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# Local/Distortional/Global Buckling Mode Interaction on Thin Walled Lipped Channel Columns

**Abstract**

The buckling behaviour of pin ended cold-formed steel lipped channel columns affected by local / distortional / global buckling mode interaction under axial loading is simulated by using finite element software ABAQUS, and the results are compared with the test results available in the literature. The comparisons show that the proper analysis model can simulate the buckling behaviour and ultimate capacity of cold-formed steel columns. Then a parametric analysis is carried out for 15 column geometries of 3 yield stress values of different dimension, thickness and lengths. The cross sectional dimensions and length of the specimen have been chosen such that to have almost equal local, distortional and global critical buckling stresses by using the CUFSM software. The selected sections also satisfied the limitations given for pre-qualified sections in Direct Strength Method (DSM). After comparing the FEM column ultimate loads with the estimates predicted by the current Direct Strength Method (DSM) design curves against local, distortional and global failures, which clearly shows that they lead to inaccurate and often very unsafe ultimate load estimates. At the end, a design recommendation is made for current design practice of evaluating the ultimate strength of the lipped channel columns.

**Keywords**

Direct Strength Method, Distortional buckling, Global buckling, Lipped channel column, Local buckling, Local / Distortional / Global Interaction, etc.

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## 1 INTRODUCTION

Cold-formed steel columns are widely used in the construction industry due to their light weight, easy installation and erection, and economy. In practice, singly symmetric open sections, such as C sections, are commonly used in cold-formed design. Normally, one of the three basic types of buckling, local, distortional and overall can occur in thin-walled steel open sections. However, the basic types may interact with each other depends on the cross-section, dimension and length. The mode interaction phenomena affecting the column buckling behaviour, those nearly simultaneous occurrence of local and global buckling are better understood, as addressed by their inclusion in cold-formed steel design codes, either through the "effective width method" concept or by means of the "Direct Strength Method".

In the past, researchers have investigated the various buckling modes of commonly used cold-formed steel sections. Kwon and Hancock (1992) studied simple lipped channels and a lipped channel with

intermediate stiffener under fixed boundary conditions. They chose section geometry and yield strength of steel to ensure that a substantial post buckling strength reserve occurs in the distortional mode for the test section. The Distortional buckling behaviour of cold-formed steel columns are reported by Schafer (2000). Young and Yan (2002) studied the lipped channels columns undergoing local, distortional, and overall buckling. Local, distortional and Euler's buckling of thin walled columns is studied by Schafer (2002). The interaction between local and distortional buckling of the cold-formed steel sections were reported by Yang and Hancock (2004), Silvestre and Camotim (2006), Dinis et al. (2007). Schafer (2002) has developed semi-analytical finite strip method for elastic buckling of cold-formed steel sections. Yap and Hancock (2008) proposed new design methods for the effects of interaction of local and distortional buckling modes for the cross-shaped section. Schafer (2008) studied the cold-form member design by direct strength method. Nearly simultaneous occurrence of local and global buckling are well understood and already covered by current cold-formed steel design codes, either through the 'Effective Width Method' or Direct Strength Method (2009).

Silvestre et al. (2009) conducted parametric studies to assess the performance of DSM for lipped channel columns affected by local-distortional interaction. Kwon et al. (2009) conducted compression tests on high strength cold-formed steel channels with buckling interaction. Dinis et al. (2011) & (2012), Camotim et al. (2011), Crisan et al. (2012) carried out numerical and experimental investigations on interaction of local, distortional and global buckling of cold-formed steel column section with different end conditions.

From the review of the literature it was found that the study of Local/Distortional/Global buckling interaction on simply supported lipped channel column was scattered and limited.

The objective of this paper is to present and discuss the results of a numerical investigation dealing with the influence of local/distortional/global buckling mode interaction on the buckling behaviour and strength of the pin-ended cold-formed steel lipped channel columns (i.e., Uniformly compressed members). The columns analysed were chosen from the preliminary study conducted by CUFSM software to exhibit the cross-section dimensions ensuring equal local, distortional and global critical stresses. A nonlinear finite element model was developed and validated using fixed-ended column tests available in the literature. In order to fully understand the structural behaviour an extensive parametric study has been carried out for 15 column geometries of 3 yield stress values of different dimension, thickness and lengths. The accuracy of current design rules of the Direct Strength Method (DSM) was investigated using the ultimate capacity results from the parametric study, and suitable recommendations were made in the design rules.

## 2 Finite Element Modelling and Validation

### 2.1 General

Commercial finite element software package ABAQUS version 6.10 was used for the numerical studies. Prior to analysing the post-buckling behaviour of the structure, a linear buckling analysis is performed on the specimens to obtain its buckling mode shape. Following this nonlinear post-buckling analysis is carried out to study the load versus end-shortening characteristics and to predict the ultimate load capacity. The model was based on the centre line dimensions of the cross-sections. However, the residual stresses of the channel sections were not included in the model, since its effect on the ultimate load was less as stated by Schafer and Pekoz (1998). The details of finite element modelling and analysis (FEA) are discussed in the following sections.

### 2.2 Element type and mesh

S4R5 shell elements in ABAQUS are used. The S4R5 element is a four-node doubly curved shell element with reduced integration and hourglass control using five degrees of freedom per node.

Convergence studies have been carried out on the column in order to determine a suitable finite element model for the analysis. The aspect ratio (length to width) of 1.0 for the flange and web elements was used. The sizes of the elements were 10 mm x 7.5 mm (length by width) for the lip and 10 mm x 10 mm for the flange and web.

### 2.3 Boundary condition and Loading

The fixed end conditions were used in the verification model but in the parametric study models used the pinned end conditions. The pin-end conditions of the columns were modelled with both the ends prevented from rotation about the z-axis. The translation in x, y and z direction was arrested at the unloaded end. But in the loaded end was prevented from translation in x and y directions.
These boundary conditions were applied to the independent node of the rigid fixed MPC (Multi Point Constraint) located at the upper and lower end of the model. Rigid fixed MPC consists of an independent node and any number of dependent nodes. Dependent nodes are connected to the independent node using rigid beams and all six structural degrees of freedoms are rigidly attached to each other. In this model, the independent node was located at the geometric centre of the cross-section. This MPC acted as a rigid surface that was rigidly connected to the upper and lower end of the columns as shown in Fig.1 and Fig.2. The displacement control load was applied in increments to the master node using the modified RIKS method available in the ABAQUS library.

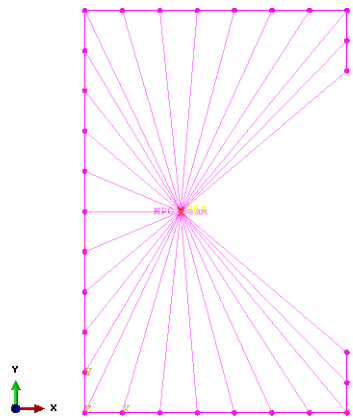



**Figure.1** Boundary conditions applied to the MPC at the upper end

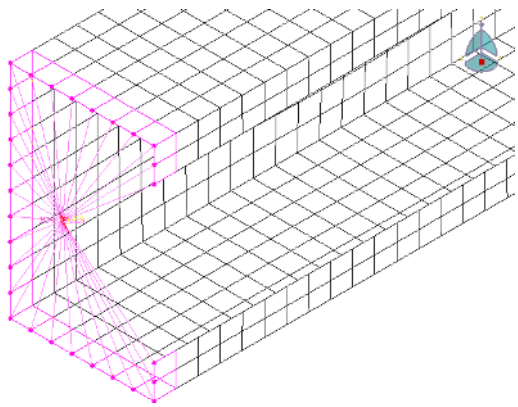


**Figure.2** Column ends modelled using MPC

### 2.4 Material properties and Geometric Imperfections:

The geometric and material non linearity was used in the model. All the specimens were modelled based on isotropic strain-hardening behaviour. In order to account for the Elasto-plastic behaviour, a bilinear stress-strain curve is adopted, having a tangent modulus ($E_t$) of 2000 N/mm$^2$ for the verification model. Elastic perfectly plastic material model was used for parametric study.

The initial geometric imperfections were included in the non-linear analysis. Eigenmode 1 was scaled by a factor of 25% of the plate thickness of the sections was used in modelling the geometric imperfections of the columns in the verification model. But for the sections on parametric study undergone three buckling modes it needs to incorporate all the three imperfections. From the detailed considerations of the literature, the local and distortional imperfection was taken equal to 0.25t and 1t respectively as recommended by Schafer and Pekoz, in addition the overall imperfection was taken 1/1000 of the full length of the column at the mid-height section for lipped channels were used in the parametric study models to initiate the nonlinear analyses.

### 2.5 Non-Linear Analysis

The bifurcation buckling analysis is first performed to determine the elastic buckling loads and modes followed by the nonlinear analysis to determine the ultimate loads and deformations, including post-local buckling reserve strength was carried out using ABAQUS.

### 2.6 Validation

Finite element analysis (FEA) is increasingly used for research purposes since it shows many advantages when compared with experimental studies. Valuable time and physical resources can be saved by using finite element analysis rather than experiments. However, the validity of FEA must be established before undertaking detailed parametric studies.

The developed finite element models were validated by comparing ultimate capacity results with experimental results of lipped channel columns reported by Young and Yan (2002). Six specimens were subjected to axial load from their work were modelled using the finite element computer package. The ultimate load computed by the finite element model has been compared with the values derived experimentally. The comparison between the test results of the ultimate load of the tested specimens, and those computed by the finite element model is presented in Tables 1 and showed a reasonable agreement between the finite element results and test results.

**Table 1** Comparison of finite element and experimental results of lipped channel columns tested by Young and Yan (2002)

| Specimen ID | $P_{EXP}$ (kN) | $P_{FEM}$ (kN) | $P_{EXP}/P_{FEM}$ |
|---|---|---|---|
| L48F300 | 111.9 | 112.71 | 0.993 |
| L48F1000 | 102.3 | 105.22 | 0.972 |
| L48F1500 | 98.6 | 99.07 | 0.995 |
| L48F2000 | 90.1 | 92.18 | 0.977 |
| L48F2500 | 73.9 | 69.85 | 1.058 |
| L48F3000 | 54.3 | 55.26 | 0.983 |
| | | Mean | 0.996 |
| | | Standard Deviation | 0.031 |

The mean and standard deviation of the Test to FEA ultimate loads are 0.996 and 0.031 respectively. The resemblance of Fig. 3 demonstrates the reliability of the FEA predictions.

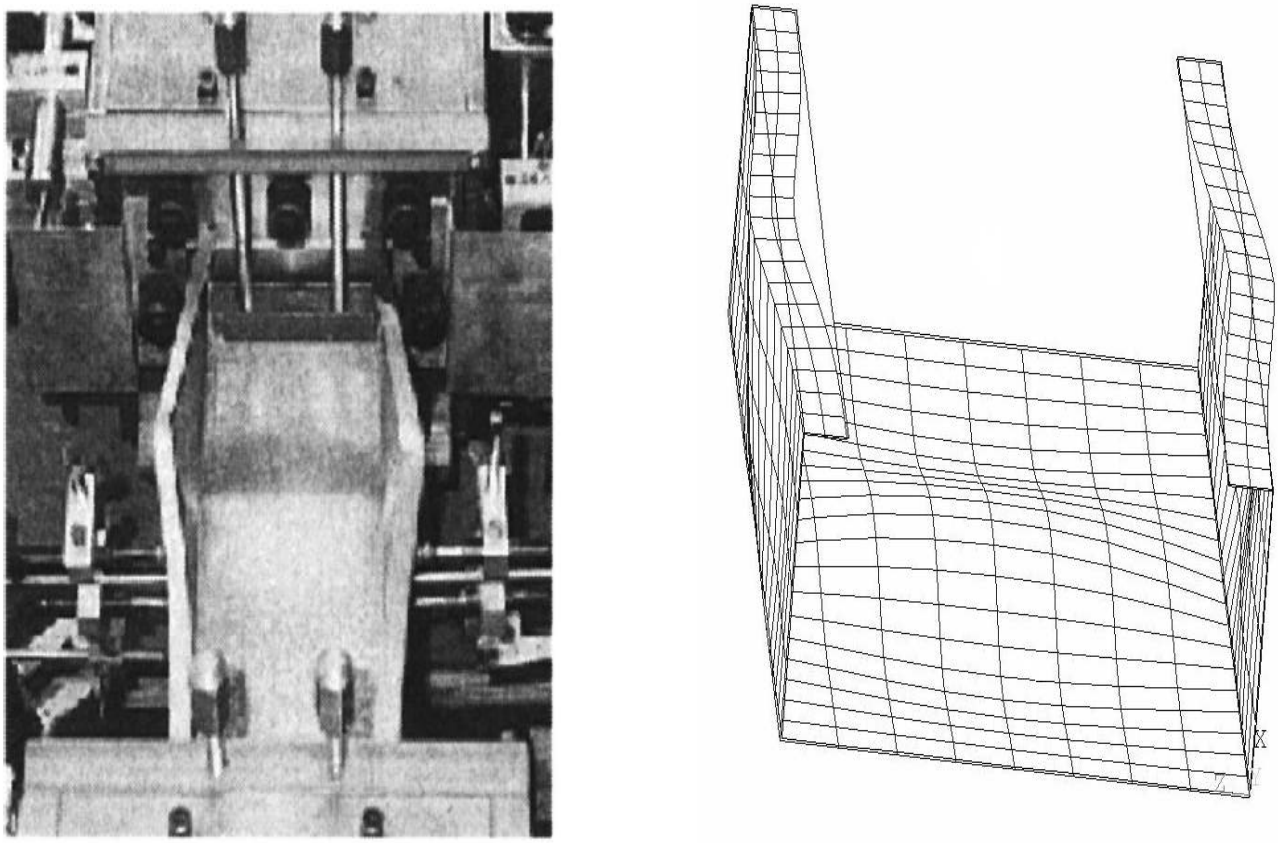


**Figure. 3** Comparison of Experimental and FEA deformed shape for Specimen LA8F0300

## 3 Section design and Parametric Studies

### 3.1 Section selection

The lipped channel column cross-section dimensions and length were suitably chosen for buckling interaction between the local, distortional and global buckling modes. To ensure nearly coincident local, distortional and global buckling stresses ($f_{crl}$, $f_{crd}$ and $f_{cre}$), it was necessary to perform educated trial-and-error buckling analysis using the CUFSM software which is based on the finite strip method. This procedure led to the identification of the fifteen column specimen geometries with the corresponding $f_{crl}$, $f_{crd}$, and $f_{cre}$ values calculated for E $=$ 2x10$^5$ Mpa and $\nu = 0.3$ given in Table 2. Where, $f_{crl}$, $f_{crd}$ and $f_{cre}$ are the local, distortional and global (flexural or flexural-torsional) critical buckling stresses respectively. The cross-sectional dimensions are also satisfies the limitations given for pre qualified sections of columns in Direct Strength method (Table 3).

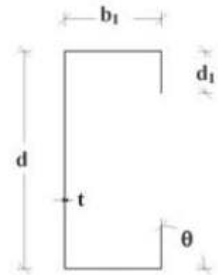
| Section | Geometric limitation |
|---|---|
|  | $d/t < 472$ |
| | $b_1/t < 159$ |
| | $4 < d_1/t < 33$ |
| | $0.7 < d/b_f < 5.0$ |
| | $0.05 < d_1/b_1 < 0.41$ |
| | $\theta = 90°$ |
| | $E/f_y > 340$ ($f_y < 593$ MPa) |



**Table 2** Section properties and Geometry

| Sl. No. | d (mm) | b1 (mm) | d1 (mm) | t (mm) | Length (mm) |
|---|---|---|---|---|---|
| 1 | 58 | 40 | 10 | 1.25 | 622.3 |
| 2 | 75 | 45 | 15 | 2.00 | 660.4 |
| 3 | 82 | 50 | 15 | 2 | 736.6 |
| 4 | 100 | 65 | 15 | 2 | 1117.6 |
| 5 | 110 | 55 | 10 | 2 | 1168.4 |
| 6 | 85 | 50 | 10 | 1.6 | 939.8 |
| 7 | 230 | 120 | 20 | 4 | 2844.8 |
| 8 | 50 | 30 | 10 | 1.25 | 457.2 |
| 9 | 65 | 35 | 12 | 1.6 | 508.0 |
| 10 | 75 | 40 | 14 | 2 | 609.6 |
| 11 | 80 | 60 | 16 | 1.6 | 914.4 |
| 12 | 78 | 45 | 15 | 2 | 685.8 |
| 13 | 85 | 53 | 15 | 2 | 812.8 |
| 14 | 108 | 68 | 15 | 2 | 1270.0 |
| 15 | 58 | 30 | 10 | 1.6 | 457.2 |

**Table 3** Geometric Limitations of Lipped Channel Sections

The Fig. 4 displays the buckling plot and modes for the column LC-100-65-15-2 for the yield stress value of 250 Mpa.

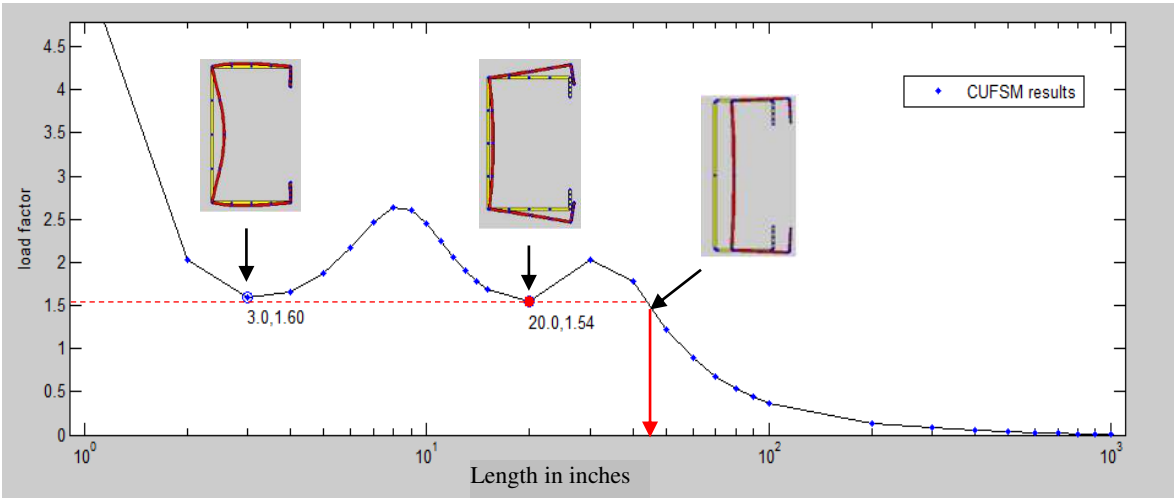



**Figure. 4** – Buckling plot and modes of the series LC-100-65-15-2

As shown in Figure 4 the CUFSM program results show that the pure local buckling occurs up to a half wavelength of 76 mm and pure distortional buckling from 203 mm to 762 mm. The minimum elastic distortional buckling stress occurs when the half wave length equals 508 mm. The behaviour changes to a pure global buckling mode when the half wave length is about 762 mm. The pure global buckling occurs for the lengths greater than 762 mm. However, according to the results from CUFSM analyses, showed that local, distortional and global buckling occurs when the column length is between 762 mm to 1117.6 mm. Therefore, the column length was selected as 1117.6 mm for the cold-formed lipped C-section so that three buckling mode interaction would occur. By the same way all 15 cross-section geometries were selected.

### 3.2 Parametric study

It is shown that the FEM closely predicted the column strengths and the behaviour of the tested columns. Hence the model was used for an extensive parametric study of 15 cross-section geometries with 3 yield stress values. The plate thickness was varied at 1.25 and 4.0 mm. The column length ranged from 457 to 2,845 mm. In total 45 finite element analyses were conducted using the column models. The specimens were labelled such that the type of channels, the width of the web, the width of the flange, the lip length and plate thickness could be identified from the label. Fig. 6 explains a typical specimen label for parametric study.

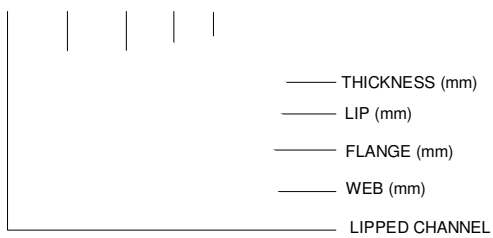
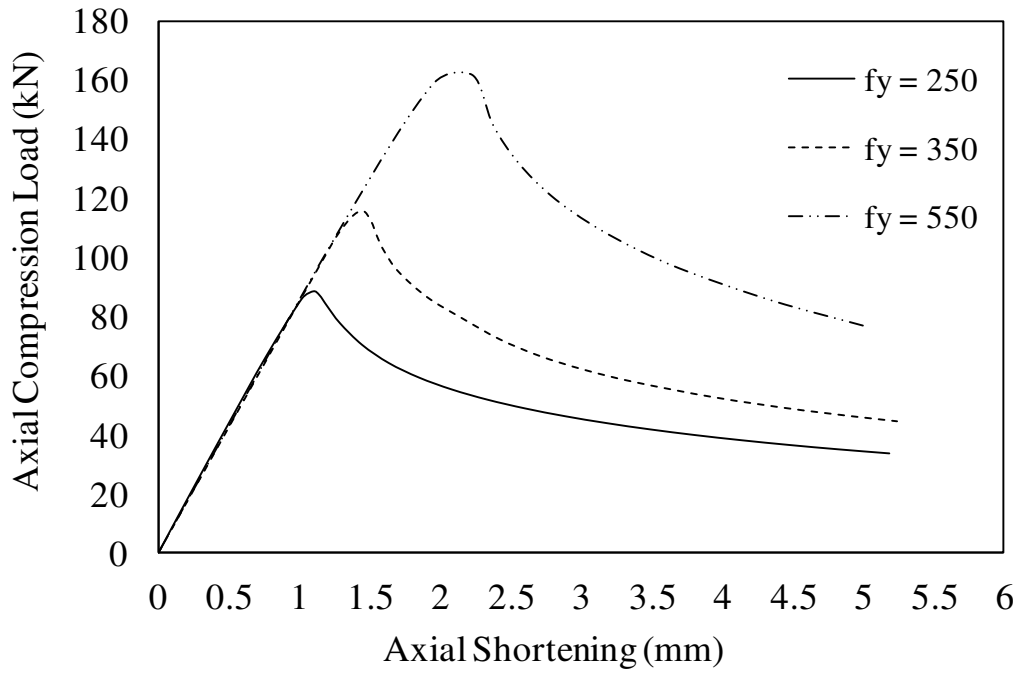



**Figure. 5**– Typical specimen labelling

Fig. 6 shows the FEA parametric study results for the series LC- LC-100-65-15-2 of 3 yield stress values.




## 4 Comparison of Ultimate capacities from FEA with design column strengths

The column strengths (PFEA) obtained from the FEA compared with design strength predicted by DSM is shown in Table 4.

The mean and standard deviation for the ratios of finite element analysis results to the DSM results were calculated based on the thicknesses, grades and section types. The mean and standard deviation of the PFEA to PDSM ultimate loads are 0.821 and 0.061 respectively. The DSM design strengths are unconservative for all channel columns.

## 5 Design Recommendation Proposed

A comparison of numerical results with the direct strength results is presented in Fig. 7. Most of the results are on the unsafe side. It shows that design procedure accounting for interaction of local, distortional and global buckling for lipped channel columns are required. A regression analysis was conducted for the results associated with the 45 analysis.

**Table 4** Comparison of Finite element results with DSM results

| Specimen ID | Yield stress (mm) | $P_{DSM}$ (kN) | $P_{FEA}$ (kN) | $P_{FEA}$ / $P_{DSM}$ |
|---|---|---|---|---|
| | 250 | 44.63 | 35.99 | 0.806 |
| LC-58-40-10-1.25 | 350 | 55.94 | 49.11 | 0.878 |
| | 550 | 73.43 | 72.43 | 0.986 |
| | 250 | 95.31 | 76.24 | 0.800 |
| LC-75-45-15-2 | 350 | 127.43 | 103.83 | 0.815 |
| | 550 | 175.15 | 152.18 | 0.869 |
| | 250 | 103.18 | 80.57 | 0.781 |
| LC-82-50-15-2 | 350 | 135.19 | 108.4 | 0.802 |
| | 550 | 179.63 | 156.29 | 0.870 |
| | 250 | 114.11 | 88.63 | 0.777 |
| LC-100-65-15-2 | 350 | 141.1 | 115.68 | 0.820 |
| | 550 | 184.3 | 162.2 | 0.880 |
| | 250 | 100.43 | 78.01 | 0.777 |
| LC-110-55-10-2 | 350 | 126.34 | 101.24 | 0.801 |
| | 550 | 159.78 | 133.25 | 0.834 |
| | 250 | 69.31 | 55.57 | 0.802 |
| LC-85-50-10-1.6 | 350 | 87.03 | 71.2 | 0.818 |
| | 550 | 110.82 | 100.3 | 0.905 |
| | 250 | 275.2 | 225.76 | 0.820 |
| LC-230-120-20-4 | 350 | 301.03 | 256.36 | 0.852 |
| | 550 | 303.46 | 273.24 | 0.900 |
| | 250 | 39.64 | 30.82 | 0.777 |
| LC-50-30-10-1.25 | 350 | 52.07 | 40.66 | 0.781 |
| | 550 | 71 | 61.04 | 0.860 |
| LC-60-35-12-1.6 | 250 | 60.2 | 48.37 | 0.803 |

| | | | | |
|---|---|---|---|---|
| | 350 | 81.63 | 65.7 | 0.805 |
| | 550 | 111.38 | 97.12 | 0.872 |
| | 250 | 89.71 | 68.44 | 0.763 |
| LC-75-40-14-2 | 350 | 122.52 | 91.25 | 0.745 |
| | 550 | 168.44 | 129.71 | 0.770 |
| | 250 | 81.46 | 65.43 | 0.803 |
| LC-80-60-16-1.6 | 350 | 102.33 | 97.12 | 0.949 |
| | 550 | 131.56 | 129.71 | 0.986 |
| | 250 | 96.77 | 74.43 | 0.769 |
| LC-78-45-15-2 | 350 | 128.73 | 102.25 | 0.794 |
| | 550 | 176.28 | 148.53 | 0.843 |
| | 250 | 106 | 75.89 | 0.716 |
| LC-85-53-15-2 | 350 | 135.77 | 99.29 | 0.731 |
| | 550 | 174.65 | 145.12 | 0.831 |
| | 250 | 116.08 | 83.85 | 0.722 |
| LC-108-68-15-2 | 350 | 141.71 | 113.05 | 0.798 |
| | 550 | 185.15 | 158.01 | 0.853 |
| | 250 | 54.18 | 42.32 | 0.781 |
| LC-58-30-10-1.6 | 350 | 74.26 | 58.06 | 0.782 |
| | 550 | 102.9 | 85.21 | 0.828 |
| | | | Mean | 0.821 |
| | | | Standard Deviation | 0.061 |

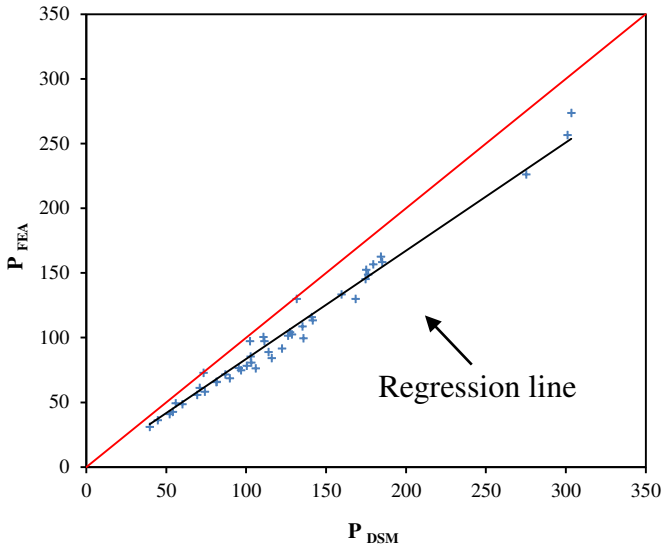



**Figure. 7** PFEA Versus PDSM curve

From the regression analysis it is proposed that a modification factor of 0.836 is to be applied to the ultimate strength of cold-formed steel lipped channel columns calculated by DSM for the sections undergoing Local/Distortional/Global mode interaction. The accuracy of the proposed equation was verified by comparing its results to selected available experimental studies of Kwon et al. (2008). The ultimate strength estimates provided by the proposed equation and Direct Strength Method is presented in Table 5.

**Table 5** Comparison of Experimental and Proposed equation

| Specimen ID | Test Ultimate load ( $P_{TEST}$) | Ultimate load by DSM ($P_{DSM}$) | Ultimate load by Proposed Equation ($P'_{FEA}$) | $\frac{P'_{FEA}}{P_{DSM}}$ | $\frac{P'_{FEA}}{P_{TEST}}$ |
|---|---|---|---|---|---|
| A-6-1 | 39.9 | 49.45 | 41.34 | 0.836 | 1.036 |
| A-6-2 | 42.0 | 49.11 | 41.06 | 0.836 | 0.978 |
| | | Mean | | 0.836 | 1.007 |

The Direct Strength Method overestimate the ultimate strength of CFS lipped channel column undergone Local/Distortional/Global mode interaction. Thus proposed equation gives best results compared to the Direct Strength method. This equation is reasonably predicted the ultimate strength of the sections which are undergoing Local, Distortional & Global mode interaction. The application of this procedure, to the results in the literature, proves the effectiveness of the proposed approach.

## 6 SUMMARY & CONCLUSION

This work deals with a theoretical and numerical investigation concerning the behaviour of pinned cold-formed steel lipped channel columns affected by local/Distortional/global buckling mode interaction. Finite Element Analysis were performed using the simulation software ABAQUS 6.10. Theoretical study was carried out using the Direct Strength Method. Material properties and geometric

imperfections were included in the Finite element analysis. The FEA results were verified against the experimental results available in the literature. The section profiles were chosen based on the geometric limitations for the pre-qualified sections provided in the AISI S100-2007 code. CUFSM software was used to derive the buckling plots of selected section profiles. From the plots the ratio between the local and distortional load factor, distortional and global and the local and global were taken as simultaneous or nearly simultaneous. A series of parametric studies was also carried out by varying the yield strength (250, 350 and 550 Mpa), thickness and length of the columns. The column strengths obtained from the finite element analysis are compared with the design column strengths calculated using the Direct Strength Method from AISI specifications for cold-formed steel structures. Then within the limitations of the present study the following conclusions are drawn.

- The interaction between local, distortional and global buckling reduced significantly the ultimate strength of the intermediate length cold-formed steel columns.
- Lipped channel sections undergoing interaction between local / distortional / global buckling displayed a significant post-buckling strength reserve, regardless of the critical buckling mode, local or distortional or global.
- The finite element analysis predictions were generally in good agreement with the failure modes of the lipped channel columns.
- Comparison of the results show that the current design rules in AISI 100-2007 (DSM) are unconservative therefore the new design expression is proposed.
- The simple design equation proposed for lipped channel section can account for the interaction of local, distortional and global buckling mode, though the proposed method gives a lower bound of test results.

Further experimental investigations to validate the finding and recommendation presented herein are greatly encouraged.

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