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Post-limit stiffness prediction of semi-rigid joints using genetic algorithms

L. R. O. de Lima[a,*], P. C. G. da S. Vellasco[a], J. G. S. da Silva[b], L. A. C. Borges[c] and L. A. P. S. da Silva[d]

[a] Structural Engineering Department, State University of Rio de Janeiro, Brasil
[b]Mechanical Engineering Department, State University of Rio de Janeiro, Brasil
[c]Steel Structures Laboratory – ICOM, École Polytechnique Fédérale de Lausanne, EPFL
[d]Civil Engineering Department, University of Coimbra, Portugal

Abstract

This paper proposes the use of genetic algorithms to calibrate the post limit stiffness of the various joint components that, following the Eurocode 3 component models, allow the prediction of the nonlinear behaviour of steel joints. NASCon, a specialized computer software purposely developed for the analysis of steel joints, was used to calibrate the post-limit stiffness of each individual joint component. To achieve this objective, an automatic iterative procedure was implemented that combined the component method solver (NASCon) with the genetic algorithm software, Evolver, to yield reliable estimates of the post-limit stiffness of each component from available experimental test results for the global joint behaviour.

Keywords: semi-rigid joints, steel structures, genetic algorithms and non-linear analysis.

1 Introduction

Past evidence has proved that semi-rigid connections provide adequate solutions, reducing the final cost. However, their safe use requires an accurate estimate of its structural behaviour [12]. Several investigations have attempted to determine the components post-limit stiffness from experimental data, various results being summarized in Faella et al. [16]. This painstaking procedure has proved to be difficult, since each post-limit stiffness had to be manually calibrated when the component reached the plastic phase.

This was the main motivation for the use of genetic algorithms to automatically determine the most suitable post-limit component stiffness, used to represent each component, into the mechanical model proposed in Eurocode 3 - Part 1.8 [7].

Over the last few years, various authors have used artificial intelligence techniques in order to solve complex problems in structural engineering. These techniques may be expressed by neural networks, genetic algorithms and fuzzy logic or by a combination of them.

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Considering the behaviour of joints, few investigations using artificial neural networks were found in the literature. Abdalla and Stavroulakis [4] and Stavroulakis and Abdalla [23] have used neural networks to predict the global moment versus rotation curve of single web angle beam-to-column joints. Anderson *et al.* [5] described the use of neural networks to predict a bilinear approximation of the moment versus rotation curves of minor axis beam-to-column endplate joints. Lima *et al.* [13] used an artificial neural networks system to predict the flexural resistance and initial stiffness of semi-rigid beam-to-column joints. This structural engineering problem is characterized by the influence of several physical and geometric parameters and for the great difficulty to generate new data based on experimental tests.

Ramires *et al.* [21] focusing on the objective of minimizing the effort spent in the structural joint analysis, proposed an automatic optimisation procedure based on genetic algorithms, implemented to evaluate the most suitable joint layout, i.e. maximizing the bending moment resistance and its the initial stiffness and, obviously, reducing costs.

When fuzzy logic system are considered, it is fair to mention the investigations made by Muthukumaran *et al.* [20] and Zhao and Chen [25]. Vamvakeridou and Lyroudias [24] considered a combination of neural networks and a fuzzy logic system [22] to develop an expert mathematical model for the restoration of the wall faces of the Parthenon, Greece.

2 The end plate beam-to-column connection model

The endplate beam-to-column joints investigated in this paper were designed according to the component method philosophy proposed in Part 1.8 of the Eurocode 3 [7]. In this procedure, the joint overall response can be determined based on the force-deformation properties of its parts (components). Subsequently, the components are assembled into a mechanical model, consisting of extensional springs and rigid links, as shown in Figure 1. The relevant components are: (1) column web panel in shear, (2) column web in compression, (3) column web in tension, (4) column flange in bending, (5) end-plate in bending, (7) beam flange in compression, (8) beam web in tension and (10) bolts in tension.

In general, each component (spring) is characterized by a non-linear force-displacement (F x Δ) curve, adequately represented by a bi-linear approximation depicted in Figure 2 when only the resistance and the initial stiffness of the connection is required [10]. In this procedure, the post-limit stiffness of the components is disregarded since a perfect elastic-plastic behaviour is assumed [9]. However, this method was shown to be unconservative, whenever the joint ductility is a critical issue. The application of the component method to steel joints requires the following steps:

- Selection of the relevant (active) components from a global list of components (20 different components currently codified, for example, in Eurocode 3 - Part 1.8 [7]);
- Evaluation of the force-deformation response for each component;

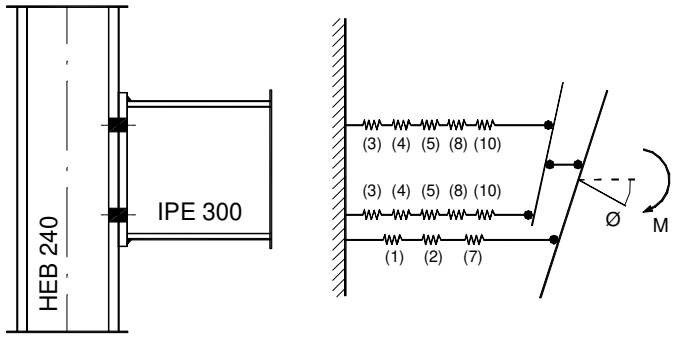

Figure 1: Flush endplate beam-to-column joint mechanical model.

- Assembly of the active components for the evaluation of the joint moment-rotation response by using a representative mechanical model.

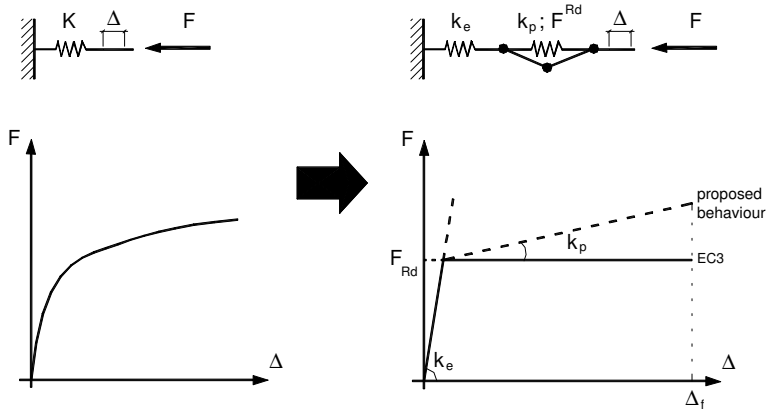

Figure 2: F x Δ curve for a generic component.

3 Computer implementation of the component method – NASCon

This section presents a brief description of the software NASCon (Non-linear Analysis of Steel Connections) [6] and [17]. This software was developed using Borland Delphi 6 development tool (Object Pascal) [3] and is illustrated in Figure 3. It was developed in order to provide a user-friendly tool that can help researchers and civil engineers to easy implement the component method procedures. Additionally, since the program has been developed for a specific use, it is possible to control every step of the design procedure and add extra functions as the Genetic Modules.

This software simulates the joint behaviour by numerically calculating its generalized force-deformation response from the complete characterization of the relevant components. It allows the user to control either loadings or displacements, i.e., the two different strategies usually adopted in a joint test [6]. It further allows comparison with experimental results and the evaluation of the corresponding error.

Steel joints may present a wide range of geometries, with different numbers of bolt rows and connecting parts. This wide variety required the conception of a standard way to model the joint so that NASCon could properly evaluate the component method model. The model file can be written directly in an ASCII text file (using any text editor) or by means of a Connection Assistant Tool (a user-friendly pre-processor for writing the model file). This file contains the information concerning the joint configuration followed by the number of bolt rows and the corresponding individual component behaviour.

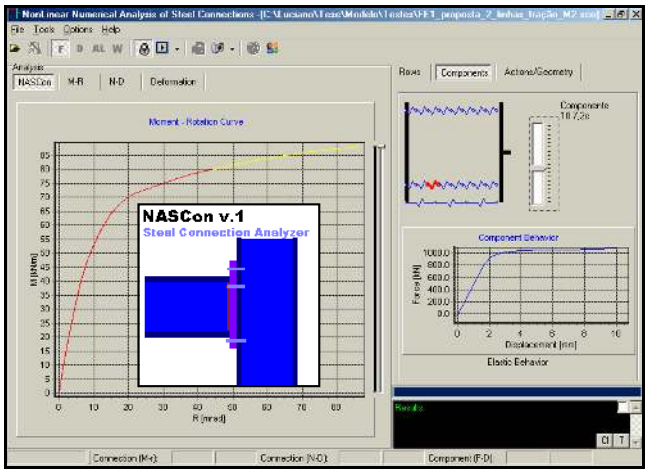

Figure 3: NASCon - Nonlinear Analysis of Steel Connections software.

4 Genetic algorithms - GA

During the last thirty years there has been a growing interest in problem solving systems based on principles of evolution and hereditary: such as systems maintain a population of potential solutions. They have some selection process based on fitness of individuals, and some genetic operators that are inspired on the Darwin's principle of the species evolution and genetics [19]. In 1990, Koza [18] proposed an evolution based system, Genetic Programming, to search for the most fit computer program to solve a particular problem. This computational intelligence technique was used by the authors to calibrate and forecast the post-limit stiffness of the column web out of plane component presented in minor axis beam-to-column joints [15].

Genetic algorithms (GA) use a vocabulary borrowed from natural genetics. It is common to talk about *individuals* (or *genotypes, structures*) in a population; quite often these individuals are called also *strings* or *chromosomes*. Chromosomes are made of units – *genes* (also *features, characters, or decoders)* – arranged in linear succession where every gene controls the inheritance of one or several characters.

Figure 4 presents the structure of a genetic algorithm where each individual represents a potential solution to the problem at hand. The first population is randomly generated. After creating an initial population, each member of the population is evaluated by computing the representative objective and constraint functions later to be compared to the other members. Each of these solutions is evaluated to give some measure of its *fitness*. Afterwards, a new population (iteration $t+1$) is formed by selecting the most fit individuals. Some members of the new population undergo transformations by means of genetic operators: crossover and mutation. Crossover is a genetic operator which forms a new chromosome by combining parts of two parental chromosomes, Figure 5. Mutation is a genetic operator that forms a new chromosome by making (usually small) alterations to the gene values creating a copy of the single parent chromosome. This process of going from the current population to the next constitutes one generation in the genetic algorithm evolution process. After some generations the program converges – to a feasible solution where the best individual represents a near-optimum solution.

```
procedure evolution program
begin
   t ← 0                                   // first generation
   initialise P(t)                         // initialise random population
   evaluate P(t)                           // evaluate the fitness of each individual
   while (not termination-condition) do
   begin
      t ← t + 1                            // next generation
      select P(t) from P(t - 1)
      alter P(t)                           // crossover and mutation
      evaluate P(t)                        // evaluate the fitness of each individual
   end
end
```

Figure 4: The structure of a genetic algorithm [19]

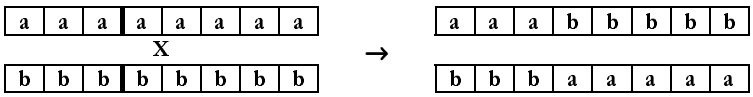

Figure 5: Genetic operator - crossover

4.1 The proposed genetic algorithm system

In order to evaluate the joint global response, their full geometrical and mechanical properties have to be considered. The joint mechanical model can be characterised according to the type of connection. Finally, all the component properties are evaluated and the joint moment *versus* rotation curve can be obtained. As an example, Table 1 summarises the properties of the flush endplate joint, Figure 7, evaluated according to EUROCODE 3 recommendations where only the upper bolt row was considered to be in tension (T) [11].

Table 1: Flush endplate joint components behaviour - FE1 test [11]

	Component		$\mathbf{F}_{Rd}$ **(kN)**	$\mathbf{k}_e$ **(kN.m)**	$\mathbf{k}_p/\mathbf{k}_e$	$\mathbf{k}_p$ **(kN.m)**
T	3,1	column web in tension	458.7	1.48E+06	**0.0340**	5.02E+04
	4,1	column flange in bending	397.2	8.03E+06	**0.0013**	1.00E+04
	5,1	endplate in bending	321.7	2.80E+06	**0.0050**	1.40E+04
	8,1	beam web in tension	455.2	1.00E+12	**0.0365**	3.65E+10
	10,1	bolts in tension	441.0	1.63E+06	**0.0050**	8.15E+03
C	1	column web in shear	-490.3	1.58E+06	**0.0176**	2.78E+04
	2	column web in compression	-530.7	2.18E+06	**0.0485**	1.06E+05
	7	beam flange in compression	-503.6	1.00E+12	**0.0489**	4.89E+10
					(chromosome)	

The column containing the k_p/k_e ratio represents the chromosome used in the genetic algorithm. Within these values, NASCon evaluates the moment *versus* rotation curve by successive comparison to the experimental curve. The solution fitness is evaluated according to the distance between these two curves. The procedure continues with the genetic algorithm operating the changes in the chromosome (crossover and mutation) and a new iteration is performed. This interactive procedure was implemented with the aid of genetic algorithmic software, Evolver [2], a plug-in for Microsoft Excel [1], as can be observed in Figure 6.

Several configurations of the genetic algorithm parameters were performed such as crossover rate, mutation rate and population size. The best results were achieved with:

- crossover rate: 0.30
- mutation rate: 0.008
- population size: 50

5 Experimental test data

The five experimental data used in this paper are described in Table 2. The first test, a flush endplate beam-to-column joint, was tested at the Civil Engineering Department of the University

of Coimbra presenting the components parameters depicted in Table 1 [14].

In this test, the column were simply-supported at both ends and consisted of a HEB240 profile while the beams were made of an IPE240 and a 15 mm thick end-plate, all manufactured from a S275 steel. The bolts were M20, class 10.9. The joint layout is presented in the Figure 7 while the test setup is illustrated in Figure 8 where the bending moment was applied by a hydraulic jack acting on the cantilever.

The other beam-to-column joint tests, two extended endplate joints with backing plates (Humer, 109.005 and 109.006) and two welded joints (Klein, 105.018 and Braun, 106.002) were obtained from the SERICON II Database [8]. The test data are summarized in Table 2 while the mechanical properties are presented in Table 3.

The component resistance for all tests were evaluated according to EUROCODE 3 using the measured material properties. The results are presented in Table 4 to Table 7 where the values of k_p/k_e were obtained for the best genetic algorithm solution.

Table 2: Experimental test data

Test ID	**Author**	**Joint Type**	**Beam**	**Column**	**Endplate**	**Bolts/Welds**
FE1	Lima *et. al*	Flush endplate	IPE240	HEB240	15mm	4 M20 10.9
109.005	Humer, C.	Extended endplate	IPE450	HEB240	41mm	6 M24 10.9
109.006	Humer, C.	Extended endplate	IPE600	HEB240	41mm	6 M24 10.9
105.018	Klein, H.	Welded	IPE450	HEB240	-	10.1mm
106.002	Braun, H.	Welded	IPE400	HEM180	-	12.1mm

Table 3: Tests mechanical characteristics (in MPa)

Test		**FE1**		**109.005**		**109.006**		**105.018**		**106.002**	
specimen		$\mathbf{f}_y$	$\mathbf{f}_u$	$\mathbf{f}_y$	$\mathbf{f}_u$	$\mathbf{f}_y$	$\mathbf{f}_u$	$\mathbf{f}_y$	$\mathbf{f}_u$	$\mathbf{f}_y$	$\mathbf{f}_u$
column	**flange**	343	449	276	401	275	398	269	-	265	-
	web	372	477	307	445	309	449	310	-	270	-
beam	**flange**	340	448	285	413	288	418	284	-	358	-
	web	363	454	317	398	294	427	338	-	457	-
endplate		369	503	323	-	325	-	-	-	-	-
bolts		900	1000	900	1000	900	1000	-	-	-	-

6 Results

This section summarizes the GA results according to the joint type to identify and enhance each component behaviour.

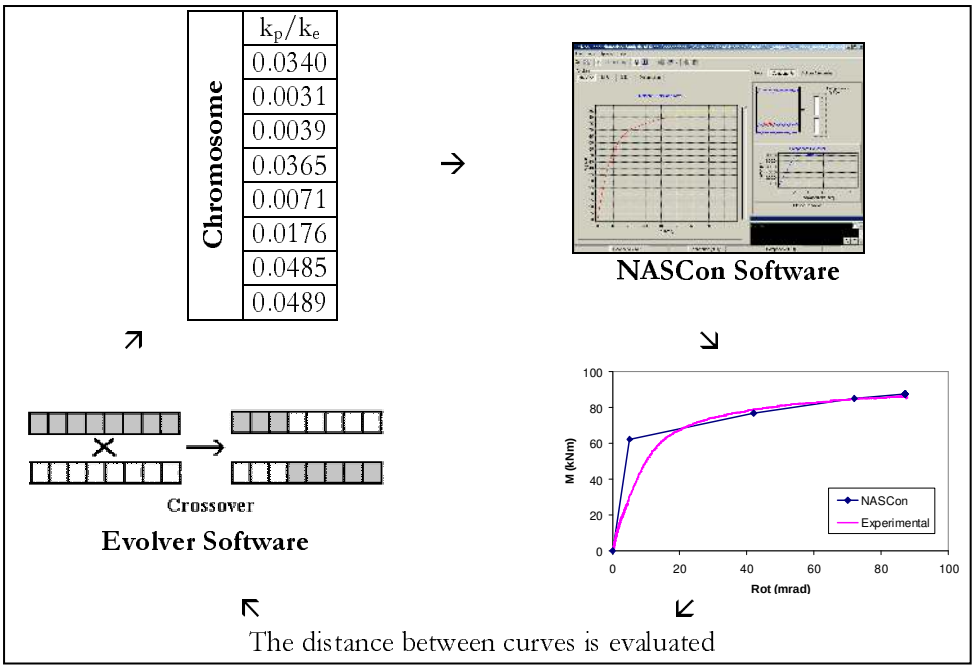

Figure 6: NASCon and Evolver softwares interactive procedure.

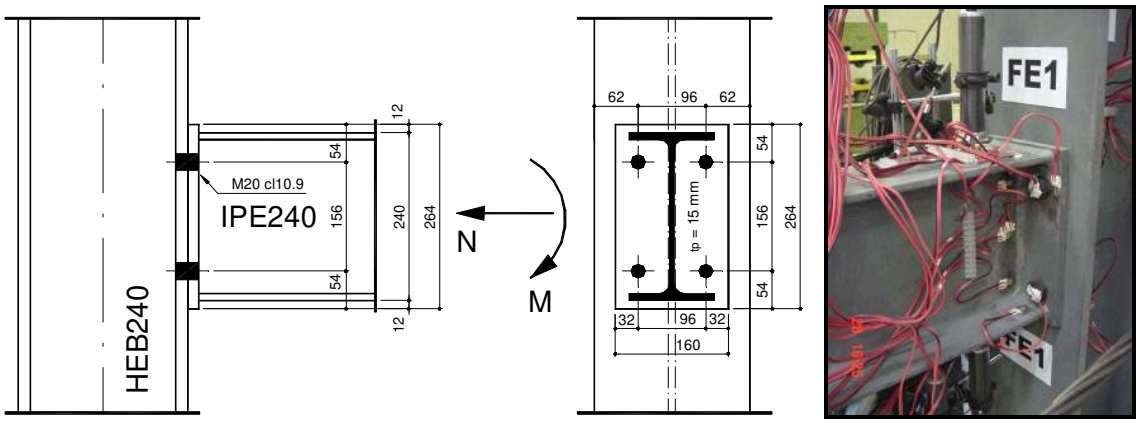

Figure 7: Flush endplate joint layout.

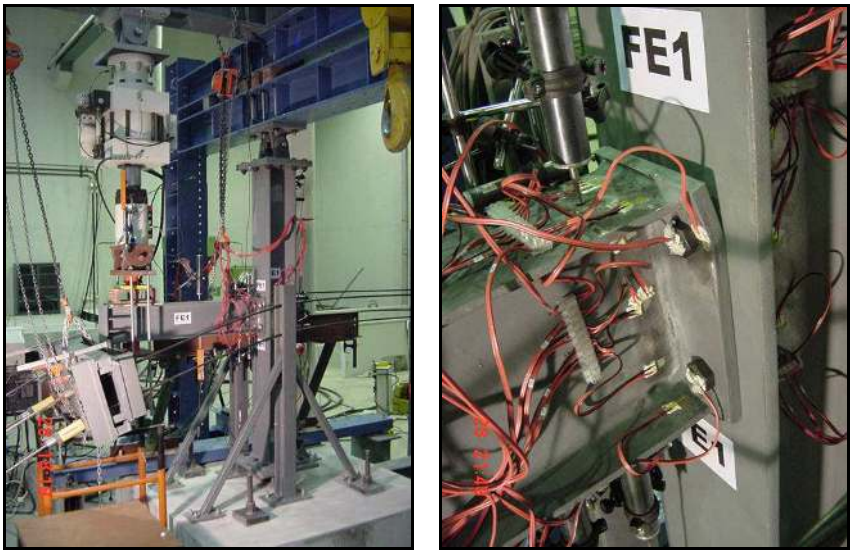

Figure 8: Flush endplate test setup and deformed joint.

Table 4: Extended endplate joint components behaviour, Humer, 109.005 [8]

	Component		$\mathbf{F}_{Rd}$ **(kN)**	$\mathbf{k}_e$ **(kN.m)**	$\mathbf{k}_p/\mathbf{k}_e$	$\mathbf{k}_p$ **(kN.m)**
	3,1	column web in tension	544.11	1.50E+06	**0.0025**	3.74E+03
	4,1	column flange in bending	460.50	3.75E+06	**0.1500**	5.62E+05
	5,1	endplate in bending	635.40	2.41E+07	**0.0018**	4.29E+04
	10,1	bolts in tension	635.00	1.31E+06	**0.0800**	1.05E+05
T	3,2	column web in tension	228.37	1.50E+06	**0.0025**	3.74E+03
	4,2	column flange in bending	411.35	3.75E+06	**0.1500**	5.62E+05
	5,2	endplate in bending	635.40	2.45E+07	**0.0018**	4.38E+04
	8,2	beam web in tension	909.90	1.00E+12	**0.0205**	2.05E+10
	10,2	bolts in tension	635.00	1.31E+06	**0.0800**	1.05E+05
	1	column web in shear	-543.83	6.11E+05	**0.0400**	2.44E+04
C	2	column web in compression	-602.31	2.46E+06	**0.0500**	1.23E+05
	7	beam flange in compression	-1203.22	1.00E+12	**0.3236**	3.24E+11

Table 5: Extended endplate joint components behaviour, Humer, 109.006 [8]

	Component		$\mathbf{F}_{Rd}$ (kN)	$\mathbf{k}_e$ (kN.m)	$\mathbf{k}_p/\mathbf{k}_e$	$\mathbf{k}_p$ (kN.m)
T	3,1	column web in tension	579.80	1.64E+06	**0.1500**	2.45E+05
	4,1	column flange in bending	270.06	2.50E+06	**0.1000**	2.50E+05
	5,1	endplate in bending	635.40	2.56E+07	**0.0184**	4.70E+05
	10,1	bolts in tension	635.00	1.30E+06	**0.0049**	6.38E+03
	3,2	column web in tension	424.93	1.64E+06	**0.1500**	2.45E+05
	4,2	column flange in bending	401.31	2.50E+06	**0.1000**	2.50E+05
	5,2	endplate in bending	635.40	1.71E+07	**0.0184**	3.15E+05
	8,2	beam web in tension	909.90	1.00E+10	**0.0250**	2.50E+08
	10,2	bolts in tension	635.00	1.30E+06	**0.0049**	6.38E+03
C	1	column web in shear	-548.50	4.68E+05	**0.0178**	8.32E+03
	2	column web in compression	-612.12	2.55E+06	**0.0928**	2.37E+05
	7	beam flange in compression	-1203.22	1.00E+10	**0.0087**	8.71E+07

Table 6: Welded joint components behaviour, Klein, 105.018 [8]

	Component		$\mathbf{F}_{Rd}$ (kN)	$\mathbf{k}_e$ (kN.m)	$\mathbf{k}_p/\mathbf{k}_e$	$\mathbf{k}_p$ (kN.m)
T	3	column web in tension	555.6	1.62E+03	**0.0309**	5.01E+01
	4	column flange in bending	787.6	1.00E+08	**0.0406**	4.06E+06
C	1	column web in shear	-535.4	6.09E+02	**0.0488**	2.98E+01
	2	column web in compression	-555.6	1.62E+03	**0.0307**	4.98E+01
	7	beam flange in compression	-1110.2	1.00E+08	**0.0312**	3.12E+06

Table 7: Welded joint components behaviour, Braun, 106.002 [8]

	Component		$\mathbf{F}_{Rd}$ (kN)	$\mathbf{k}_e$ (kN.m)	$\mathbf{k}_p/\mathbf{k}_e$	$\mathbf{k}_p$ (kN.m)
T	3	column web in tension	605.4	3.24E+03	**0.0250**	8.10E+01
	4	column flange in bending	870.0	1.00E+09	**0.0925**	9.25E+07
C	1	column web in shear	-482.6	7.10E+02	**0.0700**	4.97E+01
	2	column web in compression	-605.4	3.24E+03	**0.0563**	1.82E+02
	7	beam flange in compression	-1210.6	1.00E+07	**0.0044**	4.35E+04

6.1 Flush endplate joint – FE1

In test FE1, the yielding sequence observed in the mechanical model was: (5) endplate in bending, (4) column flange in bending and finally (10) bolts in tension. In this test, the convergence was more difficult because the experimental initial stiffness was slightly different from the value evaluated according to EUROCODE 3. Despite this fact, the slope of the obtained post-limit curve from the mechanical model was close to the experimental curve, Figure 9, where the post-limit stiffness values are also presented.

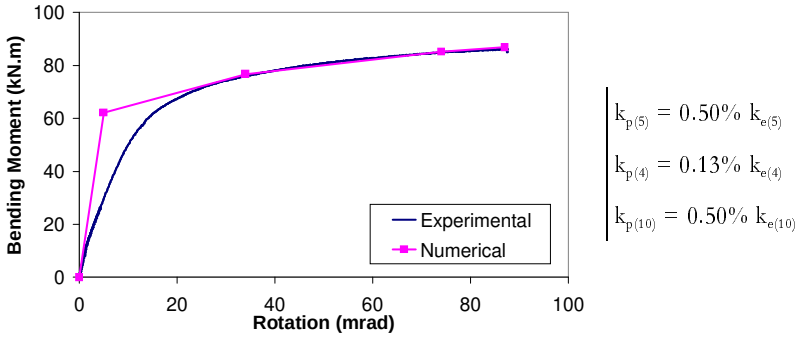

Figure 9: Moment *versus* rotation curves - FE1, Lima et al. [21]

7 Extended endplate joints – Humer 109.005 and 109.006

For both extended endplate joints, the genetic algorithm optimisation results presented a good agreement with the experiments. On the first test, Humer 109.005, the observed yield sequence was: (1) column web in shear, (4) column flange in bending and (3) column web in tension in first bolt row, (2) column web in compression and (3) column web in tension in the second bolt row whose k_p/k_e ratios are depicted in Figure 10. The yield sequence for the second test, Klein 109.006, was: (4) column flange in bending in the first bolt row, (1) column web in shear, (2) column web in compression and (3) column web in tension in the first bolt row - Figure 11.

When the results of these two tests are compared, Table 4 and Table 5, it can be observed that the tests critical components are equal, i.e., the column flange in bending (4,1) in the tension zone and the column web in shear (1) in the compression zone. The difference found between the post-limit stiffness for a single component can be explained by the yielding sequence that can affect the slope of its respective post-limit stiffness. In the first test, the first component to yield was the critical component in the compressive zone while in the second test, the critical component occurred in the tension zone.

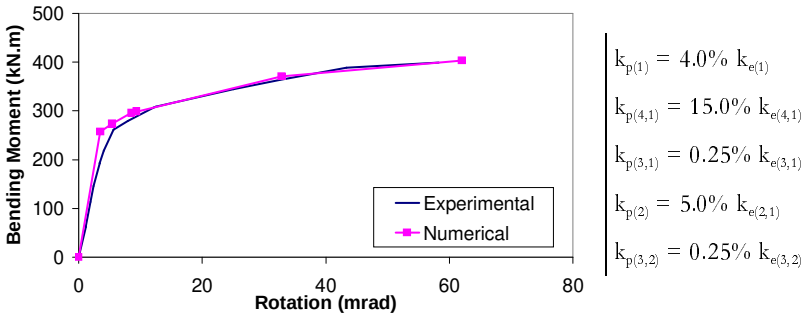

Figure 10: Moment *versus* rotation curves - Humer, 109.005

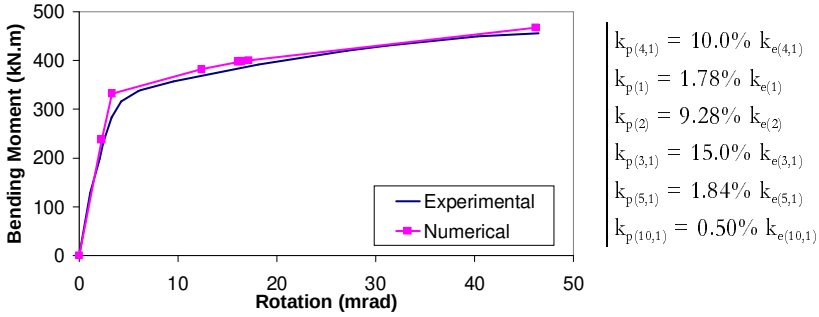

Figure 11: Moment *versus* rotation curves - Humer, 109.006

7.1 Welded joints – Klein 105.018 and Braun 106.002

The results for the welded joints also presented a good agreement to the experimental curves. In this type of joint, the calibration of the reduced number of components obviously made the convergence process faster. The values found for the component post-limit stiffness were also similar for both tests, as seen in Figure 12 and Figure 13, respectively.

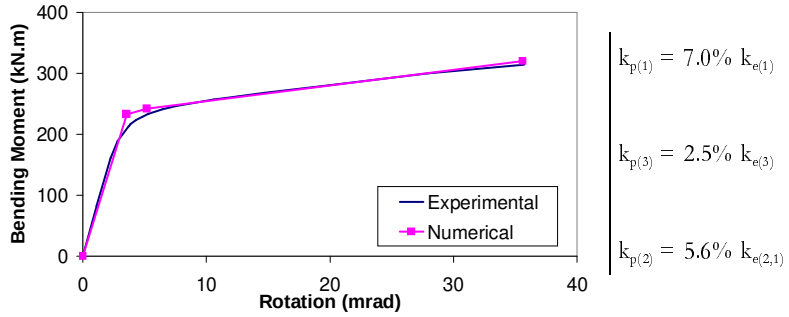

Figure 12: Moment *versus* rotation curves - Klein, 105.018

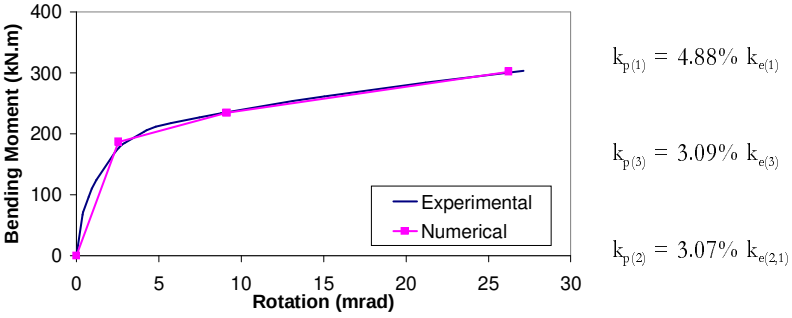

Figure 13: Moment *versus* rotation curves - Braun, 106.002

8 Concluding remarks

The use of semi-rigid connections has been significantly increased over the last few years. In order to represent the connections actual behaviour, many models were proposed. The EUROCODE 3 mechanical model adopted in this paper can be applied to any kind of joint as a set of components (springs) assembled in series or/and parallel.

The potential use of computational evolutionary techniques for the evaluation of structural and civil engineering problems has been fulfilled by the trustworthy results obtained with the semi-rigid joints. The use of the Genetic Algorithm procedure substantially simplified the cali-

bration of the components against the experimental data.

The main contribution of this work is to present the use of genetic algorithm as a design aid to predict the full response of the beam-to-column joints considering when appropriate, the components post-limit stiffness. It is also important to mention that before the implementation of the Genetic Algorithm procedure the components post limit stiffness determination was only possible by means of an user-dependent time-consuming manual process.

Five joints were investigated: one flush endplate joint, two extended endplate and two welded joints. The results enable the determination of the post-limit stiffness for each relevant component, by calibrating their values against experimental evidence.

The best results were obtained for the welded joints although all the results proved to be reliable when compared to the available tests. Despite this fact the results indicated that the Genetic Algorithm procedure can be safely use to evaluate the required joint components post limit stiffness. The major differences between the predicted and tested joint responses were due to the adopted Eurocode 3 joint model. A refined model (including, for example, other bolt rows) could, in principle, better represent the tests. This strategy would also increase the number of variables to be predicted (and also the required computer time) with a small improvement of the system performance and acuracy.

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