 - (d + h)] e^{\iota\xi(x_1 - ct)}, d < x_3 < (d + h) \tag{34}$$

Nonleaky and leaky Lamb waves are distinguished by selecting the functions ϕ_{L_1} and ϕ_{L_2} in such a way that the acoustical pressure is zero at $x_3 = \mp(d + h)$. This shows that ϕ_{L_1} and ϕ_{L_2} are solutions of standing wave and travelling wave for nonleaky Lamb waves and leaky Lamb waves respectively.

5 DISPERSION EQUATIONS

We apply the formal solutions of previous section to study the specific situations with inviscid fluid.

5.1 Leaky Lamb Waves

We consider an isotropic thermoelastic micropolar plate with two temperatures completely immersed in a inviscid liquid as shown in fig. 1(a).

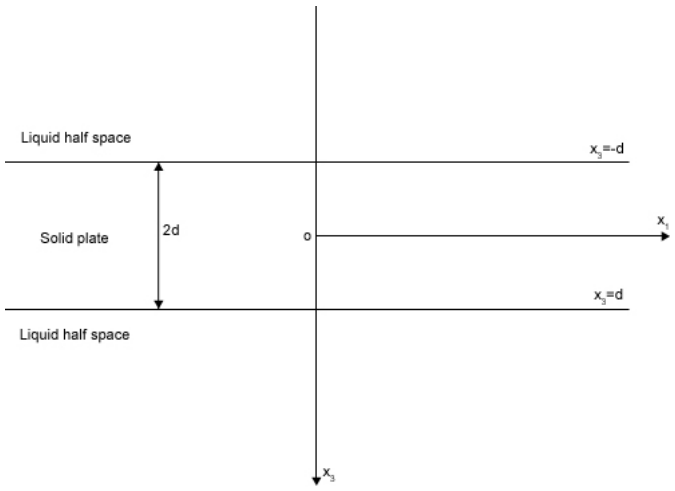

Figure 1(a) Geometry of leaky Lamb waves

The thickness of the plate is $2d$ and thus the lower and upper portions of the fluid extend from d to ∞ and $-d$ to $-\infty$ respectively. In this case, the partial waves exist in the plate as well as the fluid. The appropriate formal solutions for the plate and fluids are those given by eqs. (20)-(23), (31) and (32). By applying the boundary conditions (26)-(30) at $x_3 = \pm d$ and subsequently selecting nontrivial values of the partial wave amplitudes E_k and F_k $(k = 1,2,3,4)$; E_5, F_6 and $\gamma_L \neq 0$, we arrive at the characteristic dispersion equations as

$$\left[\frac{T_1}{T_3}\right]^{\pm 1} - \left[\frac{T_2}{T_3}\right]^{\pm 1} P_1 - \left[\frac{T_1}{T_4}\right]^{\pm 1} P_2 + \left[\frac{T_2}{T_3}\right]^{\pm 1} P_1 P_2 + \frac{P_3 S}{n_4 G}\left[\frac{P_4 n_3 h_3}{h_4 n_2 [T_4]^{\pm}} - \frac{P_5}{[T_3]^{\pm}}\right] = \frac{-P_3 Q m_3 n_3 (h_4 - h_3)}{h_4} \tag{35}$$

for stress-free thermally insulated boundaries ($H \to 0$) of the plate
where

$$P_1 = \frac{n_1 h_1 m_2}{n_2 h_2 m_1}, \quad P_2 = \frac{n_3 h_3 m_6}{n_4 h_4 m_5}, \quad P_3 = \frac{(h_1 - h_2) n_1}{m_1 m_5 h_2}, \qquad P_4 = (\xi^2 c m_3 + R m_6), \quad P_5 = (\xi^2 c m_3 + R m_5)$$

and

$$\left[\frac{T_1}{T_3}\right]^{\pm 1} - \left[\frac{T_2}{T_3}\right]^{\pm 1} P_6 + \left[\frac{T_4}{T_3}\right]^{\pm 1} \frac{P_7 h_3 m_6}{n_4 Q} + \left(1 - \frac{P_4 n_3 h_3}{P_5 n_4 h_4}\left[\frac{T_4}{T_3}\right]^{\pm 1}\right) P_8 \left(\left[\frac{T_1}{T_3}\right]^{\pm 1} - \left[\frac{T_2}{T_3}\right]^{\pm 1} P_6\right) = \frac{P_7 h_4 m_5}{n_3} \tag{36}$$

for stress-free isothermal boundaries ($H \to \infty$) of the plate.
where

$$P_6 = \frac{n_2 h_1}{n_1 h_2}, \quad P_7 = \frac{(m_1 h_2 - m_2 h_1)}{n_1 h_2 m_3 (h_4 - h_3)}, \quad P_8 = \frac{P_5 h_4 S}{n_3 m_3 QG(h_3 - h_4)}$$

5.2 Nonleaky Lamb Waves

We consider an isotropic thermoelastic micropolar plate with two temperatures bordered with layers of inviscid liquid on both sides as shown in Fig. 1(b).

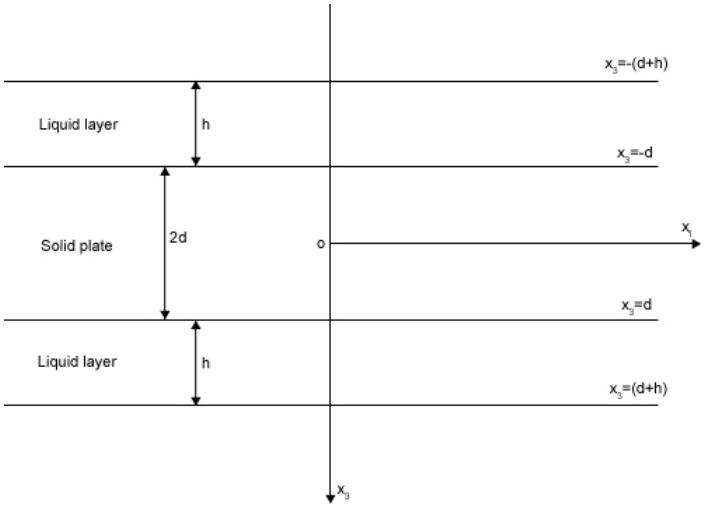

Figure 1(b) Geometry of nonleaky Lamb waves

The appropriate formal solutions for the plate and fluids are given by eqs. (20)-(23), (33) and (34). By applying the boundary conditions (26)-(30) at $x_3 = \pm d$ and subsequently selecting non-trivial values of the partial wave amplitudes E_k and F_k $(k = 1,2,3,4)$; E_5, F_6 and $\gamma_L \neq 0$, we arrive at the characteristic dispersion equations as

$$\left[\frac{T_1}{T_3}\right]^{\pm 1} - \left[\frac{T_2}{T_3}\right]^{\pm 1} P_1 - \left[\frac{T_1}{T_4}\right]^{\pm 1} P_2 + \left[\frac{T_2}{T_3}\right]^{\pm 1} \frac{P_2 n_1 h_1 m_6}{n_2 h_2 m_2} + \frac{P_3 S}{n_4 G} T_5 \left[\frac{P_4 n_3 h_3}{h_4 n_2 [T_4]^{\pm}} - \frac{P_5}{[T_3]^{\pm}}\right] = -\frac{P_3 Q m_3 n_3 (h_4 - h_3)}{h_4} \tag{37}$$

for stress-free thermally insulated boundaries ($H \to 0$) of the plate.

$$\left[\frac{T_1}{T_3}\right]^{\pm 1} - \left[\frac{T_2}{T_3}\right]^{\pm 1} P_6 + \left[\frac{T_4}{T_3}\right]^{\pm 1} \frac{P_7 h_3 m_6}{n_4 Q} + \left(1 - \frac{P_4 n_3 h_3}{P_5 n_4 h_4}\left[\frac{T_4}{T_3}\right]^{\pm 1}\right) P_8 T_3 T_5 \left(\left[\frac{T_1}{T_3}\right]^{\pm 1} - \left[\frac{T_2}{T_3}\right]^{\pm 1} P_6\right) = \frac{P_7 h_4 m_5}{n_3} \tag{38}$$

for stress-free isothermal boundaries ($H \to \infty$) of the plate.

Here the superscript +1 refers to skew-symmetric and -1 refers to symmetric modes. All the coefficients and other quantities are recorded in Appendix.

Eqs. (35) and (38) are the general dispersion relations involving wave number and phase velocity of various modes of propagation in a micropolar thermoelastic plate bordered with layers of inviscid liquid or half-spaces on both sides.

5.3 Special cases

The removal of the liquid layers or half-spaces on the both sides, provide the wave propagation in micropolar thermoelastic solid with two temperatures. Analytically, if we take $\rho_L = 0$ in eqs. (35) and (37) then the secular equations for stress-free thermally insulated boundaries ($H \to 0$) for the said case reduce to

$$(T_1T_3)^{\pm}N_1 + (T_1T_4)^{\pm}N_2 + (T_2T_3)^{\pm}N_3 + (T_2T_4)^{\pm}N_4 + (T_3T_4)^{\pm}N_5 = 0 \tag{39}$$

where

$$N_1 = h_2h_3n_2n_3m_1m_6G, \quad N_2 = -h_2h_4n_2n_4m_1m_5G, \quad N_3 = -h_1h_3n_1n_3m_2m_6G,$$
$$N_4 = h_1h_4n_1n_4m_2m_5G, \quad N_5 = n_1n_2n_3n_4m_3QG(h_4 - h_3)(h_2 - h_1)$$

If we take $a = 0$ in eq. (30), then we obtain the secular equations in micropolar generalized thermoelastic plate.

6 AMPLITUDES OF DILATATION, MICROROTATION AND HEAT FLUX

The amplitudes of dilatation, microrotation and heat flux for symmetric and skew-symmetric modes have been computed for micropolar thermoelastic plate. By using eq. (17) in (12)-(16) and then using eqs. (20)-(25), we obtain

$$(e)_{sy} = [-M_1 \cos n_1x_3 + \frac{M_2h_1n_1S_1}{h_2n_2S_2}\cos n_2x_3]A_1e^{\iota\xi(x_1-ct)}, \tag{40}$$

$$(e)_{asy} = [-M_1 \sin n_1x_3 + \frac{M_2h_1n_1C_1}{h_2n_2C_2}\sin n_2x_3]B_1e^{\iota\xi(x_1-ct)}, \tag{41}$$

$$(\phi_2)_{sy} = [h_3 \sin n_3x_1 - \frac{h_3n_3C_3}{n_4C_4}\sin n_4x_3]B_3e^{\iota\xi(x_1-ct)}, \tag{42}$$

$$(\phi_2)_{asy} = [h_3 \cos n_3 x_3 - \frac{h_3 n_3 S_3}{n_4 S_4} \cos n_4 x_3] A_3 e^{\iota\xi(x_1 - ct)}, \tag{43}$$

$$(q)_{sy} = [n_1 h_1 K^* \sin n_1 x_3 - \frac{h_1 n_1 S_1}{n_2 S_2} n_1 K^* \sin n_2 x_3] A_1 e^{\iota\xi(x_1 - ct)}, \tag{44}$$

$$(q)_{asy} = [-n_1 h_1 K^* \cos n_1 x_3 + \frac{h_1 n_1 C_1}{C_2} K^{*'} \cos n_2 x_3] B_1 e^{\iota\xi(x_1 - ct)}. \tag{45}$$

7 NUMERICAL RESULTS AND DISCUSSION

For numerical computation we select Magnesium crystal (micropolar thermoelastic solid). The physical data for this medium is given below:

(i) The values of micropolar constants are taken from Eringen (1984):

$$\lambda = 9.4\times10^{10}\, Nm^{-2}, \quad \mu = 4.0\times10^{10}\, Nm^{-2}, \quad \gamma = 0.779\times10^{-9}\, N,$$
$$K = 1.0\times10^{10}\, Nm^{-2}, \quad \rho = 1.74\times10^{3}\, kgm^{-3}, \quad j = 2.0\times10^{-20}\, m^2;$$

(ii) and thermal parameters are taken from Dhaliwal and Singh (1980):

$$\nu = 0.268\times10^{7}\, Nm^{-2}K^{-1}, \quad c^* = 1.04\times10^{3}\, m^2 sec^{-2}K^{-1},$$
$$T_0 = 0.298K, \quad K^* = 1.7\times10^{2}\, Nsec^{-1}K^{-1}, \quad a = 0.5,$$
$$\tau_0 = 0.613\times10^{-12}\, sec, \quad \tau_1 = 0.813\times10^{-12}\, sec, \quad \omega = 1, \quad d = 1.0m$$

For numerical calculations, water is taken as liquid and the speed of sound in water is given by

$$c_L = 1.5\times10^{3}\, m/\sec$$

In general, wave number and phase velocity of the waves are complex quantities, therefore, the waves are attenuated in space. If we write

$$C^{-1} = V^{-1} + \iota\omega^{-1}Q, \tag{45}$$

then $\xi = R + \iota Q$, where $R = \omega/V$ and Q are real numbers. This shows that V is the propagation speed and Q is the attenuation coefficient of waves. Using eq. (45) in secular eqs. (35) and (37), the value of propagation speed V and attenuation coefficient Q for different modes of propagation can be obtained.

In figures 2 to 9, GLS and GNLS refer to leaky and nonleaky symmetric waves in micropolar thermoelastic solid with two temperatures, GLSK and GNLSK refer to leaky and nonleaky skew-symmetric waves in micropolar thermoelastic solid with two temperatures, GALS and GANLS refer to leaky and nonleaky symmetric waves in micropolar thermoelastic solid, GALSK and GANLSK refer to leaky and nonleaky skew-symmetric waves in micropolar thermoelastic solid. In figures 10 to 15, GT represents the amplitude for micropolar thermoelastic solid with two temperatures and TS represents the amplitude for micropolar thermoelastic solid.

7.1 Phase velocity

Leaky Lamb Waves

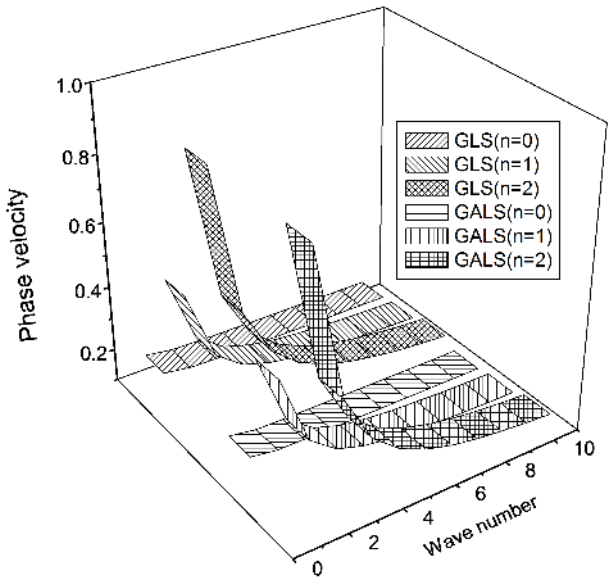

Figure 2 Variations of phase velocity for symmetric leaky Lamb waves

It is depicted from fig. 2 that the phase velocity for lowest symmetric mode of propagation for GLS and GALS coincide. The magnitude of phase velocity for GALS is greater than GLS for (n=1) symmetric mode of propagation for $\xi d = 1$, $3 \le \xi d \le 6$ and in the remaining range the behavior is reversed. It is noticed that for (n=2) mode, the phase velocities for GALS remain more than the values for GLS for $1 \le \xi d \le 3$, $\xi d = 6$ and in the remaining region, the behavior is reversed.

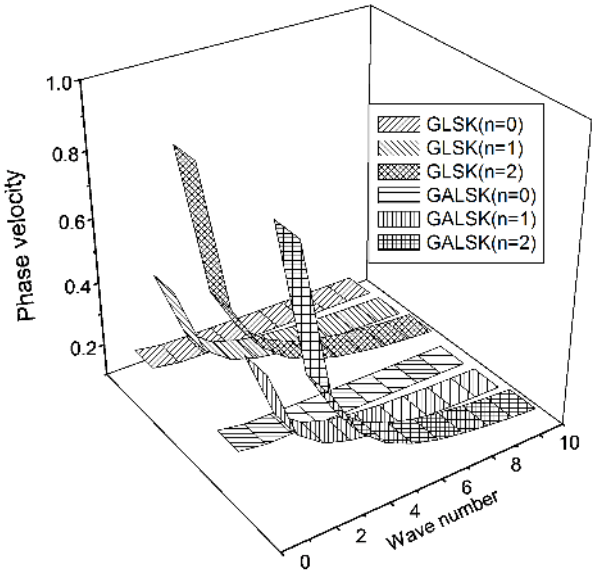

Figure 3 Variations of phase velocity for skew-symmetric leaky Lamb waves

It is noticed from figs. 3 that the phase velocities for (n = 0) and (n = 1) skew-symmetric leaky Lamb wave mode of propagation for GLSK and GALSK coincide. The velocities for GALSK are greater than the velocities for GLSK for wave number $\xi d = 2$ and in the remaining region, the velocities coincide.

Leaky Lamb Waves

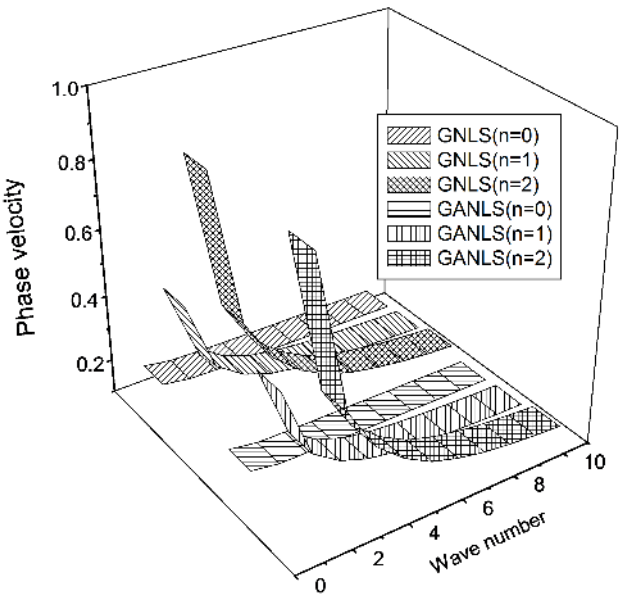

Figure 4 Variations of phase velocity for symmetric nonleaky Lamb waves

Fig. 4 shows that for (n=0) symmetric mode of propagation, the velocities for GANLS are greater than GNLS for $\xi d = 2, 4 \le \xi d \le 6, 8 \le \xi d \le 10$ and in the remaining region, the behavior is opposite. It is also noted that for (n=1) mode, the phase velocities for GANLS are greater than the phase velocities for GNLS in the whole region. The values for GANLS are greater than the values for GNLS in the whole region, except for $\xi d = 1$.

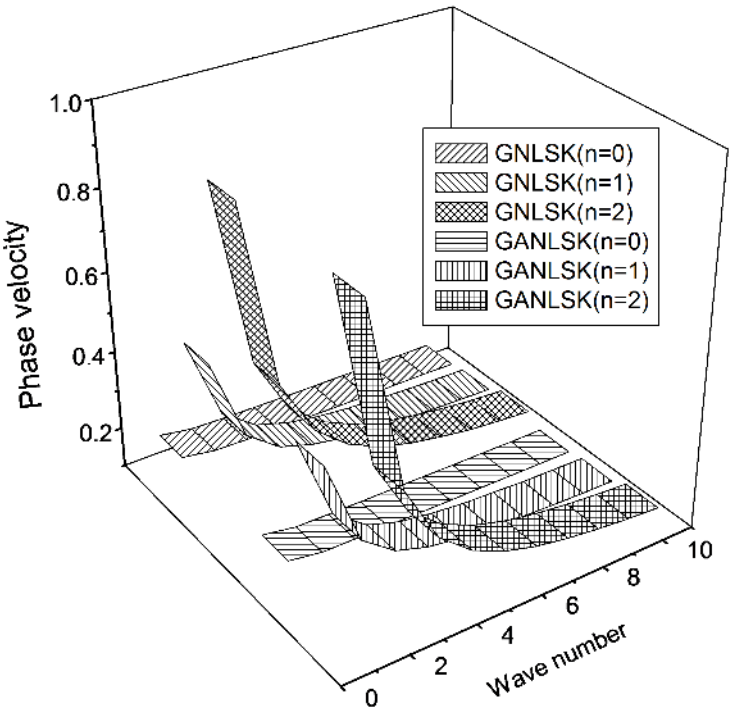

Figure 5 Variations of phase velocity for skew-symmetric nonleaky Lamb waves

Fig. 5 depicts that for (n=0) skew-symmetric mode of propagation, the velocities for GNLSK and GANLSK coincide. It is evident that for (n=1) mode, the phase velocities for GANLSK and the values for GNLSK differ near the vanishing wave number and with increase in wave number they coincide. For (n=2) skew-symmetric mode, the phase velocities for NLSK and ANLSK coincide in the whole region, except for $\xi d = 3$.

7.2 Attenuation Coefficients

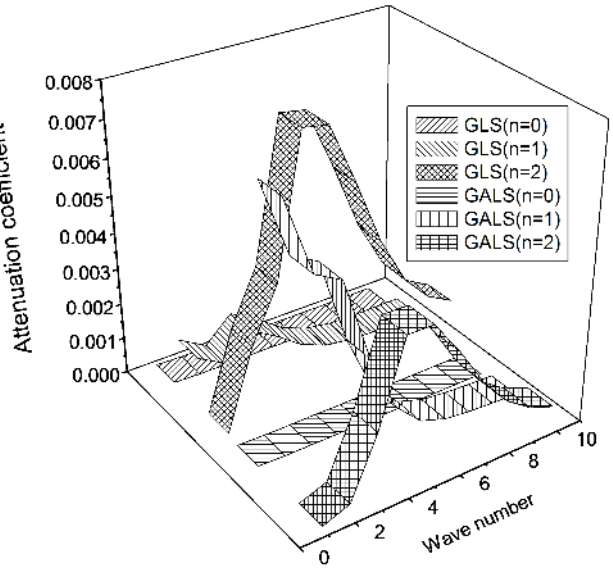

Figure 6 Variations of attenuation coefficient for symmetric leaky Lamb waves

Fig. 6 depicts that for symmetric leaky Lamb wave mode (n=0), the attenuation coefficient for GLS remain more than the attenuation coefficient for GALS when $\xi d = 1$ and $3 \le \xi d \le 6$, while in the remaining region, the behavior is reversed. For (n=1), the values for GLS oscillate and attain maximum value at $\xi d = 10$. It is noticed that for (n=2), the attenuation coefficient for GALS remain more than the values for GLS in the whole region, except in the region $7 \le \xi d \le 10$, where the behavior is reversed.

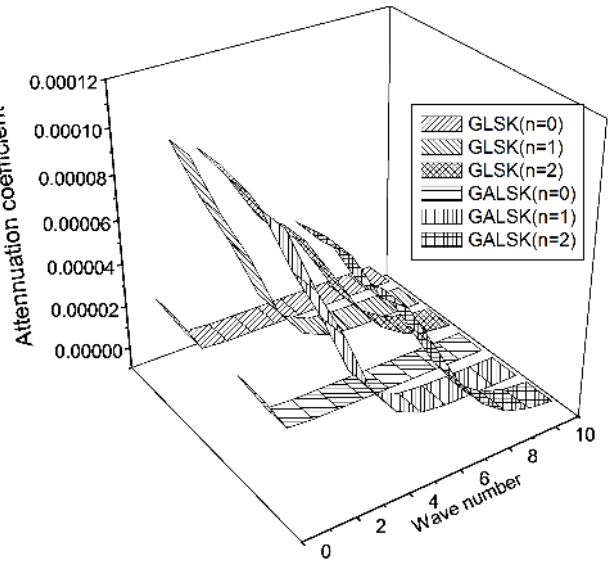

Figure 7 Variations of attenuation coefficient for skew-symmetric leaky Lamb waves

Fig. 7 shows that for (n=0) mode, the magnitude of attenuation for GLSK and GALSK attain maximum value at $\xi d = 1$. For (n=1) skew-symmetric mode, the values for GLSK decrease with increase in wave number. It is noticed that for (n=2) mode, the magnitude of attenuation coefficient for GALSK remain more than in case of GLSK in the whole region, except for $\xi d = 1$, where the values coincide.

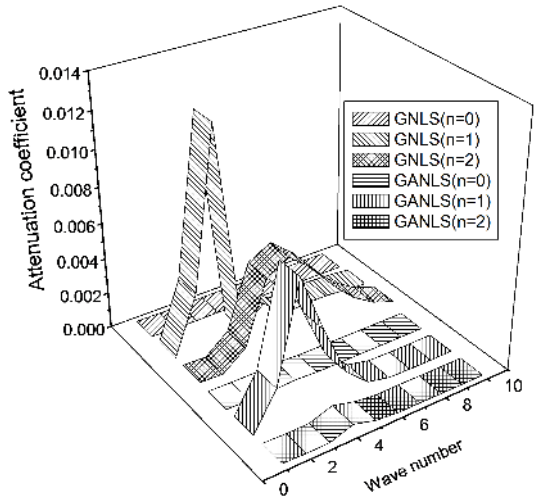

Figure 8 Variations of attenuation coefficient for symmetric nonleaky Lamb waves

It is evident from fig. 8 that for symmetric nonleaky Lamb wave mode (n=0), the attenuation coefficient for GNLS and GANLS attain maximum value at $\xi d = 1$. It is noticed that the magnitude of attenuation coefficient for GNLS and GANLS attain maximum value 0.01212 and 0.00756 at $\xi d = 3$ respectively for (n=1) mode. For (n=2) mode, the values for GNLS and GANLS attain maximum value at $\xi d = 5.$

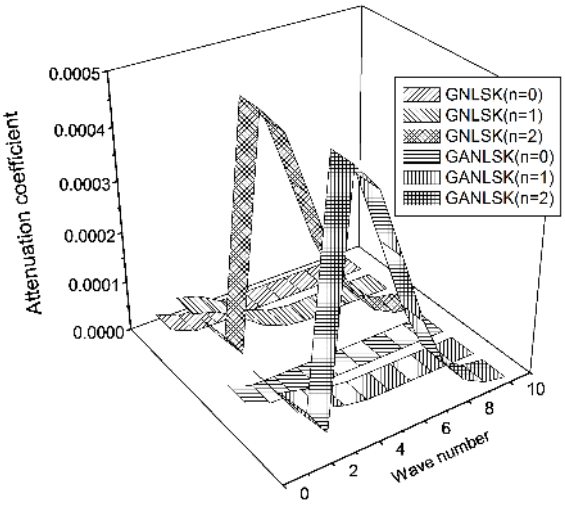

Figure 9 Variations of attenuation coefficient for skew-symmetric nonleaky Lamb waves

Fig. 9 depicts that for (n=0) skew-symmetric nonleaky Lamb wave mode of propagation, the magnitude of attenuation coefficient for GNLSK and GANLSK decrease with increase in wave number. It is depicted that for (n=1) mode, the magnitude for GNLSK and GANLSK attain maximum value at $\xi d = 1$. For (n=2) mode, the values for GANLSK are greater than GNLSK in the whole region except for $\xi d = 2,10$.

7.3 Amplitudes

In figures 10 to 15, GT represents the amplitude for micropolar thermoelastic solid with two temperatures and TS represents the amplitude for micropolar thermoelastic solid.

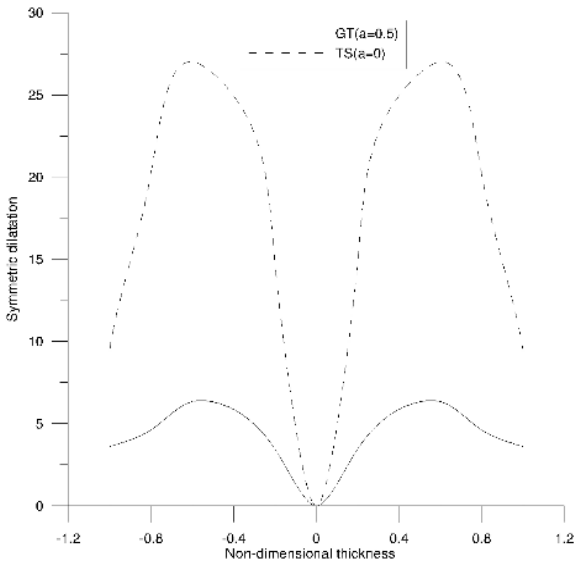

Figure 10 Amplitude of symmetric dilatation

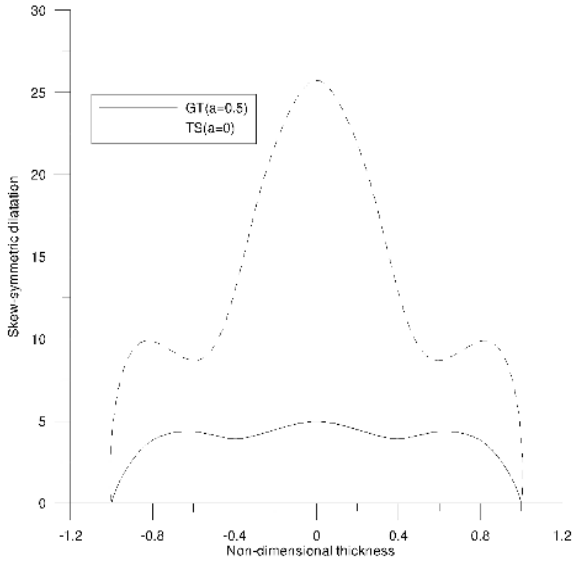

Figure 11 Amplitude of skew-symmetric dilatation

Figs. 10 and 11 depict the variations of symmetric and skew-symmetric amplitudes of dilatation for G-L theory for stress-free thermally insulated boundary. The dilatation is having minimum value at the centre and maximum value between the centre and the surfaces for symmetric mode and maximum value at the centre for skew-symmetric mode. It is observed that the dilatation for TS remain more than the dilatation for GT in the whole region for both symmetric and skew-symmetric modes.

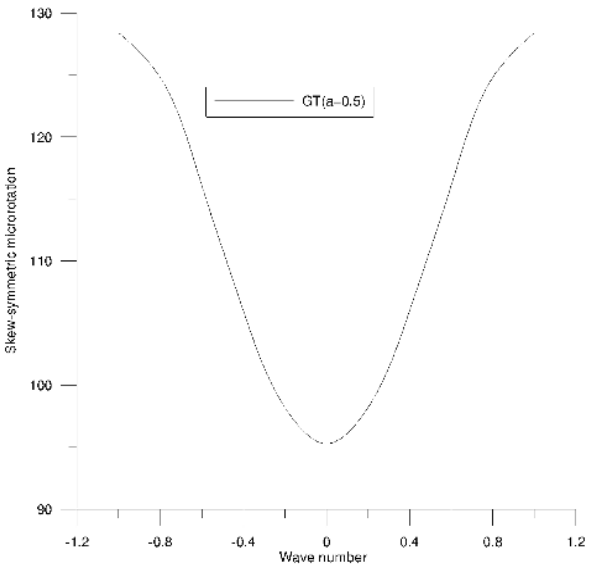

Figure 12 Amplitude of symmetric microrotation

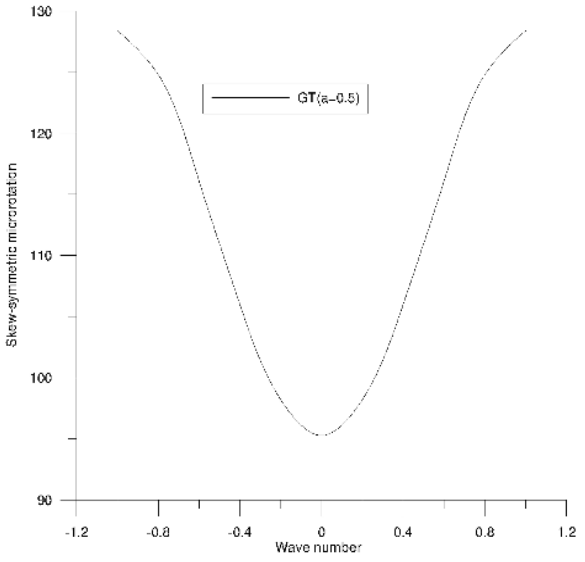

Figure 13 Amplitude of skew-symmetric microrotation

It is evident from figs. 12 and 13 that the amplitude of symmetric and skew-symmetric microrotation is minimum at the centre and obtain maximum value at the surfaces.

The amplitude of symmetric and skew-symmetric heat flux is oscillatory. For symmetric mode and skew-symmetric mode, the amplitude of heat flux attain maximum value at the sufaces and minimum value at the centre, as shown in figures 14 and 15.

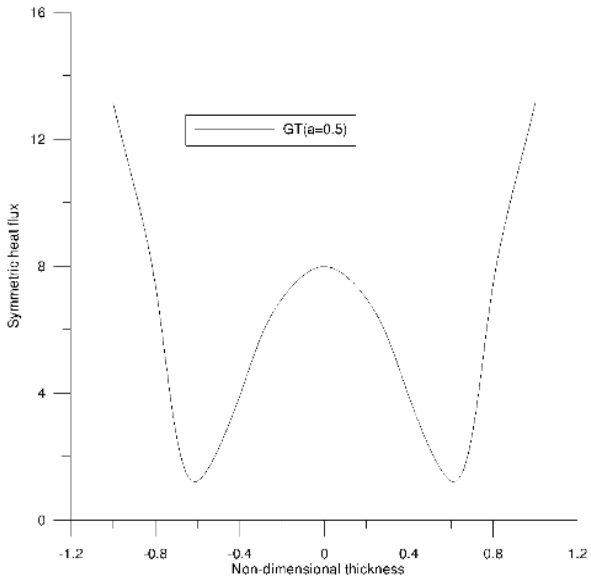

Figure 14 Amplitude of symmetric heat flux

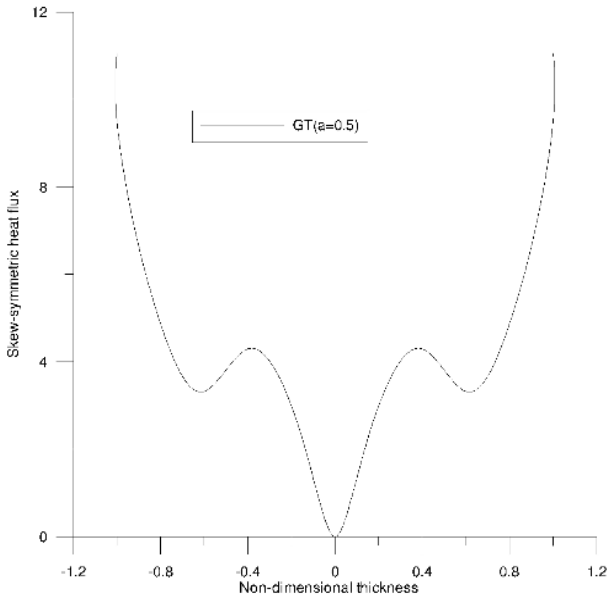

Figure 15 Amplitude of skew-symmetric heat flux

8 CONCLUSION

It is observed that the variation of phase velocities of lowest symmetric and skew-symmetric mode for leaky and nonleaky Lamb waves almost coincide with increase in wave number. The phase velocities for higher symmetric and skew-symmetric mode attain maximum value at vanishing wave number and as wave number increase the phase velocities get decreased sharply. For (n=2) symmetric mode, the attenuation coefficient for GLS is greater than the values for GALS in the whole region . It is noticed that the values of attenuation coefficient for lowest symmetric and skew-symmetric mode for leaky and nonleaky Lamb waves are very small as compared to the values for highest mode. The values of symmetric and skew-symmetric dilatation for TS are higher in comparison to GT that reveals the effect of two temperatures.

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Appendix

$$A=\frac{-aa_6\tau_0^{'}+\frac{1}{\xi^2c^2}+\frac{a_6\tau_0^{'}}{\xi^2c^2}+\frac{a_7\tau_0^{'}}{\xi c}(\frac{1}{\xi c}-\iota\tau_1)}{\frac{-aa_6\tau_0^{'}}{\xi^2c^2}+\frac{1}{\xi^4c^4}-\frac{aa_7}{\xi c}\tau_0^{'}(\frac{1}{\xi c}-\iota\tau_1)},\qquad B=\frac{a_6\tau_0^{'}}{\frac{-aa_6\tau_0^{'}}{\xi^2c^2}+\frac{1}{\xi^4c^4}-\frac{aa_7}{\xi c}\tau_0^{'}(\frac{1}{\xi c}-\iota\tau_1)},$$

$$C=\xi^2c^2[(a_5+a_2)+\frac{a_1a_3-a_4}{\xi^2c^2}],\quad D=a_2(a_5-\frac{a_4}{\xi^2c^2})\xi^4c^4,\quad \tau_0^{'}=\tau_0+\frac{\iota}{\xi c},$$

$$\gamma_L{}^2=\xi^2(1-\frac{c^2}{\delta_L^2}),$$

$$h_i = \frac{(r_1^2 - n_i^2)}{(1 + a\xi^2 + an_i^2)(1 - \iota\tau_1\xi c)}, \qquad h_j = \frac{-(r_2^{\,2} - n_j^2)}{a_1}, \qquad i = 1,2 \quad j = 3,4,$$

$$n_i^2 = \xi^2(c^2\alpha_i^2 - 1), \quad i = 1,2,3,4, \quad r_1^2 = \xi^2(c^2 - 1), \quad r_2^2 = \xi^2(c^2 a_2 - 1),$$

$$\alpha_i^2 = \frac{-k_0 \pm \sqrt{k_0^2 - 4a_6\tau_0^{'}k_1}}{2k_1^{'}},$$

$$\alpha_j^2 = \frac{1}{2}[(a_5 + a_2) + \frac{a_1a_3 - a_4}{\xi^2c^2} \pm \sqrt{[(a_5 + a_2) + \frac{a_1a_3 - a_4}{\xi^2c^2}]^2 - 4(a_5 - \frac{a_4}{\xi^2c^2})a_2}\,], \quad i = 1,2,\ j = 3,4,$$

$$k_0 = -aa_6\tau_0^{'} + \frac{1}{\xi^2c^2} + [\frac{a_6\tau_0^{'}}{\xi^2c^2} + \frac{a_7}{\xi c}\tau_0^{'}(\frac{1}{\xi c} - \iota\tau_1)], \quad k_1 = \frac{-aa_6\tau_0^{'}}{\xi^2c^2} + \frac{1}{\xi^4c^4} - aa_7\tau_0^{'}(\frac{1}{\xi^2c^2} - \frac{\iota}{\xi c}\tau_1),$$

$$k_1^{'} = \xi^2c^2k_1$$

$$M_i = \xi^2 + n_i^2, \quad i = 1, 2,$$

$$m_i = [d_1l_i + d_2n_i^2 + b_ih_i], m_3 = (2d_3 + d_4)\iota\xi, m_k = (d_3 + d_4)n_j^2 - d_3\xi^2 - d_4h_j,\ i = 1,2,\ j = 3,4,\ k = 5,6,$$

$$S = \frac{\rho_L}{\rho}\xi^2c^2, \quad R_1 = \iota\xi c, \quad g_1 = \iota\xi d_2, \quad l_i = \xi^2 + n_i^2, \quad b_i = (1 - \iota\tau_1\xi c)(1 + a\xi^2 + an_i^2),$$

$$S_i = \sin n_id, \quad S_j = \sin n_jd, \quad C_i = \cos n_id, \quad C_j = \cos n_jd, \quad T_5 = \tanh\gamma_Lh,$$

$$d_1 = \frac{\lambda}{\rho c_1^2}, \quad d_2 = \frac{(2\mu + K)}{\rho c_1^2}, \quad d_3 = \frac{\mu}{\rho c_1^2}, \quad d_4 = \frac{K}{\rho c_1^2},$$

$$T_i = \tan n_id, \qquad i = 1,2,3,4$$