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Vibration of Gold Nano Beam in Context of Two-Temperature Generalized Thermoelasticity Subjected to Laser Pulse

Abstract

In the present work, the model of vibration of gold nano- beam induced by laser pulse heating is developed in the context of two-temperature generalized thermoelasticity and non-Fourier heat conduction. The analytic solution has been derived in the Laplace transform domain. The inverse Laplace transform has been calculated numerically and the numerical results have been presented graphically in two and three dimensions figures with some comparisons to stand on the effects of the two-temperature parameter and the laser pulse parameters on all the studying fields and which one of that parameters plays a vital role in the damping of the energy which has been generated inside the beam.

Keywords

Two-temperature thermoelasticity, Euler–Bernoulli equation, gold nano-beam, state-space approach, laser pulse

Hamdy M. Youssef 12 [a]

Alaa A. El-Bary 2[b]

Khaled A. Elsibai [c]

[a] Faculty of Eng.-Mech. Dept.-Umm Al-Qura Uni.-P.O. 5555-Makkah-Saudi Arabia
Math. Dep., Facu. Educ., Alexandria Univ. Egypt.
email: youssefanne2005@gmail.com

[b] Basic and Applied Science Department-Arab Academy for Science and Technology- Po. Box 1029-Alexandria, Egypt.
email: aaelbary@aast.edu.

[c] Faculty of Appl. Sci.- Math. Dept.
Umm Al-Qura University-Makkah-Saudí Arabia.
email: drkhaledelsibai@yahoo.com

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1 INTRODUCTION

With the rapid development of imaging technology, the perspectives of biomedical research have turned from subcellular structures in micro-scale (chromosomes, organelles, cytoskeleton, etc.) to biomolecules in nano-scale (nucleic acids, proteins, etc.). Studies on how bio macro molecules assemble, coordinate, transmit signals, and execute function are very meaningful and important, because they can facilitate the research on nano-biointeractions and promote the progress in biological detection, diagnosis and treatment techniques Wang et al. (2013).

Gold Nano-Beams based nano-carriers have great potential in biomedical fields, such as biological monitoring, imaging, thermotherapy and multifunctional nano-complex diagnose, and all these bring ideas and hopes to the development of biomedicine. In connection with structure, property and bio-effects of Gold Nano-Beams, developing real-time, sensitive, high through put detection, and analysis methods could be the important consults to rational design of Gold Nano-Beams based nano-carriers. Ultimately, through surface modification and functionalization, it could improve targeting of Gold Nano-Beams, reduce immune response and other negative effects, these Gold Nano-Beams-based multifunctional nano-carriers will play crucial role in future biocatalysis, disease diagnosis, imaging and therapy Wang et al. (2013).

Many works and articles have been made recently to investigate the elastic properties of nanostructured materials by atomistic simulations. Diao et. al. (2004) studied the effect of free surfaces on the structure and elastic properties of gold nanowires by atomistic simulations. Although the atomistic simulation is a terrific way to calculate the elastic constants of nanostructured materials, it is only applicable to homogeneous nanostructured materials, for example, nanoplates, nanobeams and nanowires with a limited number of atoms. Moreover, it is difficult to obtain the elastic properties of the heterogeneous nanostructured materials using atomistic simulations. For these and other reasons, it is prudent to seek a more practical approach. One such approach would be to extend the classical theory of elasticity down to the nanoscale by including in it the hitherto neglected surface/interface effect. For this, it is necessary first to cast the latter within the framework of continuum elasticity.

Nano-mechanical beams have attracted considerable attention recently due to their many significant technological applications. Accurate analysis of various effects on the characteristics of beams, such as resonant frequencies and quality factors, is crucial for designing high-performance components. Many authors have studied the vibration and heat transfer process of beams. Kidawa (2003) has studied the problem of transverse vibrations of a beam induced by a mobile heat source. The analytical solution to the problem was obtained using the Green's functions method. However, Kidawa did not consider the thermoelastic coupling effect. Boley (1972) analyzed the vibrations of a simply supported rectangular beam subjected to a suddenly applied heat input distributed along its span. Manolis and Beskos (1980) examined the thermally induced vibration of structures consisting of beams, exposed to rapid surface heating. They have also studied the effects of damping and axial loads on the structural response. Al-Huniti et. al. (2001) investigated the thermal induced displacements and stresses of a rod using the Laplace transformation technique. Ai Kah Soh et al. (2008) studied the vibration of micro/nanoscale beams induced by ultra-short-pulsed laser by considering the thermoelastic coupling term. Sun et. al. (2008, 2006) constructed a model of thermoelastic damping in micro-beams, and Fang et. al. (2006) got the analysis of the frequency spectrum of laser, induced vibration of microbeams. Eringen (1983) reduced Integro-partial differential equations of the linear theory of nonlocal elasticity to singular partial differential equations for a special class of physically admissible kernels. Civalek and Akgöz (2010) presented free vibration analysis of microtubules (MTs) based on the Euler-Bernoulli beam theory. Liew et al. (2008) simulated the flexural wave propagation in a single-walled carbon nanotube (SWCNT) by using molecular dynamics (MD) based on a second-generation reactive empirical bond order (REBO) potential. Civalek and

Demir (2011) formulated the equations of motion and bending of Euler-Bernoulli beam using the nonlocal elasticity theory for cantilever microtubules (MTs).

Youssef and Elsibai (2010) investigated the vibration of gold nano- beam induced by many types of thermal loading. Youssef (2006) investigated two-temperature generalized thermoelasticity theory together with a general uniqueness theorem and solved many applications in the context of this theory with Al-Lehaibi (2007), Harby (2007) and Bassiouny (2008).

In this work, the model of vibration of gold nano- beam induced by laser pulse heating will be developed in the context of two-temperature generalized thermoelasticity and non-Fourier heat conduction. The analytic solution will be derived in the Laplace transform domain. The inverse Laplace transform will be calculated numerically and the numerical results will be presented graphically in two and three dimensions figures with some comparisons to stand on the effects of the two-temperature parameter and the laser pulse parameters on all the studying fields and which one of that parameters plays a vital role in the damping of the energy which will be generated inside the beam.

2 FORMULATION OF THE PROBLEM

Since beams with rectangular cross-sections are easy to fabricate, such cross-sections are commonly adopted in the design of NEMS beams. Consider small flexural deflections of a thin elastic beam of length $\ell \left(0 \le x \le \ell\right)$, width $b\left(-\frac{b}{2} \le y \le \frac{b}{2}\right)$ and thickness $h\left(-\frac{h}{2} \le z \le \frac{h}{2}\right)$, for which the x, y and z axes are defined along the longitudinal, width and thickness directions of the beam, respectively. In equilibrium, the beam is unstrained, unstressed, without damping mechanism, and the temperature is T0 everywhere, Soh et al. (2008), Sun et al. (2006), Youssef and Elsibai (2010).

In the present study, the usual Euler–Bernoulli assumption is adopted, i.e., any plane cross-section, initially perpendicular to the axis of the beam remains plane and perpendicular to the neutral surface during bending. Thus, the displacements u, v, w are given by

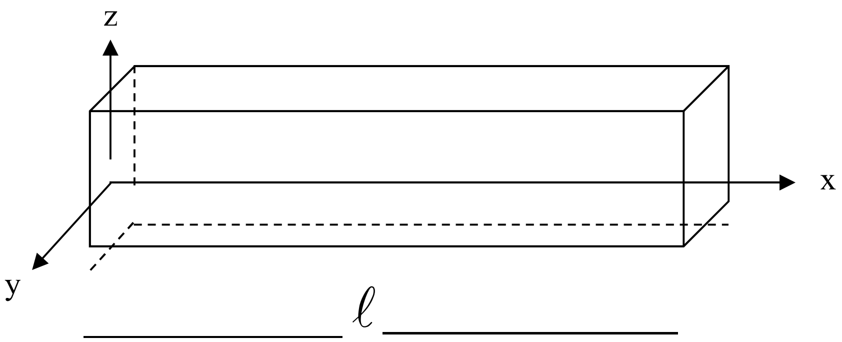

Figure 1.

$$u=-z\frac{\partial w(x,t)}{\partial x}\ ,\ \ v=0\ \ ,\ \ w(x,y,z,t)=w(x,t). \tag{1}$$

Hence, the differential equation of thermally induced lateral vibration of the beam may be expressed in the form:

$$\frac{\partial^4 w}{\partial x^4}+\frac{\rho A}{EI}\frac{\partial^2 w}{\partial t^2}+\alpha_T\frac{\partial^2 M_T}{\partial x^2}=0\,, \tag{2}$$

where E is Young's modulus, I [= $bh^3/12$] the inertial moment about x-axis, ρ the density of the beam, α_T the coefficient of linear thermal expansion, $w(x,t)$ the lateral deflection, x the distance along the length of the beam, $A = hb$ is the cross section area and t the time and M_T is the thermal moment, which is defined as:

$$M_T=\frac{12}{h^3}\int_{-h/2}^{h/2}\theta\, z\, dz\ \ , \tag{3}$$

where $\theta = T - T_0$ is the dynamical temperature increment of the resonator, in which T(x, z, t) is the temperature distribution and T0 the environmental temperature.
The Laser

$$I(t)=\frac{I_0}{t_p^2}\left(t\ e^{-\frac{t}{t_p}}\right), \tag{4}$$

where t_p is a characteristic time of the laser-pulse, I_0 is the laser intensity (the total energy carried by a laser pulse per unit cross-section of laser beam) Sun et al. (2006).
The heat source

$$Q(z,t)=\left(\frac{1-R}{\delta}\right)e^{\left(\frac{z-h/2}{\delta}\right)}I(t)=\left(\frac{1-R}{\delta}\right)\frac{I_0}{t_p^2}e^{\left(\frac{z-h/2}{\delta}\right)}\left(t\ e^{-\frac{t}{t_p}}\right), \tag{5}$$

where δ is the absorption depth of heating energy and R is the surface reflectivity.
According to Youssef model of two-temperature thermoelasticity, the non-Fourier heat conduction equation has the following form:

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial z^2}=\left(\frac{\partial}{\partial t}+\tau_o\frac{\partial^2}{\partial t^2}\right)\left(\frac{\rho C_\upsilon}{k}\theta+\frac{\alpha T_0}{k}e\right)-\left(\frac{1-R}{\delta}\right)\frac{I_0}{t_p^2}e^{\left(\frac{z-h/2}{\delta}\right)}\left(1+\tau_o\frac{\partial}{\partial t}\right)\left(t\ e^{-\frac{t}{t_p}}\right), \tag{6}$$

where C_υ is the specific heat at constant volume, τ_0 the thermal relaxation time, k the thermal conductivity, $\alpha=\frac{E\alpha_T}{1-2\nu}$ in which ν is Poisson's ratio, φ is the conductive temperature increment it satisfies the following relation Youssef (2008):

$$\varphi-\theta=\lambda\left(\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial z^2}\right), \tag{7}$$

where λ is non negative parameter (two-temperature parameter).
The volumetric strain takes the form

$$e=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z} \tag{8}$$

Since there is no heat flow across the upper and lower surfaces of the beam, then,

$$\frac{\partial\theta}{\partial z}=\frac{\partial\varphi}{\partial z}=0 \text{ at } z=\pm h/2 \tag{9}$$

For a very thin beam and assuming the temperature varies in terms of a $\sin(pz)$ function along the thickness direction, where $p=\pi/h$, gives:

$$\theta(x,z,t)=\theta_1(x,t)\sin(pz) \tag{10}$$

and

$$\varphi(x,z,t)=\varphi_1(x,t)\sin(pz) \tag{11}$$

which gives

$$(\varphi_1-\theta_1)=\lambda\left(\frac{\partial^2\varphi_1}{\partial x^2}-p^2\varphi_1\right) \tag{12}$$

Hence, equation (2) gives

$$\frac{\partial^4 w}{\partial x^4}+\frac{\rho A}{EI}\frac{\partial^2 w}{\partial t^2}+\frac{12\alpha_T}{h^3}\frac{\partial^2\theta_1}{\partial x^2}\int_{-h/2}^{h/2} z\sin(pz)\,dz=0 \tag{13}$$

and equation (6) gives

$$\begin{aligned}\frac{\partial^2\varphi_1}{\partial x^2}\sin(pz)-p^2\varphi_1\sin(pz)=&\left(\frac{\partial}{\partial t}+\tau_o\frac{\partial^2}{\partial t^2}\right)\left(\frac{\rho C_\upsilon}{k}\theta_1\sin(pz)-\frac{\alpha T_0}{k}z\frac{\partial^2 w}{\partial x^2}\right)\\&-\left(\frac{1-R}{\delta}\right)\frac{I_0}{t_p^2}e^{\left(\frac{z-h/2}{\delta}\right)}\left(1+\tau_o\frac{\partial}{\partial t}\right)\left(t\,e^{-\frac{t}{t_p}}\right)\end{aligned} \tag{14}$$

After doing the integrations, equation (13) takes the form

$$\frac{\partial^4 w}{\partial x^4}+\frac{\rho A}{EI}\frac{\partial^2 w}{\partial t^2}+\frac{24\alpha_T}{h\pi^2}\frac{\partial^2\theta_1}{\partial x^2}=0 \tag{15}$$

In equation (14), we multiply the both sides by z and integrating with respect to z from $-\frac{h}{2}$ to $\frac{h}{2}$, then we obtain

$$\frac{\partial^2 \varphi_1}{\partial x^2} - p^2 \varphi_1 = \left(\frac{\partial}{\partial t} + \tau_o \frac{\partial^2}{\partial t^2}\right)\left(\varepsilon \theta_1 - \frac{\alpha T_0 \pi^2 h}{24k} \frac{\partial^2 w}{\partial x^2}\right) - \beta T_0 \left(\tau_o + \omega t\right) e^{-\frac{t}{t_p}} \tag{16}$$

where $\varepsilon = \frac{\rho C_\upsilon}{k}$, $\omega = \frac{t_p - \tau_o}{t_p}$, $\beta = \frac{(1-R) h I_0}{2 t_p^2 T_0}\left[(2a+1)e^{-1/a} - 2a + 1\right]$, $a = \frac{\delta}{h}$

Now, for simplicity we will use the following non-dimensional variables, Youssef (2008):

$$\left(x', w', \frac{1}{p'}\right) = \varepsilon c_o \left(x, w, \frac{1}{p}\right), \left(t', \tau_o', t_p'\right) = \varepsilon c_o^2 \left(t, \tau_o, t_p\right), \sigma' = \frac{\sigma}{E}, \theta_1' = \frac{\theta_1}{T_o}, \varphi_1' = \frac{\varphi_1}{T_o}, c_o^2 = \frac{E}{\rho} \tag{17}$$

Then, we have

$$\frac{\partial^4 w}{\partial x^4} + A_1 \frac{\partial^2 w}{\partial t^2} + A_2 \frac{\partial^2 \theta_1}{\partial x^2} = 0 \tag{18}$$

$$\frac{\partial^2 \varphi_1}{\partial x^2} - A_3 \varphi_1 = \left(\frac{\partial}{\partial t} + \tau_o \frac{\partial^2}{\partial t^2}\right)\left(\theta_1 - A_4 \frac{\partial^2 w}{\partial x^2}\right) - \beta\left(\tau_o + \omega t\right) e^{-\frac{t}{t_p}} \tag{19}$$

and

$$\varphi_1 - \theta_1 = A_5 \frac{\partial^2 \varphi_1}{\partial x^2} - A_6 \varphi_1 \tag{20}$$

where $A_1 = \frac{12}{h^2}$, $A_2 = \frac{24\alpha_t T_o}{\pi^2 h}$, $A_3 = p^2$, $A_4 = \frac{\pi^2 \alpha h}{24k\varepsilon}$, $A_5 = \lambda \varepsilon c_o$, $A_6 = \lambda \varepsilon c_o p^2$

(We have dropped the prime for convenience)

3 FORMULATION THE PROBLEM IN THE LAPLACE TRANSFORM DOMAIN

We will apply the Laplace transform for equations (18) - (20), which is defined by the following formula

$$\bar{f}(s) = L[f(t)] = \int_0^\infty f(t) e^{-st}\, dt$$

Hence, we obtain the following system

$$\frac{d^4 \bar{w}}{dx^4} + A_1 s^2 \bar{w} + A_2 \frac{d^2 \bar{\theta}_1}{dx^2} = 0 \tag{21}$$

$$\frac{d^2\bar{\varphi}_1}{dx^2} - A_3\bar{\varphi}_1 = \left(s + \tau_o s^2\right)\left(\bar{\theta}_1 - A_4\frac{d^2\bar{w}}{dx^2}\right) - \frac{t_p\left(t_p\left(\omega + s\,\tau_o\right) + \tau_o\right)\beta}{\left(t_p s + 1\right)^2} \tag{22}$$

and

$$\bar{\theta}_1 = \left(1 + A_6\right)\bar{\varphi}_1 - A_5\frac{d^2\bar{\varphi}_1}{dx^2} \tag{23}$$

We will consider the function $\bar{\eta}$ as follows:

$$\frac{d^2\bar{w}}{dx^2} = \bar{\eta} \tag{24}$$

Then, we have

$$\frac{d^2\bar{\eta}}{dx^2} = -A_1 s^2\bar{w} - A_2\frac{d^2\bar{\theta}_1}{dx^2} \tag{25}$$

$$\frac{d^2\bar{\varphi}_1}{dx^2} = A_3\bar{\varphi}_1 + \left(s + \tau_o s^2\right)\bar{\theta}_1 - A_4\left(s + \tau_o s^2\right)\bar{\eta} - \frac{t_p\left(t_p\left(\omega + s\,\tau_o\right) + \tau_o\right)\beta}{\left(t_p s + 1\right)^2} \tag{26}$$

and

$$\bar{\theta}_1 = \beta_1\bar{\varphi}_1 + \beta_2\bar{\eta} + \beta_3 \tag{27}$$

where

$$\beta_1 = \frac{\left(1 + A_6 - A_3A_5\right)}{1 + \left(s + \tau_o s^2\right)A_5}, \quad \beta_2 = \frac{A_4A_5\left(s + \tau_o s^2\right)}{1 + \left(s + \tau_o s^2\right)A_5}, \quad \beta_3 = \frac{A_5 t_p\,\beta\left(t_p\left(\omega + s\,\tau_o\right) + \tau_o\right)}{\left(1 + \left(s + \tau_o s^2\right)A_5\right)\left(t_p s + 1\right)^2}$$

Hence, we have

$$\frac{d^2\bar{\varphi}_1}{dx^2} = \alpha_1\bar{\varphi}_1 + \alpha_2\bar{\eta} + \beta_4 \tag{28}$$

where

$$\alpha_1 = A_3 + \beta_1\left(s + \tau_o s^2\right),\ \alpha_2 = \left(\beta_2 - A_4\right)\left(s + \tau_o s^2\right)$$

$$\beta_4 = \beta_3\left(s + \tau_o s^2\right) - \frac{t_p\left(t_p\left(\omega + s\,\tau_o\right) + \tau_o\right)\beta}{\left(t_p s + 1\right)^2}$$

Hence, we have

$$\frac{d^2\bar{\theta}_1}{dx^2} = \frac{\alpha_1\beta_1}{\left(1 + A_2\beta_2\right)}\bar{\varphi}_1 - \frac{\alpha_2\beta_1}{\left(1 + A_2\beta_2\right)}\bar{\eta} - \frac{A_1\beta_2 s^2}{\left(1 + A_2\beta_2\right)}\bar{w} + \frac{\beta_1\beta_4}{\left(1 + A_2\beta_2\right)} \tag{29}$$

$$\frac{d^2\bar{\eta}}{dx^2} = \left[-A_1 s^2 + \frac{A_1A_2\beta_2 s^2}{\left(1 + A_2\beta_2\right)}\right]\bar{w} - \frac{\alpha_1A_2\beta_1}{\left(1 + A_2\beta_2\right)}\bar{\varphi}_1 + \frac{\alpha_2\beta_1A_2}{\left(1 + A_2\beta_2\right)}\bar{\eta} - \frac{A_2\beta_1\beta_4}{\left(1 + A_2\beta_2\right)} \tag{30}$$

$$\frac{d^2\bar{\eta}}{dx^2}=\alpha_3\bar{w}-\alpha_4\bar{\varphi}_1+\alpha_5\bar{\eta}-\beta_5 \tag{31}$$

where

$$\alpha_3=\frac{A_1A_2s^2\beta_2}{\left[1+A_2\beta_2\right]}-A_1s^2\ ,\ \alpha_4=\frac{\alpha_1\beta_1A_2}{\left(1+A_2\beta_2\right)}\ ,\ \alpha_5=\frac{\alpha_2\beta_1A_2}{\left(1+A_2\beta_2\right)}\ ,\ \beta_5=\frac{A_2\beta_1\beta_4}{\left(1+A_2\beta_2\right)}$$

Finally, we have the system

$$\frac{d^2\bar{w}}{dx^2}=\bar{\eta} \tag{32}$$

$$\left[\frac{d^2}{dx^2}-\alpha_1\right]\bar{\varphi}_1=\alpha_2\bar{\eta}+\beta_4 \tag{33}$$

and

$$\frac{d^2}{dx^2}\left[\frac{d^2}{dx^2}-\alpha_5\right]\bar{\eta}=\alpha_3\bar{\eta}-\alpha_4\frac{d^2\bar{\varphi}_1}{dx^2} \tag{34}$$

Eliminating $\bar{\varphi}_1$ from equations (33) and (34), we get

$$\left[D^6-lD^4+mD^2-n\right]\bar{\eta}=0 \tag{35}$$

Eliminating $\bar{\eta}$ from equations (33) and (34), we obtain

$$\left[D^6-lD^4+mD^2-n\right]\bar{\varphi}_1=-\alpha_3\beta_4 \tag{36}$$

where

$$D^r=\frac{d^r}{dx^r}\ ,\ l=\alpha_1+\alpha_5\ ,\ m=\alpha_1\alpha_5+\alpha_2\alpha_4-\alpha_3\quad \text{and}\ n=-\alpha_1\alpha_3$$

Now, we will consider the first end of the nano-beams x=0 is clamped and has no thermal load, which gives

$$w(0,t)=\eta(0,t)=\varphi_1(0,t)=0 \tag{37}$$

After using Laplace transform, the above conditions take the forms

$$\bar{w}(0,s)=\bar{\eta}(0,s)=\bar{\varphi}_1(0,s)=0 \tag{38}$$

Considering the other end of the beam $x=\ell$ is clamped and remains at zero increment of temperature as follows:

$$w(\ell,t)=\varphi_1(\ell,t)=\eta(\ell,t)=0 \tag{39}$$

After using Laplace transform, we have

$$\bar{w}(\ell,s)=\bar{\varphi}_1(\ell,s)=\bar{\eta}(\ell,s)=0 \tag{40}$$

After some simplifications, we get the final solutions in the Laplace transform domain as follows:

$$\bar{\eta}=\sum_{i=1}^{3}\left(\lambda_i^2-\alpha_1\right)\left(a_i e^{-\lambda_i x}+b_i e^{\lambda_i x}\right) \tag{41}$$

$$\bar{\varphi}_1=-\frac{\beta_4}{\alpha_1}+\alpha_2\sum_{i=1}^{3}\left(a_i e^{-\lambda_i x}+b_i e^{\lambda_i x}\right) \tag{42}$$

$$\bar{w}=\sum_{i=1}^{3}\frac{\left(\lambda_i^2-\alpha_1\right)}{\lambda_i^2}\left(a_i e^{-\lambda_i x}+b_i e^{\lambda_i x}\right) \tag{43}$$

where $\pm\lambda_i,\ i=1,2,3$ satisfy the following characteristic equation

$$\lambda^6-1\lambda^4+m\lambda^2-n=0 \tag{44}$$

By using the boundary conditions, we get the following system of linear equations

$$\sum_{i=1}^{3}\left(\lambda_i^2-\alpha_1\right)\left(a_i+b_i\right)=0 \tag{45-a}$$

$$\sum_{i=1}^{3}\left(\lambda_i^2-\alpha_1\right)\left(a_i e^{-\lambda_i \ell}+b_i e^{\lambda_i \ell}\right)=0 \tag{45-b}$$

$$\sum_{i=1}^{3}\left(a_i+b_i\right)=\frac{\beta_4}{\alpha_1\alpha_2} \tag{45-c}$$

$$\sum_{i=1}^{3}\left(a_i e^{-\lambda_i \ell}+b_i e^{\lambda_i \ell}\right)=\frac{\beta_4}{\alpha_1\alpha_2} \tag{45-d}$$

$$\sum_{i=1}^{3}\frac{\left(\lambda_i^2-\alpha_1\right)}{\lambda_i^2}\left(a_i+b_i\right)=0 \tag{45-e}$$

$$\sum_{i=1}^{3}\frac{\left(\lambda_i^2-\alpha_1\right)}{\lambda_i^2}\left(a_i e^{-\lambda_i \ell}+b_i e^{\lambda_i \ell}\right)=0 \tag{45-f}$$

Solving the above system, we get

$$\bar{\eta}=\sum_{i=1}^{3}b_i\left(\lambda_i^2-\alpha_1\right)\left(e^{\lambda_i(\ell-x)}+e^{\lambda_i x}\right) \tag{46}$$

$$\bar{\varphi}_1=-\frac{\beta_4}{\alpha_1}+\alpha_2\sum_{i=1}^{3}b_i\left(e^{\lambda_i(\ell-x)}+e^{\lambda_i x}\right) \tag{47}$$

$$\bar{w} = \sum_{i=1}^{3} \frac{\left(\lambda_i^2 - \alpha_1\right) b_i}{\lambda_i^2} \left(e^{\lambda_i(\ell - x)} + e^{\lambda_i x}\right) \tag{48}$$

where

$$b_1 = \frac{\beta_4 \lambda_1^2 \left(\alpha_1 - \lambda_2^2\right)\left(\alpha_1 - \lambda_3^2\right)}{\alpha_1^2 \alpha_2 \left(\lambda_1^2 - \lambda_2^2\right)\left(\lambda_1^2 - \lambda_3^2\right)\left(e^{\lambda_1 \ell} + 1\right)}$$

$$b_2 = \frac{\beta_4 \lambda_2^2 \left(\alpha_1 - \lambda_1^2\right)\left(\alpha_1 - \lambda_3^2\right)}{\alpha_1^2 \alpha_2 \left(\lambda_1^2 - \lambda_2^2\right)\left(\lambda_3^2 - \lambda_2^2\right)\left(e^{\lambda_2 \ell} + 1\right)}$$

and

$$b_3 = \frac{\beta_4 \lambda_3^2 \left(\alpha_1 - \lambda_1^2\right)\left(\alpha_1 - \lambda_2^2\right)}{\alpha_1^2 \alpha_2 \left(\lambda_1^2 - \lambda_3^2\right)\left(\lambda_2^2 - \lambda_3^2\right)\left(e^{\lambda_3 \ell} + 1\right)}$$

4 THE STRESS AND THE STRAIN-ENERGY

The stress on the x-axis, according to Hooke's law is, Fang et al. (2006):

$$\sigma_{xx}\left(x, z, t\right) = \sigma = E\left(e - \alpha_T \theta\right) . \tag{49}$$

By using the non-dimensional variables in (9), we obtain the stress in the form

$$\sigma = e - \alpha_T T_0 \theta \tag{50}$$

After using Laplace transform, the above equation takes the form:

$$\bar{\sigma} = \bar{e} - \alpha_T T_0 \bar{\theta} \tag{51}$$

The stress-strain energy which is generated on the beam is given by

$$W\left(x, z, t\right) = \sum_{i,j=1}^{3} \frac{1}{2} \sigma_{ij} e_{ij} = \frac{1}{2} \sigma e = -\frac{1}{2} z \sigma \eta \tag{52}$$

or, we can write as follows:

$$W\left(x, z, t\right) = -\frac{1}{2} z \left[L^{-1}\left(\bar{\sigma}\right)\right]\left[L^{-1}\left(\bar{\eta}\right)\right] \tag{53}$$

where $L^{-1}\left[\bullet\right]$ is the inversion of Laplace transform.

5 NUMERICAL INVERSION OF THE LAPLACE TRANSFORM

In order to determine the solutions in the time domain, the Riemann-sum approximation method is used to obtain the numerical results. In this method, any function in Laplace domain can be inverted to the time domain as Tzou (1996):

$$f(t)=\frac{e^{\kappa t}}{t}\left[\frac{1}{2}\bar{f}(\kappa)+\mathrm{Re}\sum_{n=1}^{N}(-1)^{n}\bar{f}\left(\kappa+\frac{in\pi}{t}\right)\right] \tag{54}$$

where Re is the real part and i is imaginary number unit. For faster convergence, numerous numerical experiments have shown that the value of κ satisfies the relation $\kappa t \approx 4.7$ Tzou Tzou (1996).

Numerical Results and Discussion

Now, we will consider a numerical example for which computational results are given. For this purpose, gold (Au) is taken as the thermoelastic material for which we take the following values of the different physical constants Youssef and Elsibai (2010):

$k=318\ \mathrm{W/(mK)}$, $\alpha_T=14.2(10)^{-6}\ \mathrm{K}^{-1}$, $\rho=1930\ \mathrm{kg/m^3}$, $T_0=293\mathrm{K}$, $C_\upsilon=130\ \mathrm{J/(kgK)}$, $E=180\ \mathrm{GPa}$, $\upsilon=0.44$.

The aspect ratios of the beam are fixed as $\ell/h=10$ and $b/h=1/2$, when h is varied, ℓ and b change accordingly with h.

For the nanoscale beam, we will take the range of the beam length ℓ $(1-100)\times10^{-9}\mathrm{m}$. The original time t and the ramping time parameter t_0 will be considered in the picoseconds $(1-100)\times10^{-12}\,\mathrm{sec}$ and the relaxation time τ_0 in the range $(1-100)\times10^{-14}\,\mathrm{sec}$.

The figures were prepared by using the non-dimensional variables which are defined in (9) for beam length $\ell=1.0$, $z=h/6$ and $t=5.0$.

The two-dimensional figures 2-7 and the three-dimensional figures 9-14 show the heat conduction distribution, the dynamical heat distribution, the deflection distribution, the stress distribution, the strain distribution and the strain-stress energy distribution respectively for the two cases of one and two temperature models of thermoelasticity at constant time to stand on the effect of the two temperature parameter effect on all the studied fields. We can see that the two temperature parameter has significant effects on the heat conduction distribution, the dynamical heat distribution, the stress distribution and the strain-stress energy distribution while it has a week effects on the strain and the displacement distribution.

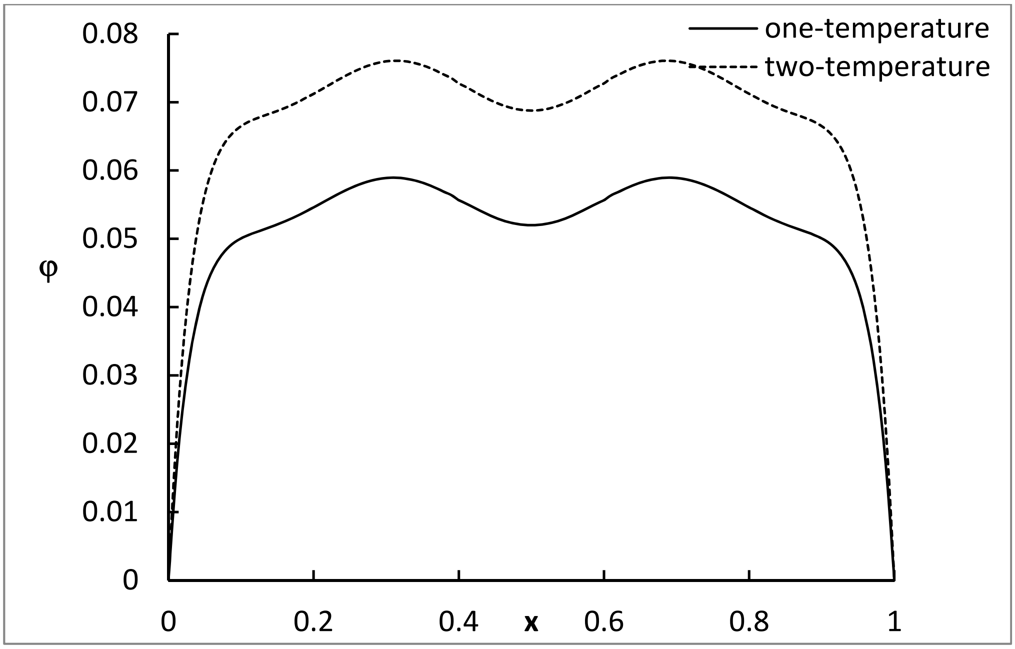

Figure 2: The heat conduction distribution.

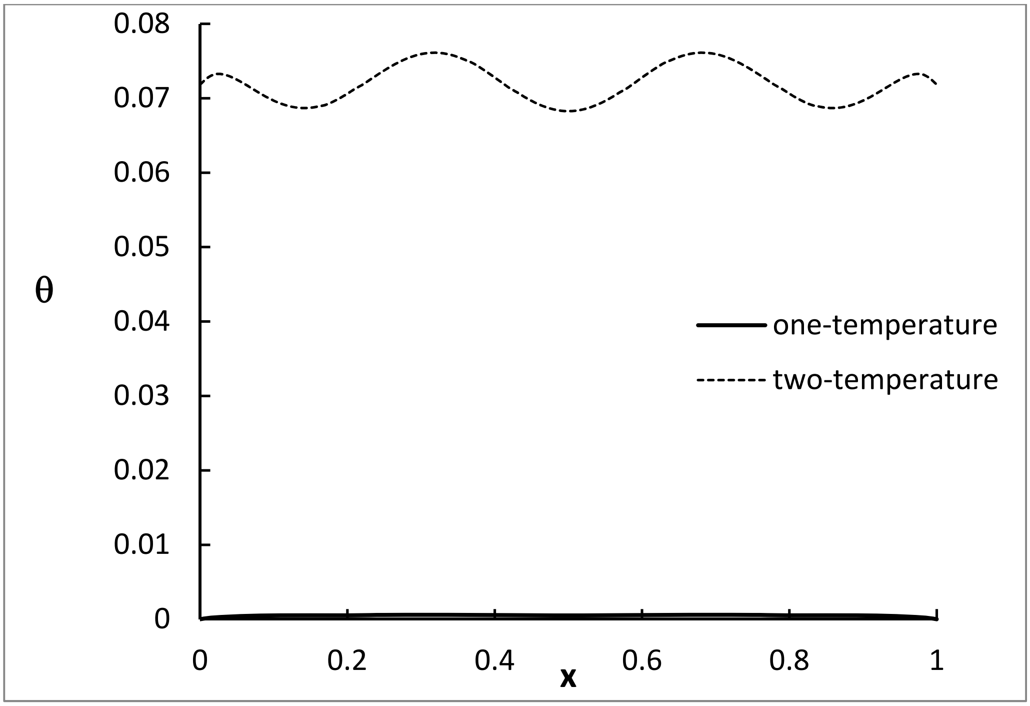

Figure 3: The dynamical heat distribution.

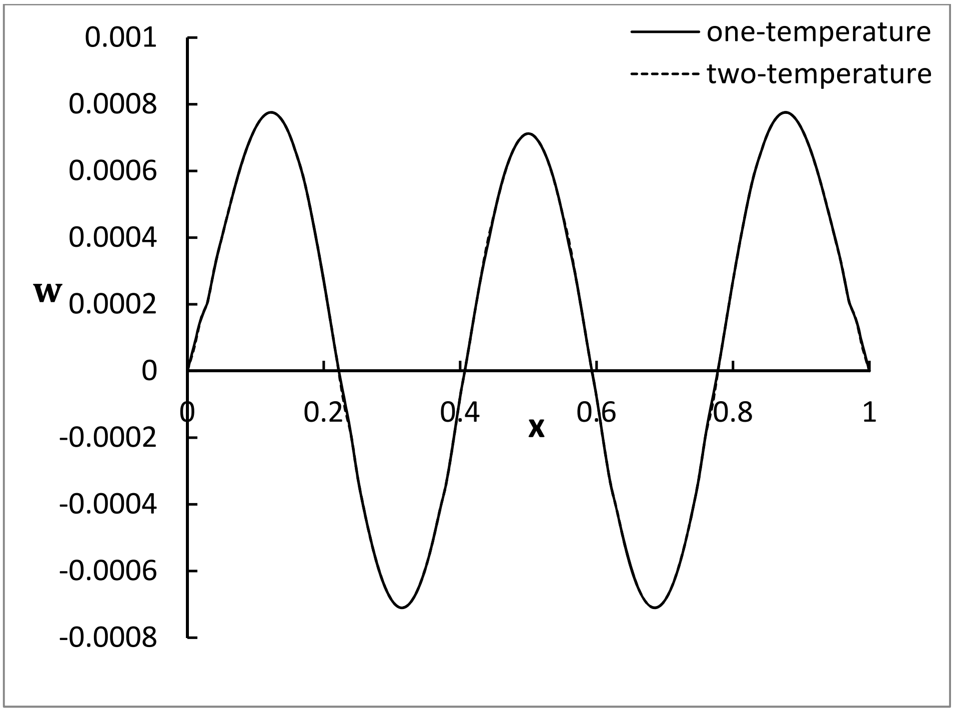

Figure 4: The deflection distribution.

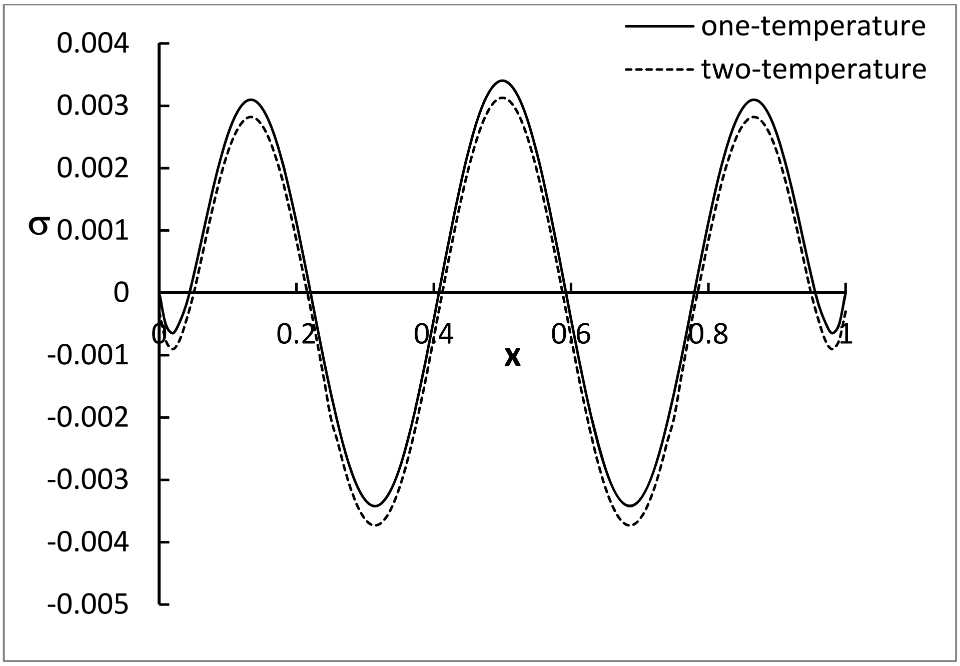

Figure 5: The stress distribution.

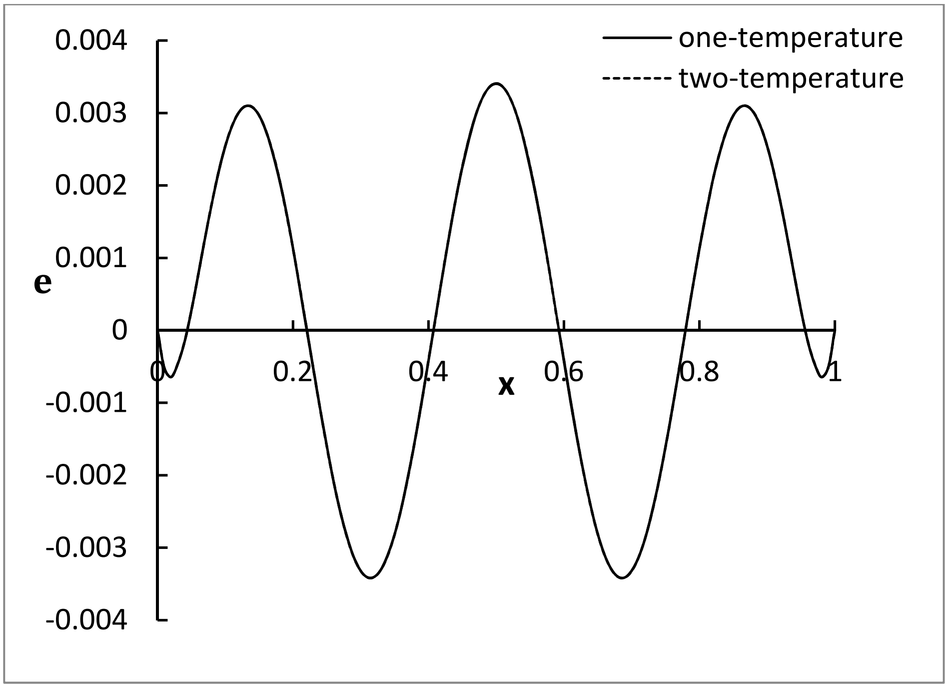

Figure 6: The strain distribution.

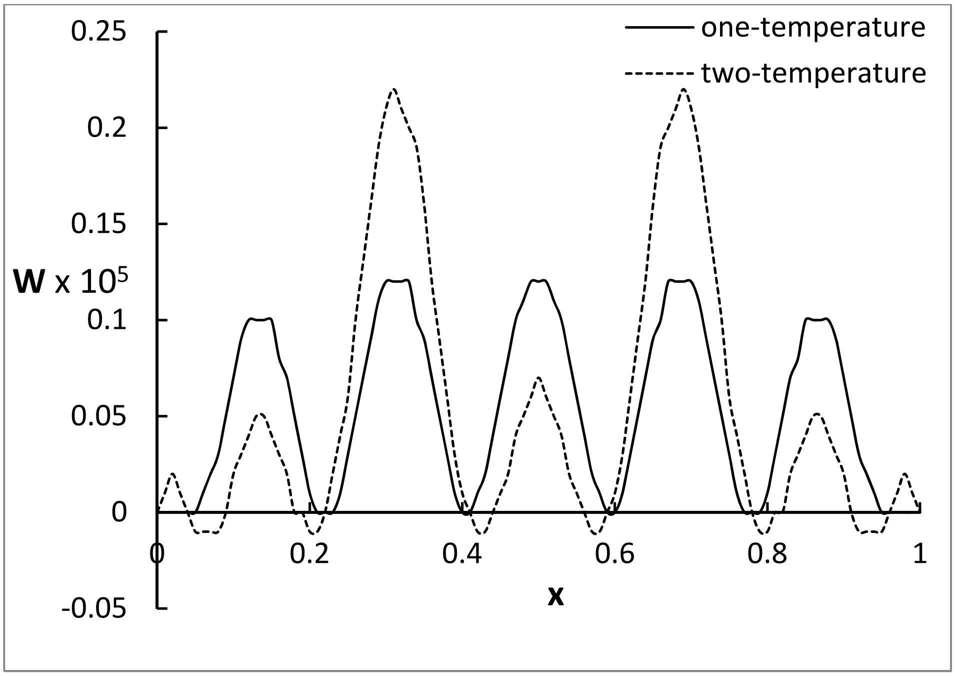

Figure 7: The strain-stress energy distribution.

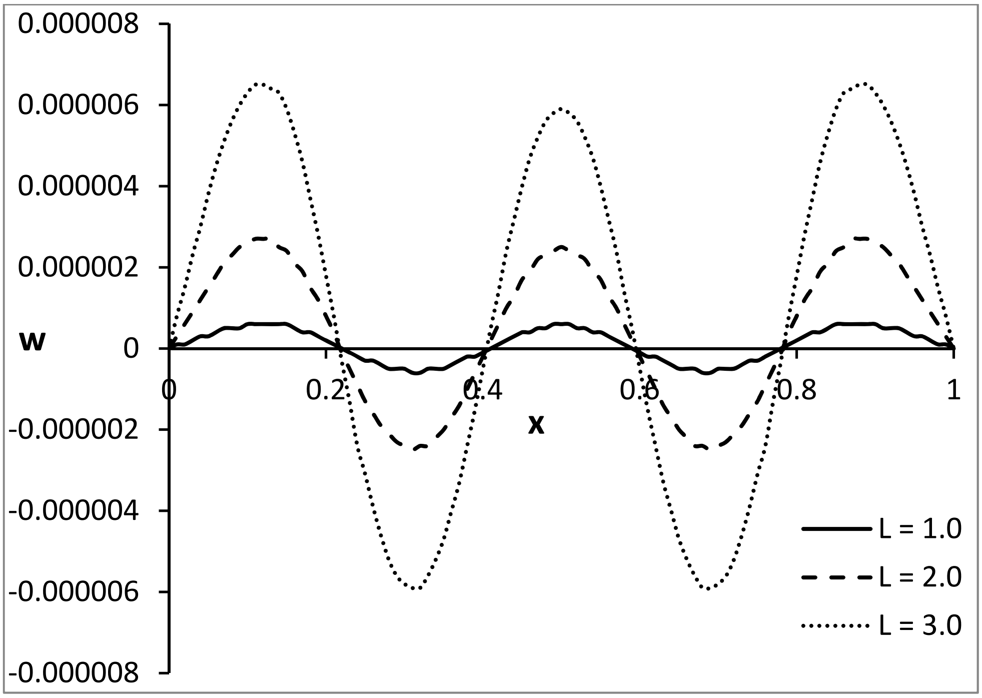

Figure 8: The deflection distribution with different values of length.

The two temperature parameter plays a vital role on the damping of the stress-strain energy where when this parameter increases that energy decreases.

Figure 8 shows the deflection distribution with different values length to stand on the effect of the scale of the beam on the deflection, and we found that, the length of the beam has a significant effect on its deflection, where the deflection increases when the length of the beam increases.

The figures 15-20 show the heat conduction distribution, the dynamical heat distribution, the deflection distribution, the stress distribution, the strain distribution and the strain-stress energy distribution respectively with constant value of the two temperature parameter and with different values of time to stand on the effect of the time on all the studied fields and we find out that, the time has significant effects on all the studied fields.

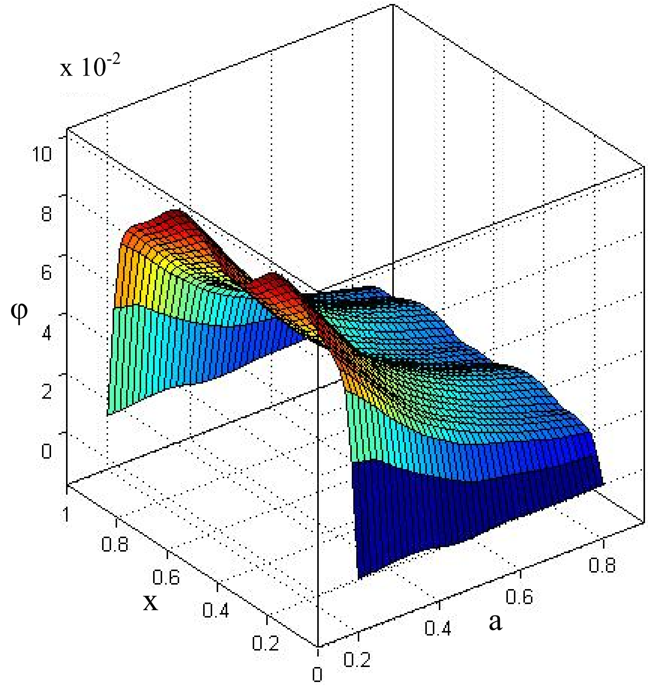

Figure 9: The heat conduction distribution at t = 5.0.

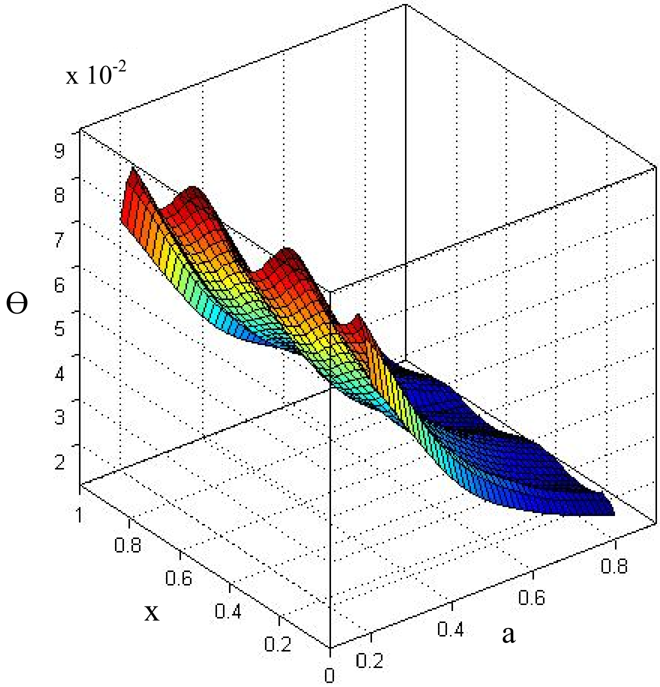

Figure 10: The dynamical heat distribution at t = 5.0.

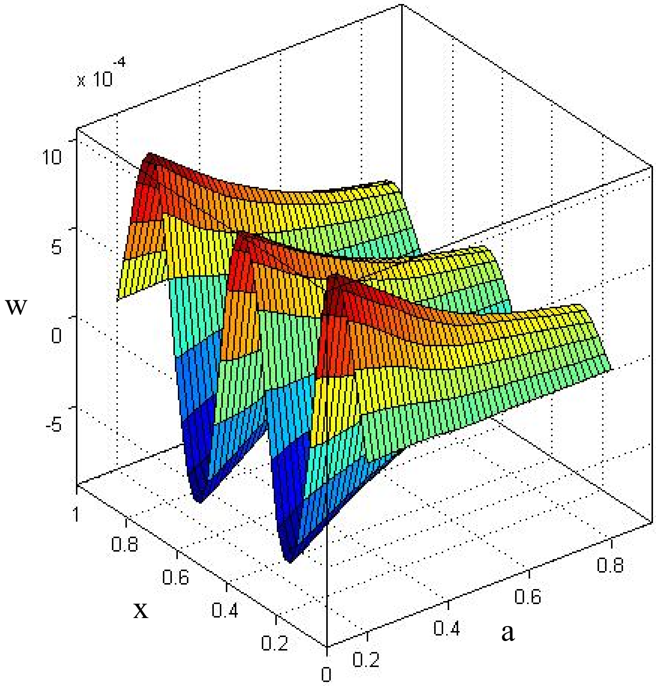

Figure 11: The deflection distribution at t = 5.0.

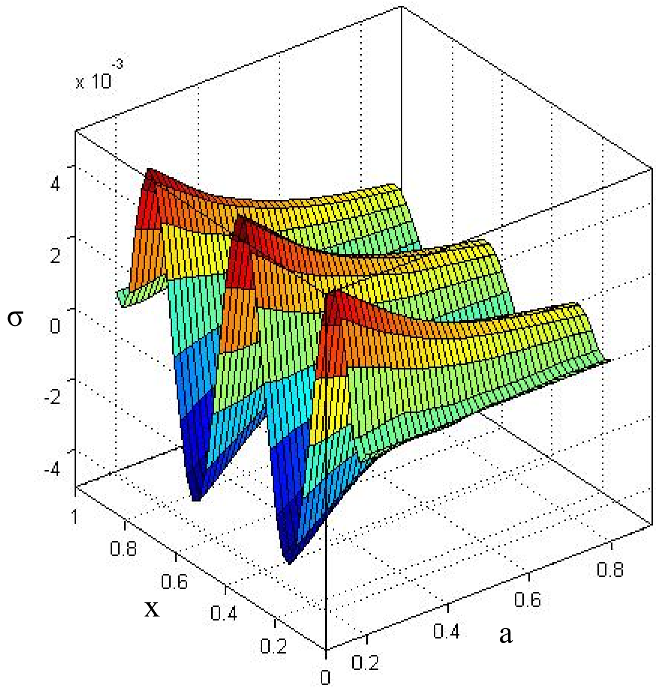

Figure 12: The stress distribution at t = 5.0.

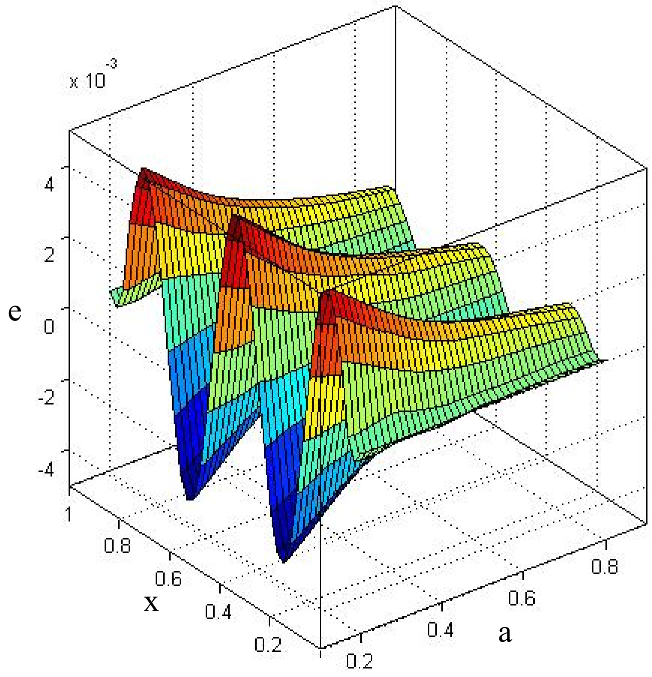

Figure 13: The strain distribution at t = 5.0.

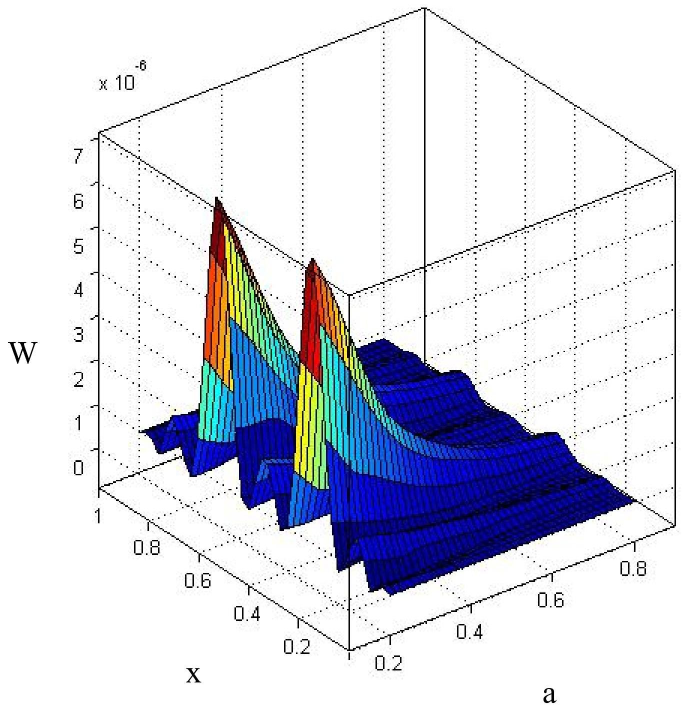

Figure 14: The strain-stress energy distribution at t = 5.0.

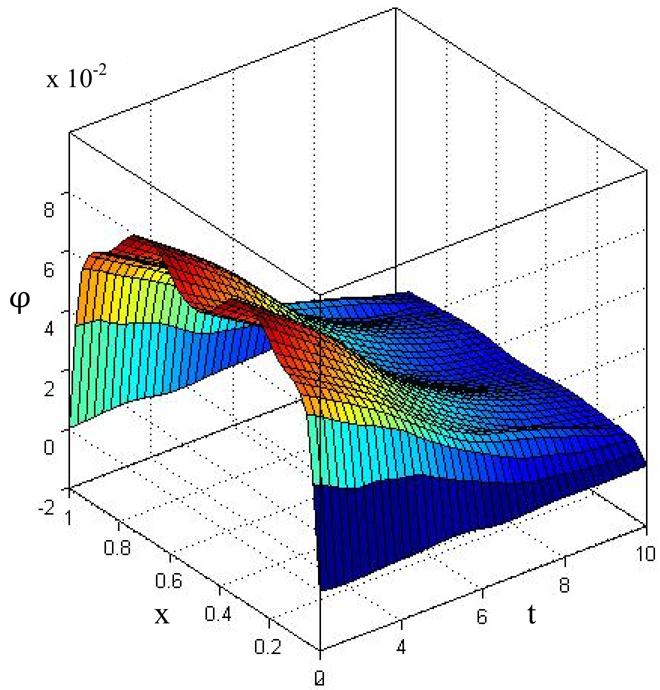

Figure 15: The heat conduction distribution at a = 0.4.

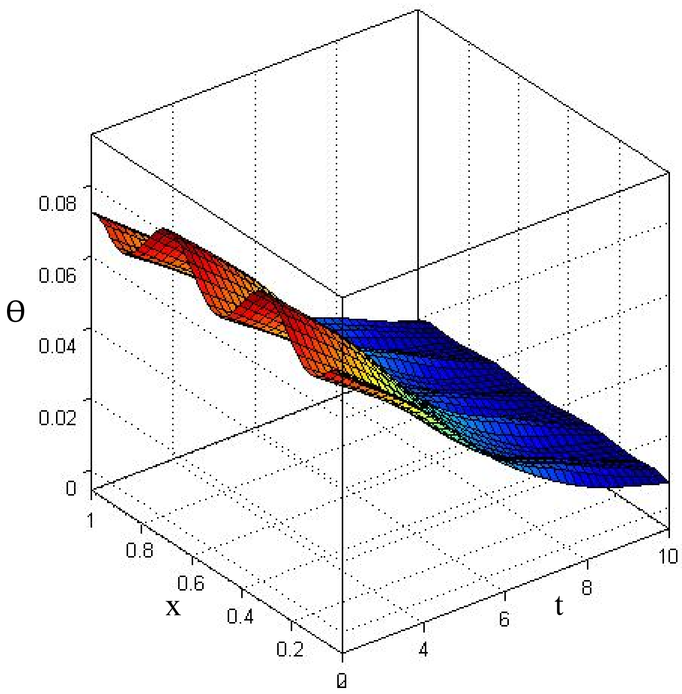

Figure 16: The dynamical heat distribution at a = 0.4.

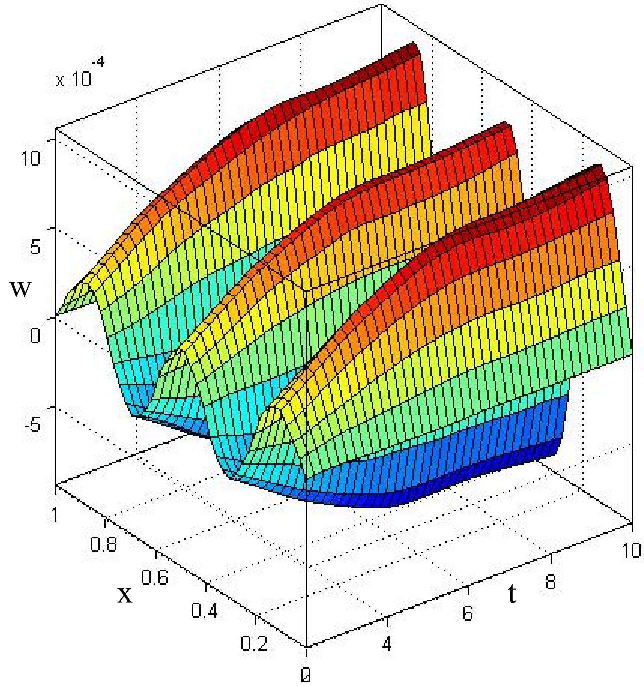

Figure 17: The deflection distribution at a = 0.4.

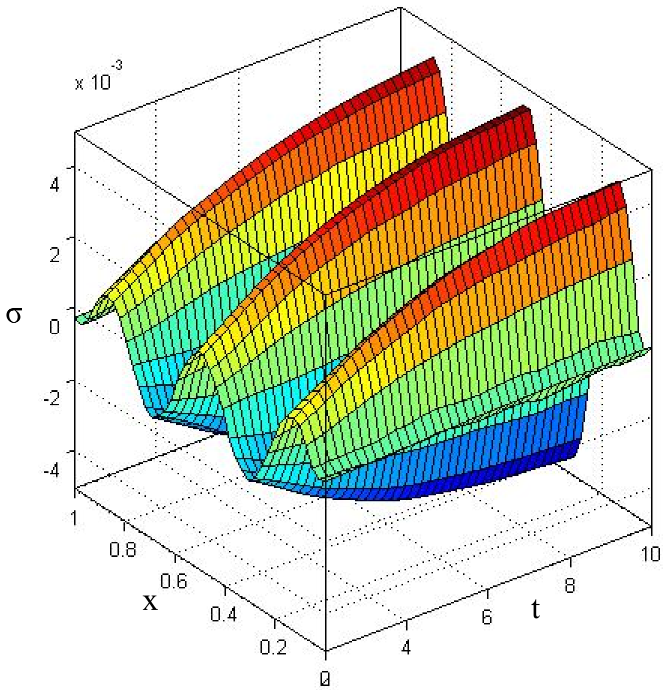

Figure 18: The stress distribution at a = 0.4.

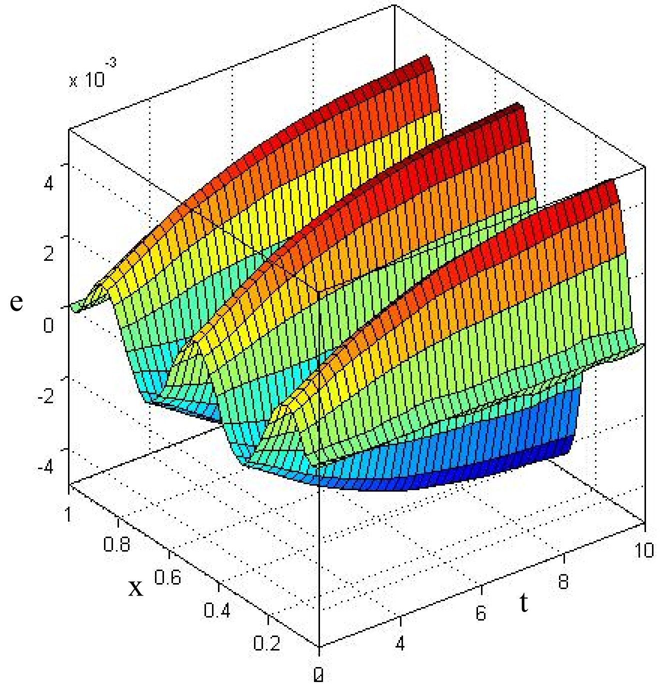

Figure 19: The strain distribution at a = 0.4.

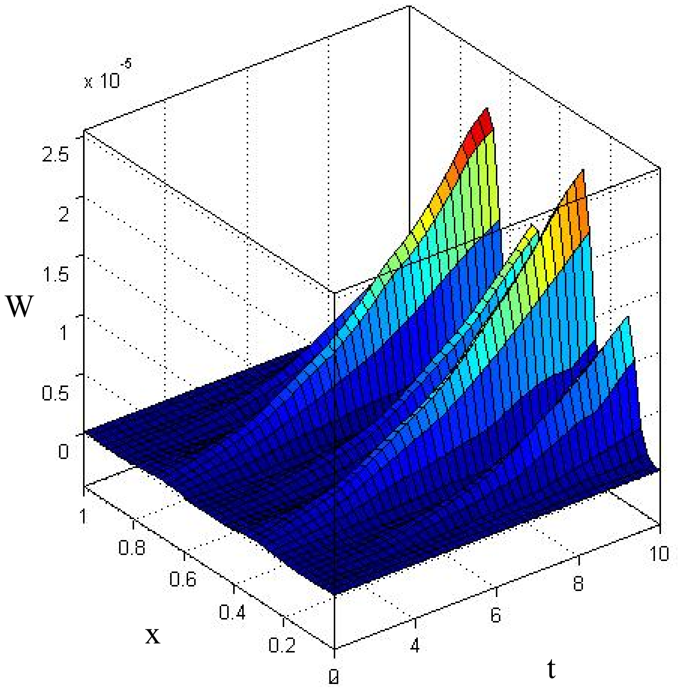

Figure 20: The strain-stress energy distribution at a = 0.4.

6 CONCLUSION

The two-temperature parameter has significant effects on the heat conduction temperature, the dynamical temperature, the stress and the stress-strain energy.
Increasing the value of the two-temperature parameter causes decreasing on the values of the stress strain energy which gives more damping of that energy.
The values of the time have significant effects on all the studied fields.
The length of the beam has a significant effect on its deflection.

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References

AL-Huniti, N.S., AL-Nimr M.A. and Naij, M. (2001).Dynamic response of a rod due to a moving heat source under the hyperbolic heat conduction model, Journal of Sound and Vibration, 242: 629–640.

Bassiouny, E. and Youssef, H. M., (2008).Two-Temperature Generalized Thermopiezoelasticity of Finite Rod Subjected to Different Types of Thermal Loading, J. Thermal Stresses, 31: 233-245.

Boley, B. A. (1972). Approximate analyses of thermally induced vibrations of beams and plates, Journal of Applied Mechanics, 39: 212–216.

Civalek, Ö., Akgöz, B., (2010). Free vibration analysis of microtubules as cytoskeleton components: Nonlocal Euler-Bernoulli beam modeling, Scientia Iranica, 17(5): 367-375.

Civalek, Ö., Demir, C., (2011). Bending analysis of microtubules using nonlocal Euler-Bernoulli beam theory, App. Math. Model., 35(5): 2053-2067.

Diao, J. K., Gall, K. and Dunn, M. L. (2004). Atomistic simulation of the structure and elastic properties of gold nanowires, Journal of Mechanics and Physics of Solids, 52: 1935–1962.

Eringen, A.C., (1983). On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves, J. Appl. Phys., 54: 4703-4710.

Fang, D. N., Sun, Y.X. and Soh, A.K. (2006). Analysis of frequency spectrum of laser-induced vibration of microbeam resonators, Chinese Physics Letters, 23 (6): 1554–1557.

Kidawa, J. (2003).Application of the Green functions to the problem of the thermally induced vibration of a beam, Journal of Sound and Vibration, 262: 865–876.

Liew, K.M. Hu Y.G., He, X.Q., (2008). Flexural wave propagation in single-walled carbon nanotubes. Journal of Computational and Theoretical Nanoscience, 5: 581-586.

Manolis, G. D. and Beskos, D.E. (1980).Thermally induced vibrations of beam structures, Computer Methods in Applied Mechanics and Engineering, 21: 337–355.

Soh A. K., Sun, Y. X. and Fang, D. (2008).Vibration of microscale beam induced by laser pulse," Journal of Sound and Vibration, 311: 243–253.

Sun, Y. X., Fang, D. N., Saka, M. and Soh A. K., (2008).Laser-induced vibrations of micro-beams under different boundary conditions, International Journal of Solids and Structures, 45: 1993–2013.

Sun, Y.X., Fang, D.N. and Soh, A.K. (2006).Thermoelastic damping in micro-beam resonators International Journal of Solids and Structures, 43: 3213–3229.

Tzou, D. (1996).Macro-to –micro heat transfer, Taylor & Francis, Washington DC.

Wang, X., Shao, M., Zhang, S., and Liu, X. (2013). Biomedical applications of gold nanorod-based multifunctional nano-carriers, Journal of Nanoparticles Research, 15:1892-1913.

Youssef, H. M. (2006).Theory of Two-Temperature Generalized Thermoelasticity, IMA J. App. Math., 71(3): 383-390.

Youssef, H. M. and AL-Harby, A. H. (2007). State-Space Approach of Two-Temperature Generalized Thermoelasticity of Infinite Body with a Spherical Cavity Subjected to Different Types of Thermal Loading, J. Archive of Appl. Mech., 77(9): 675-687.

Youssef, H. M. and AL-Lehaibi, E. A. (2007). State-space Approach of Two -Temperature Generalized Thermoelasticity of One Dimensional Problem, Int. J. Solid and Structure, 44: 1550-1562.

Youssef, H. M. and Elsibai, K. A. (2010).Vibration of Gold Nano-Beam Induced by Different Types of Thermal Loading- A State-Space Approach, J. Nanoscale and Microscale Thermophysical Engineering, 15(1): 48- 69.

Youssef, H. M., (2008).Two-Dimensional Problem of Two-Temperature Generalized Thermoelastic Half-Space Subjected to Ramp-Type Heating, J. Comp. Math. and Model., 19(2): 201-216.