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A simple procedure for reliability assessment of thin composite plates against buckling

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Abstract

Thin laminated composite plates are vulnerable to buckling under high compressive axial load due to small thickness. These plates may exhibit considerable variation in their behavior due to inherent uncertainties involved in various material and load parameters. Keeping these uncertainties in view, in the present paper an effort has been made to present a simple procedure for reliability assessment of thin antisymmetric composite laminated plates against buckling. For this purpose, limit state functions have been derived for antisymmetric cross ply and angle ply laminated plates considering buckling as serviceability failure criterion. Using these limit state functions and employing First Order Reliability Method (FORM), reliability has been computed in terms of reliability index. To study the influence of various random variables, employed in the limit state functions, sensitivity analysis has been carried out. Effect of number of layers, effect of fiber orientation, effect of longitudinal modulus, effect of transverse modulus on plate reliability have also been studied on parametric basis.

Keywords: reliability, composites, laminated plates, buckling.

1 Introduction

Laminated composites plates exhibit a considerable variation in their material properties due to involvement of number of parameters that cannot be controlled effectively during fabrication. Randomness in several factors such as fiber orientation, volume fraction, fiber-matrix interface, and curing parameters is inherent in such laminated composites plates. Despite these uncertainties, its use in engineering applications has gained increasing popularity in recent years due to advantages like, weight reduction (high strength/stiffness to weight ratio), longer life (no corrosion, low wear), fatigue endurance, and inherent damping. These plates are commonly employed in engineering applications as thin plates. However, due to small thickness these plates are vulnerable to buckling under high compressive axial load. In a deterministic sense, if nominal buckling strength of composite plate is more than the subjected compressive load, plate is considered safe. However, even if a plate is safe in deterministic sense it may not be reliable in probabilistic sense due to inherent uncertainties involved in various material and load

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parameters. Keeping these uncertainties in view, in the present paper, an effort has been made to present a procedure for reliability assessment of thin composite plates against buckling.

In the past decade some good research papers appeared on reliability analysis of composite laminated plates against buckling. Boyers et al. [2] used the first order reliability method (FORM) to study the design of composite structure for achieving a specified reliability. Lin et al. [8] proposed a procedure for failure probability evaluation of composite laminates subjected to in plane loads. They considered materials properties, fiber angles and layer thicknesses of the laminates as random variables in the reliability analysis. Lin [8] investigated the buckling failure probability of laminated composite plates subjected to different in-plane random loads. Adali [1] presented an optimal design of composite laminates under buckling load uncertainty. The laminate was subjected to biaxial compressive loads and the buckling load was maximized under worst case in-plane loading. Ibrahim et al. [4] studied the reliability of anatisymmetric composite plates against buckling. They used FORM in their reliability calculations.

A detailed review of the past investigations shows that reliability studies on anti-symmetric laminated plates are limited. Further, a sensitivity analysis which can demonstrate relative effect of various random variables on plate reliability was also not seen widely. In addition, such reliability studies in which statistics were generated using well known classical buckling equations and calculations were carried out with computationally efficient reliability calculation tools e.g. FORM are also limited. Keeping these scopes in view, a simple procedure for reliability assessment of composite laminated plates against buckling has been proposed. For this purpose, limit state functions have been derived for antisymmetric cross ply and angle ply laminated plates considering buckling as serviceability failure criterion. Using these limits state functions and employing First Order Reliability Method (FORM), reliability has been computed in terms of reliability index. To study the influence of various random variables employed in the limit state functions, sensitivity analysis has been carried out. Some parametric studies such as effect of number of layers, effect of fiber orientation, effect of longitudinal modulus, effect of transverse modulus are also included to obtain the results of practical interest.

2 Mathematical formulation

2.1 Limit state functions

Limit state function is the mathematical representation of a particular limit state of failure. In the present study, a buckled state of plate is considered as failed state i.e. mathematically a plate is identified as failed, if applied axial load exceeds from buckling strength. However, it is to be noted that such failure (buckled state) represents failure against serviceability limit state and not against the limit state of collapse, as buckled plate can still carry compressive loads in post buckling range. Considering this criterion as failure criterion the limit state functions $g(\underline{x})$ have been derived for simply supported antisymmetric cross ply and antisymmetric angle ply laminated plates and given in Eqns. (1) and (3). A derivation for these functions is presented in the Appendix II.

$$g\left(\underline{x}\right)=\left(\frac{a}{m\pi}\right)^{2}\left\{\left\{\begin{array}{l} D_{11}\left(\left(\frac{m\pi}{a}\right)^{4}+\left(\frac{n\pi}{b}\right)^{4}\right) \\ +2\left(D_{12}+2D_{66}\right)\left(\frac{m\pi}{a}\right)^{2}\left(\frac{n\pi}{b}\right)^{2}\end{array}\right\}+\left\{\frac{\begin{array}{l} 2\left(\left(A_{12}+A_{66}\right)\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)\right)\left(B_{11}\left(\frac{n\pi}{b}\right)^{3}\right)\times\left(-B_{11}\left(\frac{m\pi}{a}\right)^{3}\right)- \\ \left(A_{11}\left(\frac{n\pi}{b}\right)^{2}+A_{66}\left(\frac{m\pi}{a}\right)^{2}\right)\left(-B_{11}\left(\frac{m\pi}{a}\right)^{3}\right)^{2}- \\ \left(A_{11}\left(\frac{m\pi}{a}\right)^{2}+A_{66}\left(\frac{n\pi}{b}\right)^{2}\right)\left(B_{11}\left(\frac{n\pi}{b}\right)^{3}\right)^{2}\end{array}}{\begin{array}{l}\left(A_{11}\left(\frac{m\pi}{a}\right)^{2}+A_{66}\left(\frac{n\pi}{b}\right)^{2}\right)\left(A_{11}\left(\frac{n\pi}{b}\right)^{2}+A_{66}\left(\frac{m\pi}{a}\right)^{2}\right) \\ -\left(\left(A_{12}+A_{66}\right)\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)\right)^{2}\end{array}}\right\}\right\}-P_{c} \tag{1}$$

where, A_{ij}, B_{ij}, and D_{ij} are the extensional stiffnesses, coupling stiffnesses and flexural stiffnesses respectively; m and n are number of half sine waves in the longitudinal and transverse direction respectively; a, b are dimensions of rectangular plate; P_c is the applied compressive load (Fig. 1). The stiffnesses A_{ij}, B_{ij}, and D_{ij} are functions of reduced stiffness matrix $\overline{Q_{ij}}$ (Appendix I) that depends on the elastic constants $E_1, E_2, G_{12}, \nu_{12}, \nu_{21}$ and thickness t. Hence, Eq. (1) is the function of$E_1, E_2, G_{12}, \nu_{12}, \nu_{21}$,$t$, a, b and P_c . Therefore, vector of random variables, $\underline{x}$, can be written as:

$$\underline{x}=\left(t, a, b, E_1, E_2, G_{12}, \nu_{12}, \nu_{21}, P_c\right) \tag{2}$$

The limit state function for the antisymmetric angle ply laminate is

$$g(\underline{x})=\left(\frac{a}{m\pi}\right)^{2}\left\{\left\{\begin{array}{l} D_{11}\left(\frac{m\pi}{a}\right)^{4}+2\left(D_{12}+2D_{66}\right)\times \\ \left(\frac{m\pi}{a}\right)^{2}\left(\frac{n\pi}{b}\right)^{2}+D_{22}\left(\frac{n\pi}{b}\right)^{2}\end{array}\right\}+\frac{\left\{\begin{array}{l} 2\left(\left(A_{12}+A_{66}\right)\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)\right)\times \\ \left(-\left(B_{16}\left(\frac{m\pi}{a}\right)^{2}+3B_{26}\left(\frac{n\pi}{b}\right)^{2}\right)\right)\left(\frac{m\pi}{a}\right)\times \\ \left(-\left(3B_{16}\left(\frac{m\pi}{a}\right)^{2}+B_{26}\left(\frac{n\pi}{b}\right)^{2}\right)\right)\left(\frac{n\pi}{b}\right)- \\ \left(A_{22}\left(\frac{n\pi}{b}\right)^{2}+A_{66}\left(\frac{m\pi}{a}\right)^{2}\right)\times \\ \left(\left(-\left(3B_{16}\left(\frac{m\pi}{a}\right)^{2}+B_{26}\left(\frac{n\pi}{b}\right)^{2}\right)\right)\left(\frac{n\pi}{b}\right)\right)^{2}- \\ \left(A_{11}\left(\frac{m\pi}{a}\right)^{2}+A_{66}\left(\frac{n\pi}{b}\right)^{2}\right)\times \\ \left(\left(-\left(B_{16}\left(\frac{m\pi}{a}\right)^{2}+3B_{26}\left(\frac{n\pi}{b}\right)^{2}\right)\right)\left(\frac{m\pi}{a}\right)\right)^{2}\end{array}\right\}}{\left\{\begin{array}{l}\left(A_{11}\left(\frac{m\pi}{a}\right)^{2}+A_{66}\left(\frac{n\pi}{b}\right)^{2}\right)\left(A_{22}\left(\frac{n\pi}{b}\right)^{2}+A_{66}\left(\frac{m\pi}{a}\right)^{2}\right) \\ -\left(\left(A_{12}+A_{66}\right)\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)\right)^{2}\end{array}\right\}}\right\}-P_{c} \tag{3}$$

where $\underline{x}$ denotes the vector of random variables given by Eq. (2).

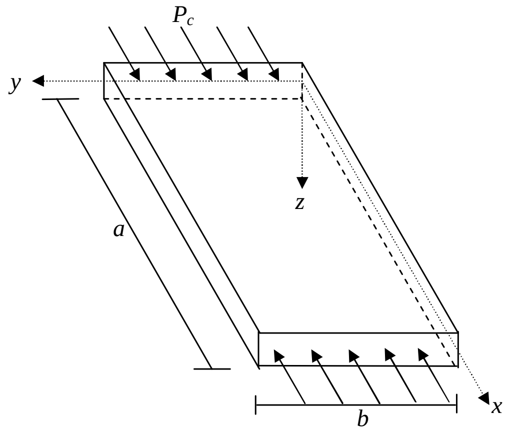


Figure 1: Laminated composite plate subjected to uniform inplane compression.

2.2 Reliability analysis

Having known the limit state functions, we have employed FORM (First Order Reliability Method, [10]) to obtain reliability of composite plates, in terms of reliability index, and probability of failure. In first order reliability method (FORM) the limit state function g($\underline{x}$) is linearized at some point on the failure surface rather than at mean, and non-normal variables are transformed into equivalent normal variables. The linearizing point is called as design point or the Most Probable Point (MPP). An algorithm for the reliability analysis using FORM is given in the following section.

2.2.1 Algorithm for reliability analysis

1. Input the limit state function g($\underline{x}$); statistical values and probability distributions for all the random variables X_i.

2. Assume the values for the initial design point $\{x_i^*\}$ for (n-1) of the random variables X_i. The limit state function g($\underline{x}$) = 0 is solved for the nth random variable. This ensures that the design point is on the failure boundary.

3. Calculate the equivalent normal mean $\mu_{x_i}^e$ and equivalent standard deviation $\sigma_{x_i}^e$ for all the non-normal distributions at the design point values x_i^* using Rackwitz-Fiessler procedure [11]. If some of the random variables are normally distributed, then the equivalent normal parameters are simply the actual parameters.

4. Determine the reduced variates $\{z_i^*\}$ corresponding to the design point $\{x_i^*\}$ using equation $z_i^* = \frac{x_i^* - \mu_{x_i}^e}{\sigma_{x_i}^e}$.

5. Compute the partial derivatives of the limit state function g($\underline{x}$)=0 with respect to the reduced variates. Define a column vector {G}, as the vector whose elements are the partial derivatives multiplied by -1, i.e. $G_i = -\left.\frac{\partial g}{\partial z_i}\right|_{\text{evaluated at design point}}$ and $\{G\} = \left\{\begin{array}{c} G_1 \\ G_2 \\ \\ G_n \end{array}\right\}$.

6. Compute the reliability index β using $\beta = \frac{\{G\}^T\{z^*\}}{\sqrt{\{G\}^T[\rho]\{G\}}}$ where,$\{z^*\} = \left\{\begin{array}{c} z_1^* \\ z_2^* \\ \\ z_n^* \end{array}\right\}$;

   $[\rho]$ = correlation matrix; and subscript T represent Transpose of the matrix.

7. Find sensitivity factors using $\{\alpha\} = \frac{[\rho]\{G\}}{\sqrt{\{G\}^T[\rho]\{G\}}}$.

8. Find a new design point in reduced variates for (n-1) of the variables using the following equation $z_i^* = \alpha_i\beta$

9. Find the corresponding design point values in original coordinates for (n-1) values, using the equation $x_i^* = \mu_{x_i}^e + z_i^*\sigma_{x_i}^e$.

10. Calculate the value of the remaining nth variable by solving the limit state function g($\underline{x}$) = 0.

11. Repeat the steps from 3 to 10 until the design point $\{x_i^*\}$ converge.

Using above algorithm, the reliability in terms of reliability indices has been obtained for various numerical studies.

3 Numerical study

In the present study, we have carried out reliability analysis of antisymmetric angle ply and cross ply composite plates against buckling. The plate is simply supported on all the four edges and subjected to axial compressive force along the edge of dimension b (Fig. 1). The stacking sequence of antisymmetric angle ply laminate is $(+45°/-45°/+45°/-45°)$ and for cross ply, it is $(0°/90°/0°/90°)$. Number of layers for both types of laminates is kept four. Necessary statistical data for reliability assessment is presented in Table 1. In this table, mean values are obtained after taking mean/nominal ratio as 1 and nominal values are taken from Nemeth [9] and Chen [3]. The coefficient of variation (COV), correlation coefficient and probability distributions are, however, assumed. This is so because information of correlation, COV and probability distribution were not available in approachable reference.

Table 1: Statistical data.

Random variables	Mean /Nominal	Distribution	Distribution parameters			
			Mean			COV
			G. E	C.E	B. E	
Thickness, t (mm)	1.0	Normal	5.0	5.0	5.0	0.10
Length, a (mm)	1.0	Normal	1000.0	1000.0	1000.0	0.05
Width, b (mm)	1.0	Normal	150.0	150.0	150.0	0.05
Long. modulus, E_1 (GPa)	1.0	Lognormal	127.55	206.84	203.95	0.12
Trans. modulus, E_2 (GPa)	1.0	Lognormal	11.31	5.17	18.48	0.12
Shear modulus, G_{12} (GPa)	1.0	Lognormal	6.0	2.59	5.58	0.15
Main Poisson ratio, ν_{12}	1.0	Lognormal	0.30	0.25	0.23	0.15
Secondary Poisson ratio,ν_{21}	1.0	Lognormal	0.0266	0.0062	0.0208	0.15
Design load, P_c (N)	1.0	Normal	500	500	500	0.15

Correlation coefficient for E_1 and E_2 is 0.5 G.E. - Graphite Epoxy
Correlation coefficient for E_2 and G_{12} is 0.6 C.E. - Carbon Epoxy
Correlation coefficient for E_1 and G_{12} is 0.6 B.E. - Boron Epoxy

Table 2: Results of the analysis.

Laminate type	Composite type	P_f	β
Antisymmetric cross ply	Graphite Epoxy	1.9×10^{-01}	0.86
	Carbon Epoxy	7.7×10^{-02}	1.43
	Boron Epoxy	3.2×10^{-02}	1.86
Antisymmetric angle ply	Graphite Epoxy	2.1×10^{-02}	2.04
	Carbon Epoxy	1.9×10^{-03}	2.89
	Boron Epoxy	9.4×10^{-04}	3.11

Table 2 shows the results of the analysis for three types of composites *viz.* graphite epoxy, carbon epoxy, and boron epoxy for antisymmetric cross ply and angle ply laminates. The results shows that the plates formed by antisymmetric cross ply are having considerably lesser reliability than their corresponding angle ply laminated plates. This is due to the lower buckling strength of cross ply laminates. In addition, we also observe that except for boron epoxy angle ply laminates, the reliability index magnitude is less than 3.0 for all other laminates. This implies that in the present study except plates made up by boron epoxy angle ply laminates, all other plates are not as reliable against buckling as desired. This is so because in structural reliability assessment, in general, any structure or its component having reliability index less than 3.0 (or probability of failure above 10^{-4}) is considered as unreliable or unsafe [12, 13]. We also observe from this table that there is a considerable difference in reliability of composite plates due to use of different composites. Boron epoxy, in general, is giving maximum reliability against buckling; however, the reliability of graphite epoxy is minimum. This nature is same for both the cross ply as well as for the angle ply laminate.

3.1 Design point or most probable point

A point on the failure surface that corresponds to the shortest distance from the origin in the reduced coordinate system is defined as the most likely failure point or design point [10]. This point can be found from the solution of the constrained optimization problem:

$$\text{Minimize } \beta(\mathbf{y}) = \left(\mathbf{y}^T\mathbf{y}\right)^{1/2} \text{ subject to } G(\mathbf{y}) = 0 \tag{4}$$

Where, $\boldsymbol{y}$ is vector of basic random variables in standard normal space; and G($\boldsymbol{y}$) is limit state function in reduced coordinate system.

Tables 3 and 4 show the values of the most likely failure point or design point on failure surface for antisymmetric cross ply and angle ply laminate respectively. These values of different random variables are necessary for reliability based probabilistic design of composite laminates. In such design partial safety factors for load and resistance variables are

Table 3: Design point values for antisymmetric cross-ply laminate.

Random variables	Design values		
	Graphite epoxy (β =3.27)	Carbon epoxy (β =3.26)	Boron epoxy (β =3.12)
Thickness, t (mm)	7.0	6.5	6.0
Length, a (mm)	1000	1000	1000
Width, b (mm)	150	150	150
Longitudinal modulus, E_1 (GPa)	127.55	206.84	203.95
Transverse modulus, E_2 (GPa)	11.31	5.17	18.48
Shear modulus, G_{12} (GPa)	6.0	2.59	5.58
Main Poisson ratio, ν_{12}	0.30	0.25	0.23
Secondary Poisson ratio,ν_{21}	0.0266	0.0062	0.0208
Design load, P_c (N)	500	500	500

determined for the target reliability (i.e. target reliability index). The desired value of the target reliability index, as generally recommended for structural components, is 3.0 [14,15]. However, the final decision for this value is to be taken by the design engineers and professionals. Having decided the target reliability index values, the safety factors are separately defined for resistance and load variables. For resistance variables, it is defined as the nominal, mean or

Table 4: Design point values for antisymmetric angle-ply laminate.

Random variables	Design values		
	Graphite epoxy (β =3.28)	Carbon epoxy (β =3.27)	Boron epoxy (β =3.11)
Thickness, t (mm)	6.0	5.3	5.0
Length, a (mm)	1000	1000	1000
Width, b (mm)	150	150	150
Longitudinal modulus, E_1 (GPa)	127.55	206.84	203.95
Transverse modulus, E_2 (GPa)	11.31	5.17	18.48
Shear modulus, G_{12} (GPa)	6.0	2.59	5.58
Main Poisson ratio, ν_{12}	0.30	0.25	0.23
Secondary Poisson ratio,ν_{21}	0.0266	0.0062	0.0208
Design load, P_c (N)	500	500	500

characteristic value divided by the design value and for load variables, as the design value divided by the nominal, mean or characteristic values.

3.2 Sensitivity analysis

This analysis has been carried out to study the influence of various random variables on plate reliability. This influence is measured in terms of sensitivity factor (α_j), which is defined for the j^{th} random variable as

$$\alpha_j = \left.\frac{\partial g}{\partial y_j}\right|_{y^*} = \frac{y_j^*}{\beta} \tag{5}$$

where y^*, point minimizing Eq. (4), usually referred to as design point, and y_j^*, value of the j^{th} random variable at this point.

In the present study, using above expression, sensitivity factors for each random variables have been determined and shown graphically in Figs. 2 and 3 for antisymmetric angle ply and cross ply respectively. The magnitude of sensitivity factor for a random variable is a direct measure of its influence on plate reliability. However, its sign determine whether the random variable is a load variable or a resistance variable. The negative value of the sensitivity factor indicates that the random variable is a resistance variable, i.e. its increase will improve the plate reliability and decrease will reduce the reliability. Similarly, positive value of sensitivity factor indicates that it is a load variable and its influence would be opposite to that of a resistance variable. The major advantage of this study is that without carrying out any separate parametric study for each variable one can directly know how a particular random variable affects the plate reliability. Figs. 2 and 3 show that thickness of laminate (X1) is the most influencing resistance parameter and applied load (X9) is the most influencing load variable. Hence to increase the reliability of composite laminate, the easiest and most efficient way is to increase its thickness. However, thickness directly affects the cost of the composite plate. Therefore, to achieve a desirable level of reliability, change of composite type or alteration of some other influencing parameter may be more attractive option. Further, the sensitivity diagram shows that out of length and width (a and b), the effect of width is more pronounced on composite reliability than its length. We also observe from this diagram that the effects of main and secondary Poisson ratios (X7 and X8) are very small as compared to elastic modulii (X4, X5 and X6).

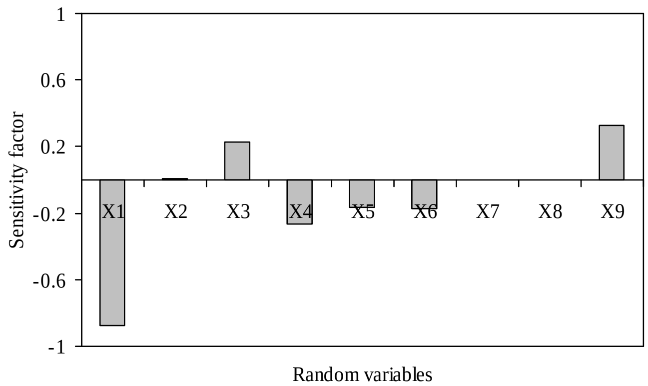
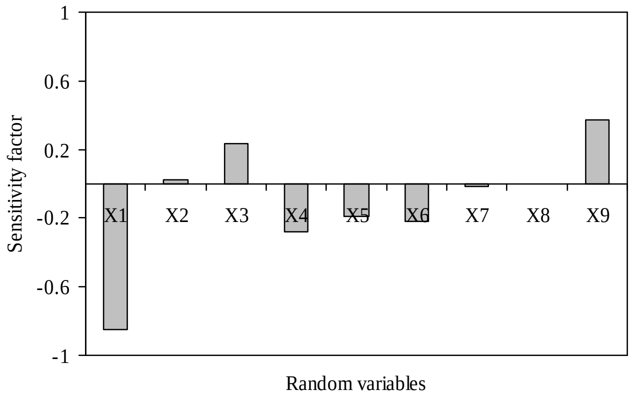


Figure 2: Sensitivity diagram for angle ply laminate.



X1: thickness of the plate (laminate);
X2: length of the plate;
X3: width of the plate;
X4: longitudinal modulus;
X5: transverse modulus;
X6: shear modulus;
X7: main Poisson ratio;
X8: secondary Poisson ratio;
X9: applied load.

Figure 3: Sensitivity diagram for cross ply laminate.

3.3 Parametric study

Table 5: Variation of reliability with correlation.

Laminate type	β	Correlation coefficient between		
		(E_1 and E_2)	(E_2 and G_{12})	(E_1 and G_{12})
Antisymmetric cross ply	0.88	0.0	0.0	0.0
	0.87	0.5	0.0	0.0
	0.87	0.0	0.6	0.0
	0.87	0.0	0.0	0.6
Antisymmetric angle ply	2.05	0.0	0.0	0.0
	2.04	0.5	0.0	0.0
	2.05	0.0	0.6	0.0
	2.05	0.0	0.0	0.6

3.3.1 Effect of correlation

Table 5 shows the variation of reliability index as a function of correlation coefficient between longitudinal modulus and transverse modulus (E_1 and E_2), transverse modulus and shear modulus (E_2 and G_{12}), and longitudinal modulus and shear modulus (E_1 and G_{12}) respectively. Table shows that there is no significant variation in the reliability index if correlation has been considered, hence, for approximate reliability calculations correlation among random variables may be neglected provided magnitude of correlation is of the same order as assumed in Table 1.

3.3.2 Effect of number of layers

Figures 4 and 5 show the effect of number of layers on reliability index for graphite and carbon epoxy composite laminated plates. Trend shows that the reliability of the laminated plate against buckling is quite small if the number of layers is two or less. However, as the number of layers increases beyond 2, reliability improves. This improvement for angle ply carbon epoxy (Fig. 5) is to such an extent that reliability reaches to the desirable range (i.e. ≥ 3). This increase in reliability with number of layers is due to the fact that as number of layers increases, bending-extension coupling decreases which in turn increases the reliability (or buckling strength) of the laminate. However, beyond 6, the effect of bending extension coupling dies out, and due to this reason, if number of layers goes beyond 6; there is no significant improvement in the reliability.

3.3.3 Effect of fiber orientation

Fig. 6 exhibits the variation of reliability index as a function of fiber orientation for an anti-symmetric angle ply laminate of graphite and carbon epoxy. As we can observe, the reliability

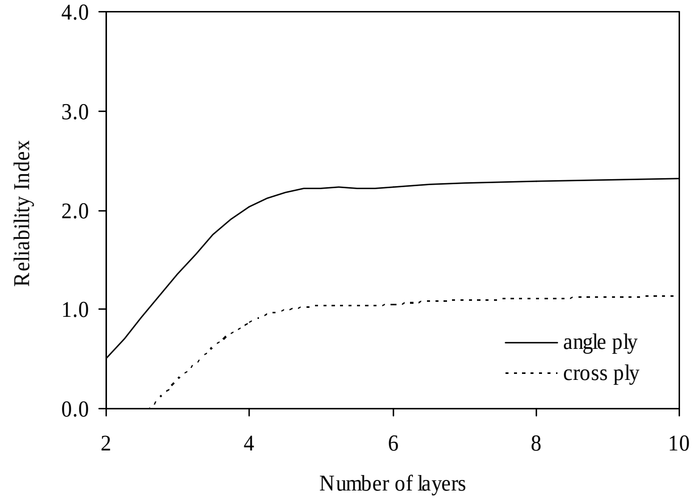


Figure 4: Effect of number of layers on reliability index for graphite epoxy.

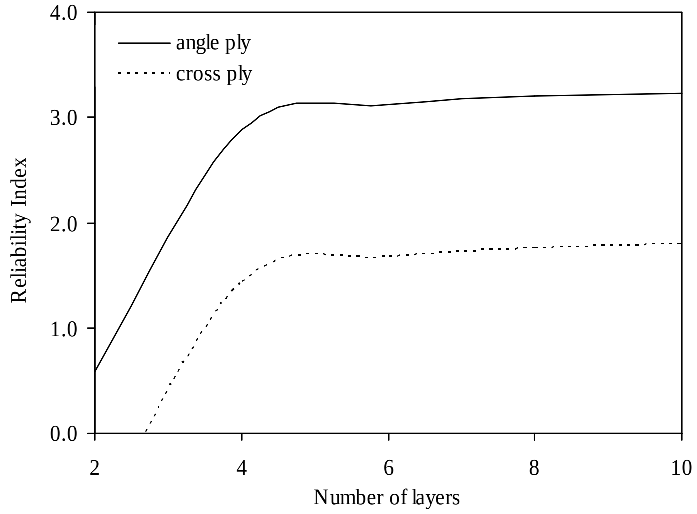


Figure 5: Effect of number of layers on reliability index for carbon epoxy.

index, for both the material reaches to a maximum at $\theta = 45°$ which is the optimal fiber angle for laminates having simply supported edges and subjected to uniaxial compression. A higher value of reliability in the region $\theta = 45°$ is due to the fact that at this angle stiffness of plate increases substantially which leads to higher buckling strength and consequently higher reliability. Further, it can be seen from this figure that trend is same for both graphite and carbon epoxy. However, except for $\theta \leq 15^o$ carbon epoxy gives a higher reliability at all other angles of lamination.

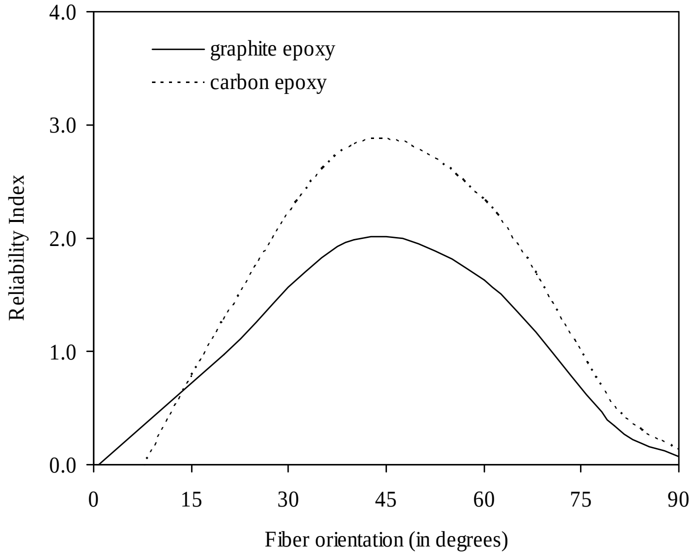


Figure 6: Effect of fiber orientation of reliability index.

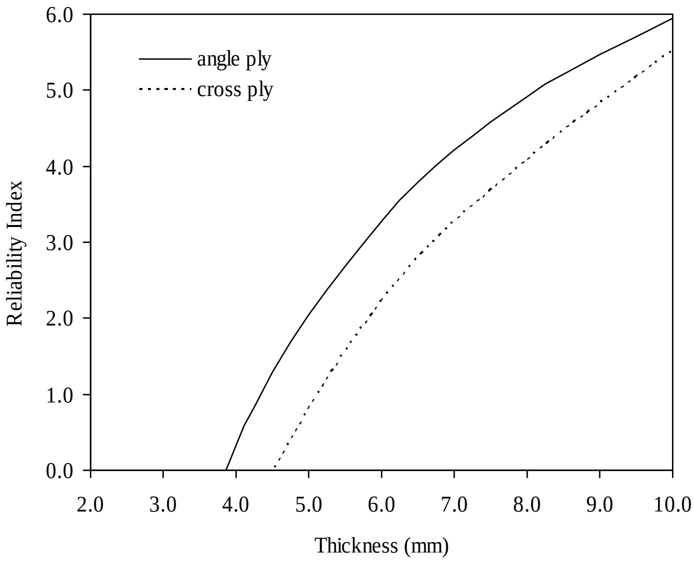


Figure 7: Effect of thickness on reliability index for graphite epoxy.

3.3.4 Effect of thickness

The thickness of composite laminate directly affects its reliability. Since it is the most influencing resistance variable (Fig. 2), its increase will improve the reliability of composite laminate dramatically. Figure 10 shows that when the thickness is 5.0 mm, reliability of graphite epoxy angle ply laminate is 2.0, however, it becomes 3.28 (desirable range of 3) when the thickness is 6.0 mm. From this figure, it can be concluded that to achieve the desired reliability index, thickness of laminate would be the simplest option, provided the cost is not the constraint.

3.3.5 Effect of length and width

Table 6 shows the variation of plate reliability with length and width of the laminated plate. Reliability increases significantly with increase in plate width while it remains almost constant with increase in the length of plate. This is due to the fact that the effect of width is more pronounced in buckling strength as compared to the length. This can also be observed in Figs. 2 and 3.

Table 6: Effect of geometrical properties for antisymmetric angle ply laminate.

Variable parameter	Aspect ratio	β
Length of plate (a)	1.0	2.03
	1.5	2.17
	2.0	2.03
	2.5	2.09
	3.0	2.03
Width of plate (b)	1.0	0.58
	1.5	2.72
	2.0	3.82
	2.5	4.69
	3.0	5.26

3.3.6 Effect of aspect ratio

The effect of aspect ratio on the reliability of composite laminated plate in terms of reliability index values is shown in Fig. 8. There is no significant change in the reliability of plate with increase of aspect ratio. This is so because with the change of aspect ratio, modes shape also changes which cause consequentially no change in critical buckling load and hence no subsequent change in the reliability index value.

3.3.7 Effect of longitudinal modulus

Sensitivity diagram (Figs. 2 and 3) show that the longitudinal modulus is a resistance variable and therefore an increase in its magnitude will improve the reliability. The same expected trend is also seen in Figs. 9 and 10. The figures show that there is a continuous increase in

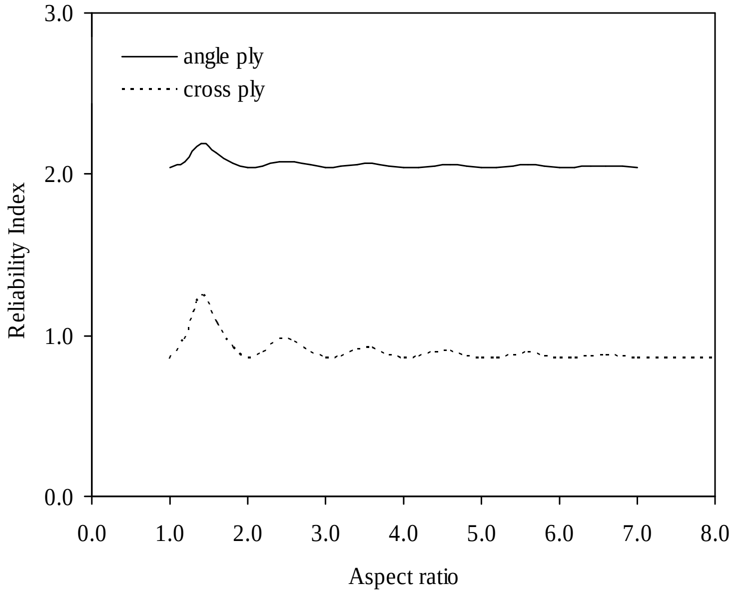


Figure 8: Effect of aspect ratio on reliability index for graphite epoxy.

the reliability with longitudinal modulus value. This is due to the fact that buckling strength directly increases with longitudinal elastic modulus.

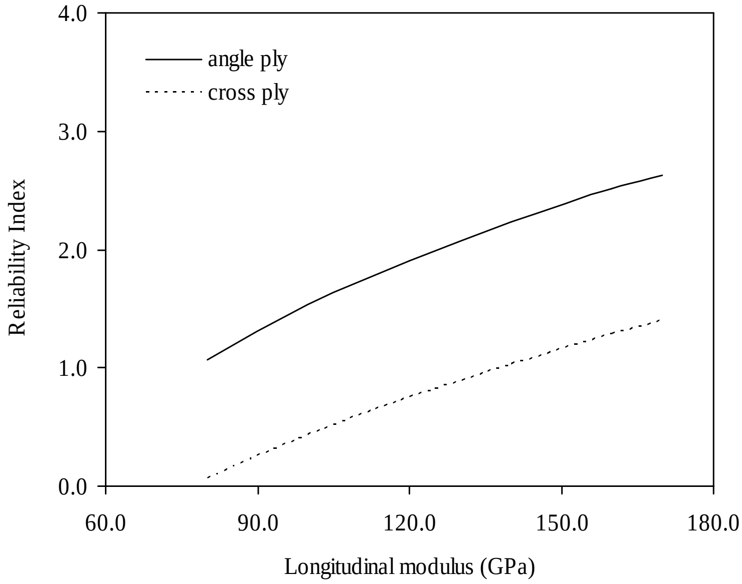


Figure 9: Effect of longitudinal modulus on reliability index for graphite epoxy.

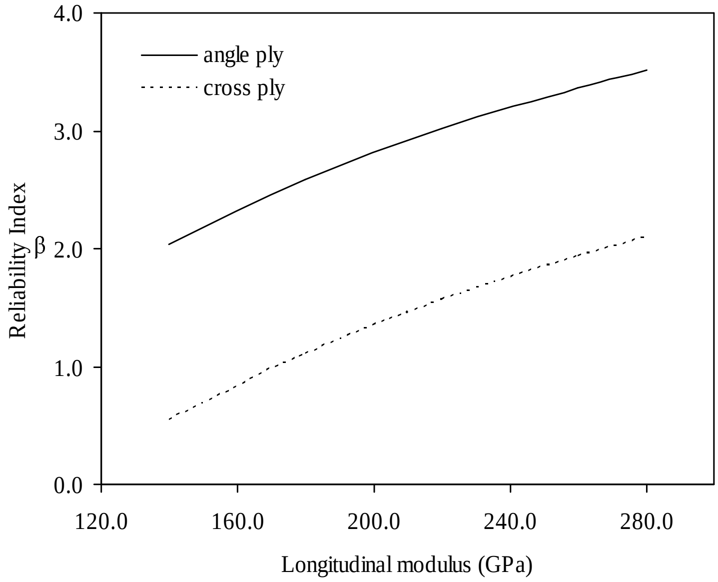


Figure 10: Effect of longitudinal modulus on reliability index for carbon epoxy.

3.3.8 Effect of transverse modulus

Transverse modulus is also a resistance variable (Figs. 2 and 3), so its increase will improve the reliability of the composite laminate. Figs. 11 and 12 exhibits this trend for both the cross ply and angle ply laminate for the graphite and carbon epoxy respectively. In terms of magnitude, 20% increase in the transverse modulus results in only 3% increase in its reliability index. This shows that its effect is not much significant as compared to the longitudinal modulus. This is so because buckling strength is not much affected by the magnitude of transverse modulus of elasticity.

3.3.9 Effect of shear modulus

Figs 13 and 14 show the effect of shear modulus on reliability index. Trend shows that there is no significant improvement in the reliability of composite plate with increase in the shear modulus of elasticity. This is so because the buckling strength is not much influenced by the change in shear modulus, and therefore, there is no subsequent change in reliability index. Moreover, it can be noted that the effect of shear modulus is slightly more for the antisymmetric cross ply laminates than their corresponding angle ply laminated plates.

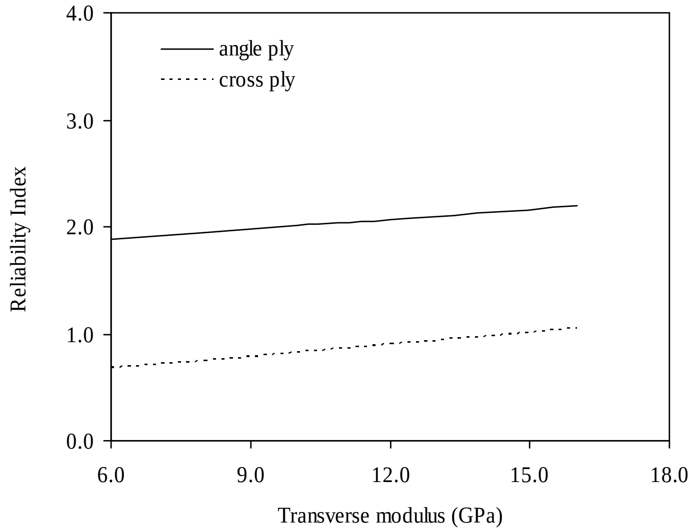


Figure 11: Effect of transverse modulus on reliability index for graphite epoxy.

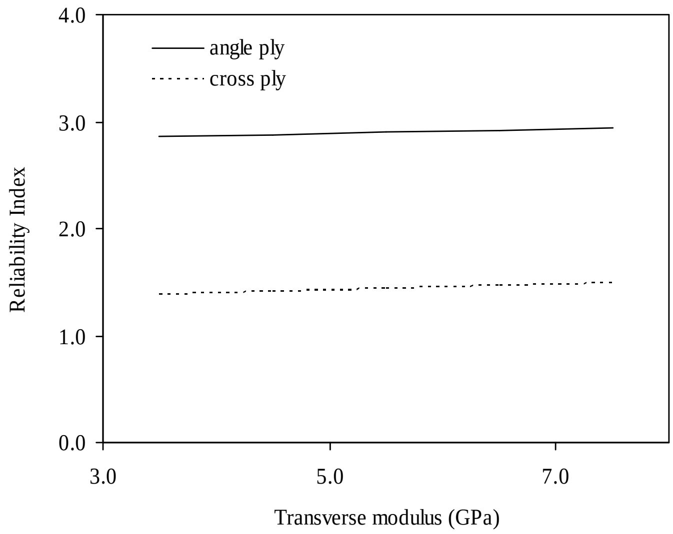


Figure 12: Effect of transverse modulus on reliability index for carbon epoxy.

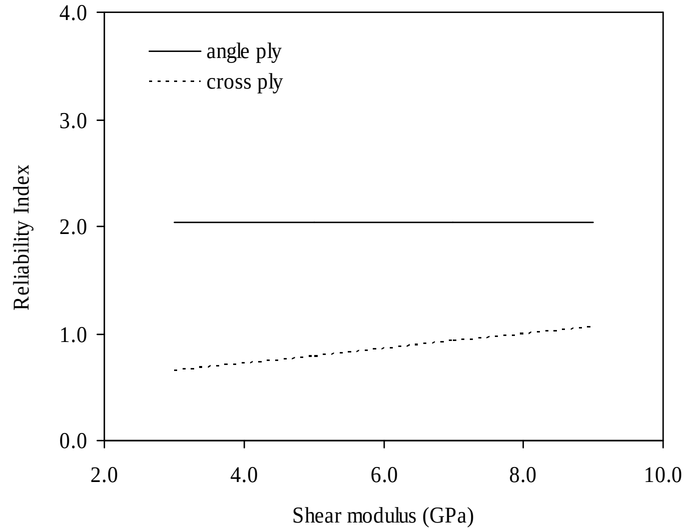


Figure 13: Effect of shear modulus on reliability index for graphite epoxy.

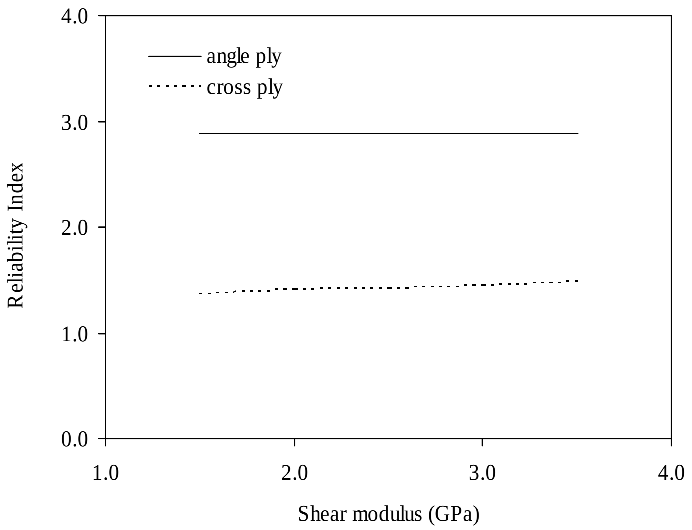


Figure 14: Effect of shear modulus on reliability index for carbon epoxy.

3.3.10 Effect of uncertainty

Figs. 15 to 17 show that as the uncertainty, measured in terms of coefficient of correlation (COV), increases there is a corresponding continuous decrease in the reliability index magnitude. This indicates that it is not only the mean value that affects the reliability of composite laminate but also the uncertainty plays a significant role in it.

An increase in the uncertainty in thickness results in significant decrease in the plate reliability (Fig. 15) while the plate reliability is not much affected with an increase in the uncertainty in length and width of the plate (Fig. 17). Fig. 16 shows the expected trend of a continuous decrease in the reliability index with increase in the uncertainty in longitudinal modulus.

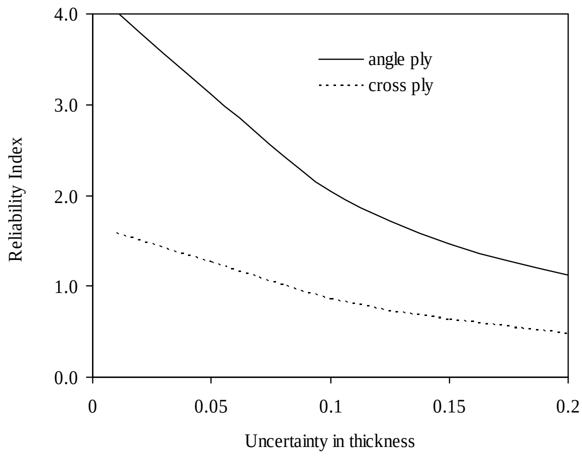


Figure 15: Effect of uncertainty in thickness on reliability index for graphite epoxy.

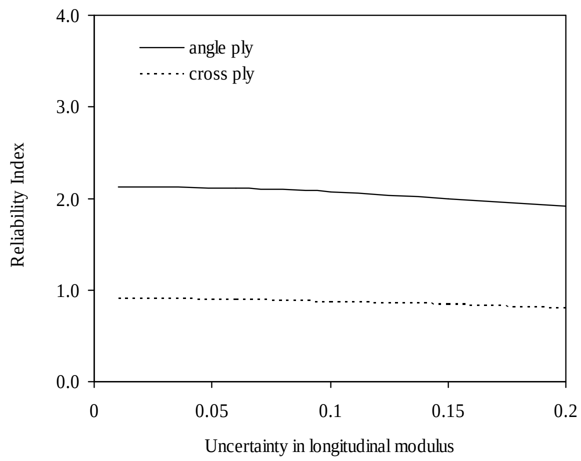


Figure 16: Effect of uncertainty in longitudinal modulus on reliability index for graphite epoxy.

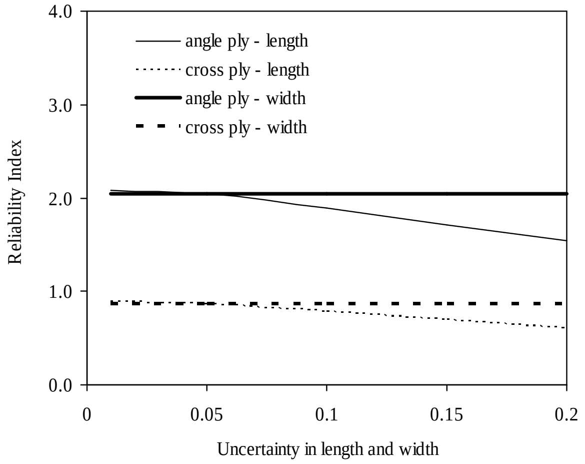


Figure 17: Effect of uncertainty in length and width on reliability index for graphite epoxy.

4 Conclusions

Based on the results obtained from the present numerical study, following conclusions may be drawn:

- Angle ply laminated plates, in general, are more reliable as compared to the same cross ply laminated composite plates.
- Correlation among various random variables may be neglected provided that the magnitude of correlation is of the same order as assumed in the present study.
- The number of layers in a composite laminated plate should be kept more than four for all design purposes. For desired reliability, preferable range for number of layers is four to eight.
- Fiber angle should be kept between $\theta = 30°$ to $\theta = 60°$ for antisymmetric angle ply laminated plates as at the extreme values of θ, reliability of composite plate decreases significantly.
- Thickness of laminated plate favors the reliability of composite plate very much.
- There is no significant change in the reliability of plate if its length is altered; however, there is a considerable change in the reliability against buckling if width of the laminated plate is changed.
- The longitudinal modulus of composites is more influencing to the plate reliability than other engineering properties of composite material.

- Effect of shear modulus is more pronounced for the cross ply laminated plates than for the angle ply laminated plates.
- It is not only the mean value that affects the reliability of plate but also the uncertainty plays a significant role. In general an increase in the uncertainty of any random variable, measured in terms of COV, reduces the reliability of laminated plates against buckling.
- If through some proper care and quality control uncertainties in random variables can be reduced, reliability of plates can be improved.

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Appendix I: Strain and stress variation in a laminate

Knowledge of the variation of stress and strain through the laminate thickness is essential to define the extensional and bending stiffness of a laminate. The laminate is presumed to consist of perfectly bonded laminae. Moreover, the bonds are presumed to be infinitesimally thin as well as non-shear deformable. That is, the displacements are continuous across lamina boundaries so that no lamina can slip relative to another. Thus, the laminate acts as a single layer with very special properties, but nevertheless acts as a single layer of material.

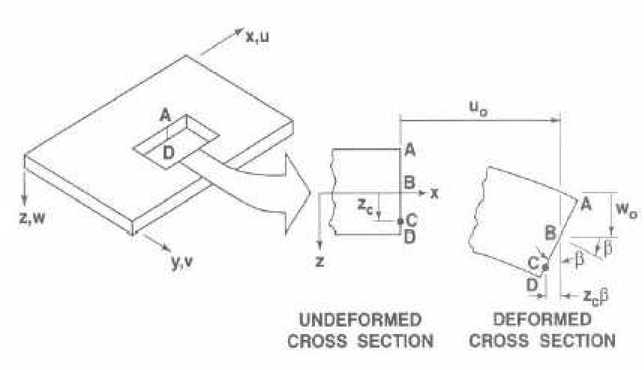

Figure A1: Deformation in xz-plane.

Figure A1 shows the deformation of the plate in the X-Z plane. Let u, v, w represent the displacements along the X-, Y-, Z-direction and u^0, v^0 the deformation of the laminate middle surface. The displacement along the X-axis of any point C at a distance z from the X-axis is

$$u = u^0 - \beta z, \tag{A1}$$

Where β is the slope of the middle surface in the X-direction, that is,

$$\beta = \frac{\partial w}{\partial x}, \tag{A2}$$

Hence the displacement u at any point z through the laminate thickness is

$$u = u^0 - z\frac{\partial w}{\partial x}, \tag{A3}$$

Similarly, we have the displacement v along the Y-axis given by

$$v = v^0 - z\frac{\partial w}{\partial y}, \tag{A4}$$

Using the strain-displacent relation

$$\varepsilon_x = \frac{\partial u}{\partial x},$$

$$\varepsilon_y = \frac{\partial v}{\partial y},$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \tag{A5}$$

we can write the strains in terms of middle surface displacements as

$$\varepsilon_x = \frac{\partial u^0}{\partial x} - z\frac{\partial^2 w}{\partial x^2},$$

$$\varepsilon_y = \frac{\partial v^0}{\partial y} - z\frac{\partial^2 w}{\partial y^2},$$

$$\gamma_{xy} = \frac{\partial u^0}{\partial y} + \frac{\partial v^0}{\partial x} - 2z\frac{\partial^2 w}{\partial x \partial y}, \tag{A6}$$

Equations (A6) can be rewritten as

$$\left\{ \begin{array}{c} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{array} \right\} = \left\{ \begin{array}{c} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{array} \right\} + z \left\{ \begin{array}{c} k_x \\ k_y \\ k_{xy} \end{array} \right\}, \tag{A7}$$

where

$$\left\{ \begin{array}{c} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{array} \right\} = \left\{ \begin{array}{c} \frac{\partial u^0}{\partial x} \\ \frac{\partial v^0}{\partial y} \\ \frac{\partial u^0}{\partial y} + \frac{\partial v^0}{\partial x} \end{array} \right\}, \tag{A8}$$

$$\left\{ \begin{array}{c} k_x \\ k_y \\ k_{xy} \end{array} \right\} = -\left\{ \begin{array}{c} \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial y^2} \\ 2\frac{\partial^2 w}{\partial x \partial y} \end{array} \right\}. \tag{A9}$$

Substituting equation (A7) in the stress-strain relations, we can express the stresses in the rth layer in terms of the laminate middle surface strains and curvature as

$$\left\{ \begin{array}{c} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{array} \right\}_r = \left[\begin{array}{ccc} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{array} \right]_r \left\{ \left\{ \begin{array}{c} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{array} \right\} + z \left\{ \begin{array}{c} k_x \\ k_y \\ k_{xy} \end{array} \right\} \right\}. \tag{A10}$$

Since the reduced stiffness matrix $\bar{Q}_{ij}$ is different for each lamina, the stress variation through the laminate thickness need not be linear, even though stress variation through the thickness is linear.

Appendix II: Buckling and limit state equations

Classical Laminate Theory, CLT [5] has been used to derive the governing buckling equations for a plate subjected to inplane load. To derive the governing equations we have considered first the equilibrium of force and then the equilibrium of moment in a way as discussed below:

The equilibrium equations in terms of the forces (Fig. A2) are

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0, \tag{A11}$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0. \tag{A12}$$

where N_x, N_y, and N_{xy} are the internal forces in normal and tangential direction.

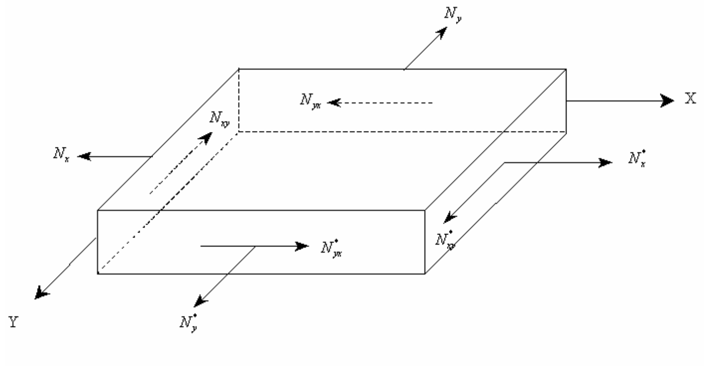

Figure A2: Forces on a laminated plate.

Again, the equilibrium equation in terms of the moments (Fig. A3) is

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} = 0, \tag{A13}$$

where, w is the displacement along z-direction.

The resultant forces $N_x, N_y N_{xy}$ and moments M_x, M_y, M_{xy} acting on a laminate are obtained by integration of the stress in each layer or lamina through the laminate thickness. Knowing the stress in terms of the displacement, we can obtain the stress resultants $N_x, N_y, N_{xy}, M_x, M_y$, and M_{xy}. The stress resultants are defined as

$$N_x = \int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_x dz, \quad N_y = \int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_y dz, \quad N_{xy} = \int_{-\frac{t}{2}}^{\frac{t}{2}} \tau_{xy} dz,$$

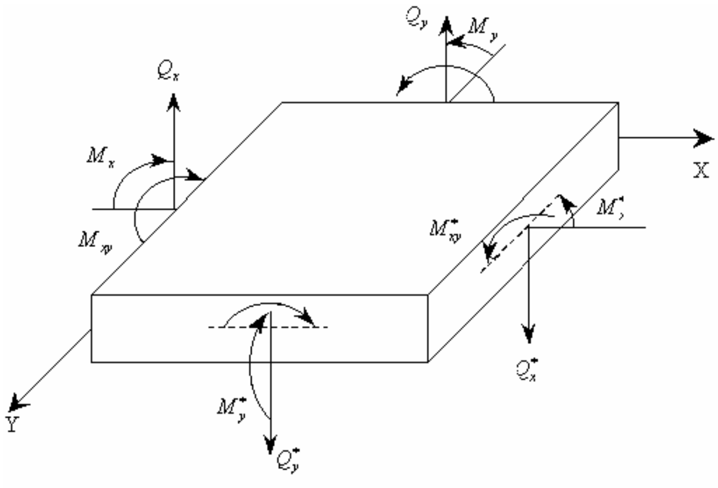

Figure A3: Shear forces and moments acting on plate element.

$$M_x = \int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_x z dz, \quad M_y = \int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_y z dz, \quad M_{xy} = \int_{-\frac{t}{2}}^{\frac{t}{2}} \tau_{xy} z dz. \tag{A14}$$

where σ_x, σ_y and τ_{xy} are normal and shear stress; t is total thickness of the laminated plate; and z is the distance measured from middle surface (Fig. A4).

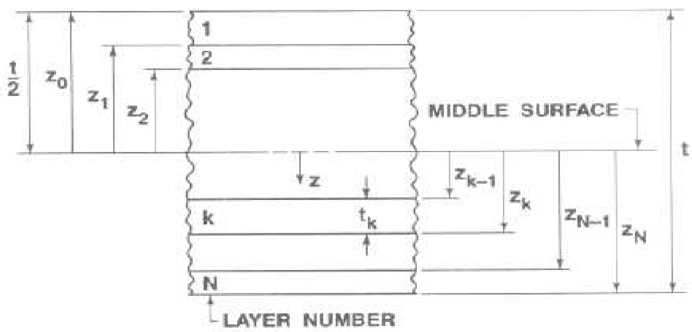

Figure A4: Geometry of an N-layered laminate.

Actually, N_x, N_y and N_{xy} are the force per unit length of the cross section of the laminate as shown in Fig. A2. Similarly, M_x, M_y, and M_{xy} are the moment per unit length as shown in Fig. A3. Thus, the forces and moments for an N-layer laminate can be defined as

$$\left\{ \begin{array}{c} N_x \\ N_y \\ N_{xy} \end{array} \right\} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \left\{ \begin{array}{c} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{array} \right\}_r dz = \sum_{r=1}^{N} \int_{z_{r-1}}^{z_r} \left\{ \begin{array}{c} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{array} \right\}_r dz, \tag{A15}$$

$$\left\{ \begin{array}{c} M_x \\ M_y \\ M_{xy} \end{array} \right\} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \left\{ \begin{array}{c} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{array} \right\}_r zdz = \sum_{r=1}^{N} \int_{z_{r-1}}^{z_r} \left\{ \begin{array}{c} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{array} \right\}_r zdz, \tag{A16}$$

where, z_r and z_{r-1} are as defined in Fig. A4. Note that $z_0 = -t/2$. Substituting for σ_x,σ_y, and τ_{xy} in Eqns. (A15) and (A16) and integrating over the thickness of each layer and adding the results so obtained for N layers, we can write the stress resultants as

$$\left\{ \begin{array}{c} N_x \\ N_y \\ N_{xy} \end{array} \right\} = \left[\begin{array}{ccc} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{array} \right] \left\{ \begin{array}{c} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{array} \right\} + \left[\begin{array}{ccc} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{array} \right] \left\{ \begin{array}{c} k_x \\ k_y \\ k_{xy} \end{array} \right\}, \tag{A17}$$

$$\left\{ \begin{array}{c} M_x \\ M_y \\ M_{xy} \end{array} \right\} = \left[\begin{array}{ccc} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{array} \right] \left\{ \begin{array}{c} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{array} \right\} + \left[\begin{array}{ccc} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{array} \right] \left\{ \begin{array}{c} k_x \\ k_y \\ k_{xy} \end{array} \right\}, \tag{A18}$$

where

$$A_{ij} = \sum_{r=1}^{N} \left(\bar{Q}_{ij}\right)_r (z_r - z_{r-1}),$$

$$B_{ij} = \frac{1}{2} \sum_{r=1}^{N} \left(\bar{Q}_{ij}\right)_r \left(z_r^2 - z_{r-1}^2\right),$$

$$D_{ij} = \frac{1}{3} \sum_{r=1}^{N} \left(\bar{Q}_{ij}\right)_r \left(z_r^3 - z_{r-1}^3\right). \tag{A19}$$

Here, A_{ij}, B_{ij} and D_{ij} are the coefficients of extensional stiffness, coupling stiffness, and flexural stiffness respectively; $\varepsilon_x^0, \varepsilon_y^0, \varepsilon_{xy}^0$ are strain components at middle surface; k_x, k_y, k_{xy} are the curvatures (for details refer appendix); and $\overline{Q_{ij}}$ (i, j = 1,2,6) are elements of the composite material reduced stiffness matrix that depend on the properties of elastic constants $E_1, E_2, G_{12}, \nu_{12}, \nu_{21}$ and thickness of the various laminas (layers) that have been stacked to form the laminate of thickness t. In the present study, we have considered regular laminate in which each lamina has the same thickness. Details about these elements may be found in any standard text on composite materials (e.g. [5]).

In the case of angle-ply laminates where the fibre orientation θ alternates from lamina to lamina as $+\theta/-\theta/+\theta/-\theta$, the force and moment resultants are

$$\left\{\begin{array}{c} N_x \\ N_y \\ N_{xy} \end{array}\right\} = \left[\begin{array}{ccc} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{array}\right] \left\{\begin{array}{c} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{array}\right\} + \left[\begin{array}{ccc} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{array}\right] \left\{\begin{array}{c} k_x \\ k_y \\ k_{xy} \end{array}\right\}, \tag{A20}$$

$$\left\{\begin{array}{c} M_x \\ M_y \\ M_{xy} \end{array}\right\} = \left[\begin{array}{ccc} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{array}\right] \left\{\begin{array}{c} \varepsilon_x^o \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{array}\right\} + \left[\begin{array}{ccc} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{array}\right] \left\{\begin{array}{c} k_x \\ k_y \\ k_{xy} \end{array}\right\}. \tag{A21}$$

Such a laminate is called an *antisymmetric angle-ply laminate.* In this type of laminate, if each lamina has the same thickness, it is then called a *regular antisymmetric angle-ply laminate.* For such a laminate, Eqns. (A20) and (A21) reduce to

$$\left\{\begin{array}{c} N_x \\ N_y \\ N_{xy} \end{array}\right\} = \left[\begin{array}{ccc} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{array}\right] \left\{\begin{array}{c} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{array}\right\} + \left[\begin{array}{ccc} 0 & 0 & B_{16} \\ 0 & 0 & B_{26} \\ B_{16} & B_{26} & 0 \end{array}\right] \left\{\begin{array}{c} k_x \\ k_y \\ k_{xy} \end{array}\right\}, \tag{A22}$$

$$\left\{\begin{array}{c} M_x \\ M_y \\ M_{xy} \end{array}\right\} = \left[\begin{array}{ccc} 0 & 0 & B_{16} \\ 0 & 0 & B_{26} \\ B_{16} & B_{26} & 0 \end{array}\right] \left\{\begin{array}{c} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{array}\right\} + \left[\begin{array}{ccc} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{array}\right] \left\{\begin{array}{c} k_x \\ k_y \\ k_{xy} \end{array}\right\}, \tag{A23}$$

There is yet another class of laminates. Here the laminae are oriented alternatively at 0^0 and 90^0. A laminate of this type is termed as a cross-ply laminate.

Substituting for N_x , N_y, N_{xy}, M_x, M_y, M_{xy} from Eqns. (A22) and (A23), after substituting for $\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0, k_x, k_y, k_{xy}$ from Eqns. (A8) and (A9), we get the governing equations as

$$\begin{array}{l} A_{11}\frac{\partial^2 u^0}{\partial x^2} + (A_{12}+A_{66})\frac{\partial^2 v^0}{\partial x \partial y} + A_{16}(\frac{\partial^2 v^0}{\partial x^2} + 2\frac{\partial^2 u^0}{\partial x \partial y}) + A_{26}\frac{\partial^2 v^0}{\partial y^2} + A_{66}\frac{\partial^2 u^0}{\partial y^2} \\ -B_{11}\frac{\partial^3 w}{\partial x^3} - 3B_{16}\frac{\partial^3 w}{\partial x^2 \partial y} - (B_{12}+2B_{66})\frac{\partial^3 w}{\partial x \partial y^2} - B_{26}\frac{\partial^3 w}{\partial y^3} = 0, \end{array} \tag{A24}$$

$$\begin{array}{l} A_{16}\frac{\partial^2 u^0}{\partial x^2} + (A_{12}+A_{66})\frac{\partial^2 u^0}{\partial x \partial y} + A_{26}\frac{\partial^2 u^0}{\partial y^2} + A_{66}\frac{\partial^2 v^0}{\partial x^2} + 2A_{26}\frac{\partial^2 v^0}{\partial x \partial y} + A_{22}\frac{\partial^2 v^0}{\partial y^2} \\ B_{16}\frac{\partial^3 w}{\partial x^3} - (B_{12}+2B_{66})\frac{\partial^3 w}{\partial x^2 \partial y} - 3B_{26}\frac{\partial^3 w}{\partial x \partial y^2} - B_{22}\frac{\partial^3 w}{\partial y^3} = 0, \end{array} \tag{A25}$$

$$\begin{array}{l} D_{11}\frac{\partial^4 w}{\partial x^4} + 4D_{16}\frac{\partial^4 w}{\partial x^3 \partial y} + (2D_{12}+4D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + 4D_{26}\frac{\partial^4 w}{\partial x \partial y^3} + D_{22}\frac{\partial^4 w}{\partial y^4} \\ -B_{11}\frac{\partial^3 u^0}{\partial x^3} - 3B_{16}\frac{\partial^3 u^0}{\partial x^2 \partial y} - (B_{12}+2B_{66})\frac{\partial^3 u^0}{\partial x \partial y^2} - B_{26}\frac{\partial^3 u^0}{\partial y^3} - B_{16}\frac{\partial^3 v^0}{\partial x^3} \\ -(B_{12}+2B_{66})\frac{\partial^3 v^0}{\partial x \partial y^2} - B_{22}\frac{\partial^3 v^0}{\partial y^3} = -N_x\frac{\partial^2 w}{\partial x^2} - N_y\frac{\partial^2 w}{\partial y^2} - 2N_{xy}\frac{\partial^2 w}{\partial x \partial y}. \end{array} \tag{A26}$$

For a general laminate, all the above three equations, i.e., Eqns. (A24), (A25) and (A26) have to be solved simultaneously as they are coupled. In the present study, we shall consider only simply supported regular antisymmetric laminated plates for reliability assessment. For such simply supported composite plates, these equations are simplified and their closed-form solutions can be obtained which can be subsequently employed to obtain limit state functions.

Antisymmetric cross ply laminated plate

Antisymmetric cross-ply laminates have extensional stiffnesses $A_{11}, A_{12}, A_{22} = A_{11}$ and A_{66}, bending-extension coupling stiffnesses B_{11} and $B_{22} = -B_{11}$, and bending stiffnesses $D_{11}, D_{12}, D_{22} = D_{11}$, and D_{66}. Because of this bending-extension coupling, the buckling differential equations are coupled:

$$A_{11}\frac{\partial^2 u^0}{\partial x^2} + (A_{12}+A_{66})\frac{\partial^2 v^0}{\partial x \partial y} + A_{66}\frac{\partial^2 u^0}{\partial y^2} - B_{11}\frac{\partial^3 w}{\partial x^3} = 0 \tag{A27}$$

$$(A_{12}+A_{66})\frac{\partial^2 u^0}{\partial x \partial y} + A_{66}\frac{\partial^2 v^0}{\partial y^2} + A_{11}\frac{\partial^2 v^0}{\partial y^2} + B_{11}\frac{\partial^3 w}{\partial y^3} = 0 \tag{A28}$$

$$D_{11}\left(\frac{\partial^4 w}{\partial x^4} + \frac{\partial^4 w}{\partial y^4}\right) + 2\left(D_{12}+2D_{66}\right)\frac{\partial^4 w}{\partial x^2 \partial y^2} - B_{11}\left(\frac{\partial^3 u^0}{\partial x^3} - \frac{\partial^3 v^0}{\partial y^3}\right) + N_x\frac{\partial^2 w}{\partial x^2} = 0 \tag{A29}$$

If a rectangular plate, simply supported on all the four edges, with dimensions a and b is subjected to axial compressive force along the edge of dimensions b (Fig. A1), we have

$$x = 0, a \quad w = 0, \quad M_x = B_{11}\frac{\partial u^0}{\partial x} - D_{11}\frac{\partial^2 w}{\partial x^2} - D_{12}\frac{\partial^2 w}{\partial y^2} = 0$$

$$u^0 = 0, \quad N_x = A_{11}\frac{\partial u^0}{\partial x} + A_{12}\frac{\partial v}{\partial y} - B_{11}\frac{\partial^2 w}{\partial x^2} = 0$$

$$y = 0, b \quad w = 0, \quad M_y = -B_{11}\frac{\partial v^0}{\partial y} - D_{12}\frac{\partial^2 w}{\partial x^2} - D_{11}\frac{\partial^2 w}{\partial y^2} = 0$$

$$v^0 = 0, \quad N_y = A_{12}\frac{\partial u^0}{\partial x} + A_{11}\frac{\partial v}{\partial y} + B_{11}\frac{\partial^2 w}{\partial y^2} = 0 \tag{A30}$$

Where u, v, w represent the displacements along the X-. Y-, Z- direction and u^0, v^0 the deformation of the laminate middle surface.

A solution of the type

$$\begin{aligned} u^0 &= u\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \\ v^0 &= v\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b} \\ w &= w\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b} \end{aligned} \tag{A31}$$

satisfies the above boundary conditions and the governing differential equations exactly if

$$N_x = \left(\frac{a}{m\pi}\right)^2 \left(T_{33} + \frac{2T_{12}T_{23}T_{13} - T_{22}T_{13}^2 - T_{11}T_{23}^2}{T_{11}T_{22} - T_{12}^2}\right) \tag{A32}$$

where,

$$T_{11} = A_{11}\left(\frac{m\pi}{a}\right)^2 + A_{66}\left(\frac{n\pi}{b}\right)^2$$

$$T_{12} = (A_{12} + A_{66})\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)$$

$$T_{13} = -B_{11}\left(\frac{m\pi}{a}\right)^3$$

$$T_{22} = A_{11}\left(\frac{n\pi}{b}\right)^2 + A_{66}\left(\frac{m\pi}{a}\right)^2$$

$$T_{23} = B_{11}\left(\frac{n\pi}{b}\right)^3$$

$$T_{33} = D_{11}\left(\left(\frac{m\pi}{a}\right)^4 + \left(\frac{n\pi}{b}\right)^4\right) + 2\left(D_{12} + 2D_{66}\right)\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 \tag{A33}$$

Here m and n are number of half sine waves in X- and Y- directions respectively. The lowest buckling load (critical buckling load) must be found by searching procedure [6, 7] involving integer values of m and n. To implement this procedure first for a given value of n (e.g. 1), m was incremented from 1 to 10 (maximum possible value of m) and at every value of m the N_x, given by Eqn. (A32), was computed. Thereafter, n was incremented to next higher integer value (e.g. 2) and again for this value of n, m was incremented from 1 to 10 and at every value of n and m N_x, was computed. This process was repeated until for all possible combinations of m and n the N_x is known. Having known the all N_x values, the minimum N_x is sorted. This minimum N_x represents the buckling load for a simply supported laminate subjected to inplane loading.

Eq. (A32) represents the characteristic equation of buckling load for a simply supported antisymmetric cross-ply laminate subjected to inplane loading. Therefore, if the applied inplane load is P_c, then the limit state function g($\underline{x}$) can be expressed by Eq. (1)

Antisymmetric angle ply laminated plate

Antisymmetric angle-ply laminates have extensional stiffnesses A_{11}, A_{12}, A_{22} and A_{66}, bending-extension coupling stiffnesses B_{16} and B_{26}, and bending stiffnesses D_{11}, D_{12}, D_{22}, and D_{66}. This type of laminate exhibits a different kind of bending-extension coupling than does the antisymmetric cross-ply laminate. The coupled buckling differential equations are

$$A_{11}\frac{\partial^2 u^0}{\partial x^2} + (A_{12} + A_{66})\frac{\partial^2 v^0}{\partial x \partial y} + A_{66}\frac{\partial^2 u^0}{\partial y^2} - 3B_{16}\frac{\partial^3 w}{\partial x^2 \partial y} - B_{26}\frac{\partial^3 w}{\partial y^3} = 0 \tag{A34}$$

$$(A_{12}+A_{66})\frac{\partial^2 u^0}{\partial x\partial y}+A_{66}\frac{\partial^2 v^0}{\partial x^2}+A_{22}\frac{\partial^2 v^0}{\partial y^2}-B_{16}\frac{\partial^3 w}{\partial x^3}-3B_{26}\frac{\partial^3 w}{\partial x\partial y^2}=0 \tag{A35}$$

$$\begin{aligned}&D_{11}\tfrac{\partial^4 w}{\partial x^4}+2\left(D_{12}+2D_{66}\right)\tfrac{\partial^4 w}{\partial x^2\partial y^2}+D_{22}\tfrac{\partial^4 w}{\partial y^4}\\&-B_{16}\left(3\tfrac{\partial^3 u^0}{\partial^2 x\partial y}+\tfrac{\partial^3 v^0}{\partial x^3}\right)-B_{16}\left(3\tfrac{\partial^3 v^0}{\partial x\partial y^2}+\tfrac{\partial^3 u^0}{\partial y^3}\right)+N_x\tfrac{\partial^2 w}{\partial x^2}=0\end{aligned} \tag{A36}$$

For simply supported edge boundary conditions, we have

$$\begin{aligned}x=0,a\quad w=0,\quad M_x&=B_{16}\left(\frac{\partial v^0}{\partial x}+\frac{\partial u^0}{\partial y}\right)-D_{11}\frac{\partial^2 w}{\partial x^2}-D_{12}\frac{\partial^2 w}{\partial y^2}=0\\
u^0=0,\quad N_{xy}&=A_{66}\left(\frac{\partial v^0}{\partial x}+\frac{\partial u^0}{\partial y}\right)-A_{16}\frac{\partial^2 w}{\partial x^2}-B_{26}\frac{\partial^2 w}{\partial y^2}=0\\
y=0,b\quad w=0,\quad M_y&=B_{26}\left(\frac{\partial v^0}{\partial x}+\frac{\partial u^0}{\partial y}\right)-D_{12}\frac{\partial^2 w}{\partial x^2}-D_{22}\frac{\partial^2 w}{\partial y^2}=0\\
v^0=0,\quad N_{xy}&=A_{66}\left(\frac{\partial v^0}{\partial x}+\frac{\partial u^0}{\partial y}\right)-B_{16}\frac{\partial^2 w}{\partial x^2}-B_{26}\frac{\partial^2 w}{\partial y^2}=0\end{aligned} \tag{A37}$$

A solution of the type

$$\begin{aligned}u^0&=u\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\\
v^0&=v\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\\
w&=w\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\end{aligned} \tag{A38}$$

satisfies the above boundary conditions and the governing differential equations exactly if

$$N_x=\left(\frac{a}{m\pi}\right)^2\left(T_{33}+\frac{2T_{12}T_{23}T_{13}-T_{22}T_{13}^2-T_{11}T_{23}^2}{T_{11}T_{22}-T_{12}^2}\right) \tag{A39}$$

where,

$$\begin{aligned}T_{11}&=A_{11}\left(\frac{m\pi}{a}\right)^2+A_{66}\left(\frac{n\pi}{b}\right)^2\\
T_{12}&=(A_{12}+A_{66})\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)\\
T_{13}&=-\left[3B_{16}\left(\frac{m\pi}{a}\right)^2+B_{26}\left(\frac{n\pi}{b}\right)^2\right]\left(\frac{n\pi}{b}\right)\\
T_{22}&=A_{22}\left(\frac{n\pi}{b}\right)^2+A_{66}\left(\frac{m\pi}{a}\right)^2\\
T_{23}&=-\left[B_{16}\left(\frac{m\pi}{a}\right)^2+3B_{26}\left(\frac{n\pi}{b}\right)^2\right]\left(\frac{m\pi}{a}\right)\end{aligned}$$

$$T_{33} = D_{11}\left(\frac{m\pi}{a}\right)^4 + 2\left(D_{12} + 2D_{66}\right)\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 + D_{22}\left(\frac{n\pi}{b}\right)^4 \qquad \text{(A40)}$$

Eq. (A39) represents the characteristic equation of buckling load for a simply supported antisymmetric angle-ply laminate subjected to inplane loading. Therefore, if the applied inplane load is P_c, then the limit state function g($\underline{\text{x}}$) can be expressed by Eq. (3).