Latin American Journal of Solids and Structures
www.lajss.org

Study of Squeeze Film Damping in a Micro-beam Resonator Based on Micro-polar Theory

Abstract

In this paper, squeeze film damping in a micro-beam resonator based on micro-polar theory has been investigated. The proposed model for this study consists of a clamped-clamped micro-beam bounded between two fixed layers. The gap between the micro-beam and layers is filled with air. As fluid behaves differently in micro scale than macro, the micro-scale fluid field in the gap has been modeled based on micro-polar theory. Equation of motion governing transverse deflection of the micro- beam based on modified couple stress theory and also non-linear Reynolds equation of the fluid field based on micropolar theory have been non-dimensionalized, linearized and solved simultaneously in order to calculate the quality factor of the resonator. The effect of micropolar parameters of air on the quality factor has been investigated. The quality factor of the of the micro-beam resonator for different values of non-dimensionalized length scale of the beam, squeeze number and also non-dimensionalized pressure has been calculated and compared to the obtained values of quality factor based on classical theory.

Keywords

MEMS, Micro-polar theory, Squeeze film damping

Mina Ghanbari [a]
Siamak Hossainpour [a*]
Ghader Rezazadeh [b]

[a] Mech. Eng. Dept., Sahand University of Technology, Tabriz, Iran
[b] Mech Eng. Dept., Faculty of Eng., Urmia University, Urmia, Iran
[*] Author's email: hossainpour@sut.ac.ir

http://dx.doi.org/10.1590/1679-78251364

Received 19.05.2014
In revised form 02.07.2014
Accepted 02.07.2014
Available online 13.10.2014

1 INTRODUCTION

Recently, progress in technology of micro-electromechanical systems (MEMS) can be seen in fabricating new devices and creating innovative applications. The fact that these systems can be produced at low cost and in large volumes, also in small size and with low-energy consumption, make them attractive and cause a great interest among scientists and engineers. Variety of MEMS devices such as accelerometers, micro-pumps, micro-sensors and micro-resonators, have been utilized in engineering, medical and science applications. But unfortunately because of complex design process of MEMS devices, they are still being designed applying trial and error method because of inherent couple-energy domains as electrostatic, thermal and mechanical forces involved in designe process. Most typical MEMS devices employ a capacitor that consists of two parallel plates in which one plate is actuated electrically and its motion is detected by capacitive changes. In order to increase the efficiency of actuation, the distance between parallel plates is minimized. Under these conditions the so-called squeeze film damping is pronounced (Nayfeh and Younis, 2004; Younis, 2004).

The squeeze film damping is the result of the movement of the fluid resisted by its viscosity that causes the fluid act as a spring and a damping force. The effect of squeeze film damping on the response of microstructures have been studied extensively in the past. The effect of damping on miniaturizing resonators was studied by Newell (1968). He divided the pressure range into three regimes. For the third regime where the pressure is high, he introduced an expression for damping based on the pressure drop in viscous flowing fluid. The quality factors of electrostatic clamped-clamped micro-beams encapsulated at low pressures were measured experimentally by Zook (1992). Experimental results were compared with both Christian's (1966) model in the molecular regime and Newell's model in the viscous regime. Legtenberg and Tilmas (1994) conducted experiments to measure the quality factors of clamed-clamped micro-beams encapsulated at very low pressures. Comparing the experimental results with those obtained by kinetic theory of gases, they found that the theory overestimates the measured quality factors by more than two orders of magnitude. Starr (1994) modeled a parallel-plate accelerometer using a linearized Reynolds equation for incompressible fluid. He calculated the pressure distribution under a plate with hole by using the finite element package ANSYS. The linearized compressible Reynolds equation was solved analytically by Blech (1983). He derived expressions for the spring and damping forces of the plates of rectangular and circular shapes. Energy dissipation was modeled using statistical thermodynamics by many researches. The Christian model was modified by Kadar (1996) and Li (1999). By comparing the theory and experimental results they found that the theory results are in good agreement with experimental ones. Bao (2002) studied the effect of a moving structure on changing the kinetic energy of the gas molecules using an energy-transfer model and derived an equation for the quality factor that was modified by a correction factor. Several models have been proposed by researchers for flexible microstructures. Yang et al. (1997) simulated the dynamic behavior of an electrostatic clamped-clamped micro-beam utilizing the finite-element package ABAQUS. A macro-model was developed by Hung and Senturia (1999) to simulate the dynamics of a clamped-clamped micro-beam by using the Galerkin method for discretizing the coupled differential equations. Damping characteristics for the first three flexural modes of vibration of the resonator were obtained by Pandey and Pratap (2007) in the case static deflection due to DC load was neglected. Younis and Nayfeh (2007) obtained bias deflection of the micro-plate under different ambient pressures by using perturbation method. Squeeze film characteristics of cantilever micro-resonators operating in different ambient pressure conditions for higher modes of vibration under large DC load were obtained by Chaterjee and Pohit (2009.; 2010). Khatami and Rezazadeh (2009) studied the dynamic response of actuators to electrostatic force and mechanical shock. They showed that the combined effect of a shock load and an electrostatic actuation makes the instability threshold much lower than the predicted threshold, considering the effect of shock force or electrostatic actuation alone.

Although several studies have been done on the dynamic behavior of the micro-structures under squeeze film damping but most of them have used the linearized Reynolds equation obtained by classical theories, for simulating the fluid field. Numerous experimental results indicate that, as fluid flow moves differently in the micro-scale than that in the macro scale, in the study of micro and nano-scale fluid mechanics, the Navier-Stokes equations derived from classical continuum, become incapable of explaining the micro scale fluid behaviour (Kucaba-Pietal, 2004).

A novel approach that was developed by Eringen (1966, 1972) includes the effect of local rotary inertia and couple stresses and offers mathematical foundation to capture the motions of the micro-scale fluids. Todays, researches show that applying micro-polar fluid theory in modeling the micro-scale fluid field can be a useful method for predicting the behaviour of the micro- scale flow [Srinivasacharya et al ,2001; Kucaba-Pietal, 2008; Deo and Shukla, 2012].

In this paper, free vibration of the micro-beam under the effect of squeeze film damping based on micro-polar theory is studied. The coupled governing equations of motion of the beam based on modified couple stress theory and pressure field of the fluid based on micro-polar theory are solved simultaneously using Galerkin based reduced order model. The effect of non-dimensionalized length scale and also coupling parameter of air on the of quality factor of the resonator is studied. Values of the guality factor of the micro-beam for different values of length to width ratio of the beam and also different values of aqueeze number amd non-dimensionalized pressure are determined and compared to those obtained based on classical theory.

2 COSSERAT (MICRO-POLAR) THEORY

The Cosserat theory of elasticity incorporates a local rotation of points as well as the translation assumed in classical elasticity; also it included a couple stress as well as the stress. Several authors developed the theory in the language of modern continuum mechanics as Mindlin and Tiersten (1962), Mindlin (1965), Eringen (1868) and Nowacki (1970). The Micro-inertia which was incorporated by Eringen and Cosserat elasticity was renamed micro-polar elasticity. Eringen (1968) introduced the concept of micro-polar fluids to characterize concentrated suspensions of neutrally buoyant rigid particles in a viscous fluid where individuality of substructures affects the physical outcome of the flow. Basically, these fluids support couple stresses and body couples furthermore, they exhibit micro-rotational and micro-inertial effects. It may be noted that micro-polar fluid theory takes care of the rotation of fluid particles by means of a kinematic vector called the micro-rotation vector which is independent from the vorticity of the fluid and is absent in classical continuum.

2.1 Kinematics

As shown in figure 1, an element ΔV is enclosed within its surface ΔS in the un-deformed body. Let the center of mass of ΔV has the position vector $\vec{X}$ .Suppose that the element ΔV contains N discrete micro-material elements $\Delta V\ (\alpha)$, $(\alpha = 1,2,\dots N)$. The position vector of the center of the mass of ΔV and displacement vector of a material point in the αth microelement in the deformed body may be expressed as:

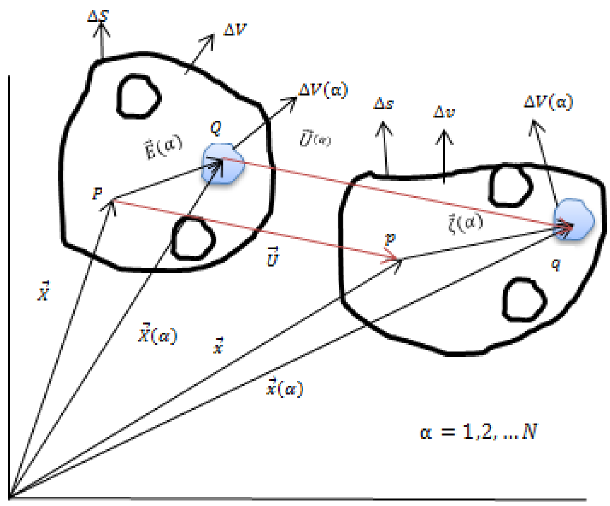

Figure 1: Deformation of a micro-volume.

$$\vec{x}_k = \vec{X}_k + \vec{U}_k \tag{1}$$

$$\vec{U}_K^{(\alpha)} = \vec{U}_K + \vec{E}_K^{(\alpha)}\phi_{kK}\ ; \qquad \phi_{KL} = -\phi_{LK} = -\varepsilon_{KLM}\phi_M \tag{2}$$

Where $\vec{E}^{(\alpha)}$is the position of a point in the microelement relative to the mass center of ΔV and ϕ_{kK} is defined as skew-symmetric micro-rotation tensor which is independent of macro-rotation in the micro-polar theory. By introducing C_{KL} as macro- deformation and $\Psi_{KL}$and Γ_{KML} as micro-deformation tensors, the macro and micro strain tensors are defined as:

$$C_{KL} \approx U_{L,K} + U_{K,L} + \delta_{KL}; \quad E_{KL} \equiv \frac{1}{2}(C_{KL} - \delta_{KL}) = \frac{1}{2}(U_{L,K} + U_{K,L}) \tag{3}$$

$$\Psi_{KL} \approx U_{L,K} + \delta_{KL} + \phi_{KL}; \quad £_{KL} \equiv \Psi_{KL} - \delta_{KL} = E_{KL} + \varepsilon_{KLM}(R_M - \phi_M) \tag{4}$$

$$\Gamma_{KML} \approx \phi_{KM,L}\ ; \qquad \Gamma_{KML} \equiv -\varepsilon_{KLN}\phi_{N,M} \tag{5}$$

Where E_{KL} is defined as the macro-strain tensor while $£_{KL}$ and Γ_{KML} are defined as micro-strain tensors, respectively.

2.2 Balance Equations

As shown in figure 2, it is assumed that the transfer of the interaction between two particles of the body through a surface element $n_l$ds occurs not only by means of a traction vector $t_l ds$ but also a moment vector $m_l$ds. Surface forces and couples are then represented by the generally skew-symmetric stress $\sigma_{lk}$and couple stress μ_{lk} tensors:

$$t_l = \sigma_{lk} n_k\ ; \qquad m_l = \mu_{lk} n_k \tag{6}$$

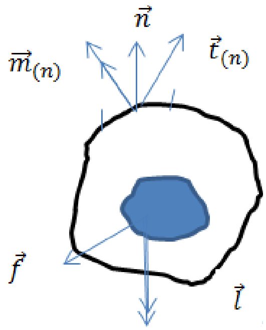

Figure 2: surface and body loads on the macro-volume.

So, the equations of the balance of momentum and balance of moment of momentum are as:

$$\sigma_{lk.k} + \rho f_l = \rho \ddot{u}_l \qquad l = 1,2,3 \tag{7}$$

$$\mu_{lk,k} + \rho l_l - \varepsilon_{lij} t_{ij} = I_f \dot{\phi}_l \qquad l = 1,2,3 \tag{8}$$

In which f_l and l_l are body forces and body couples, respectively, ρ is mass density, I_f is micro-inertia density, u_l are micro-displacements and ϕ_l are micro-rotations.

2.3 Constitutive Equations

The constitutive equations of the micro-polar media link the deformation and micro-rotations tensors to the force and couple stresses as:

$$t_{KL} = \lambda \delta_{KL} E_{KL} + (2\mu + k) E_{KL} + k \varepsilon_{KLM} (R_M - \phi_M) \tag{9}$$

$$\mu_{KL} = \alpha \delta_{KL} \frac{\partial \phi_M}{\partial X_M} + \beta \frac{\partial \phi_K}{\partial X_L} + \gamma \frac{\partial \phi_L}{\partial X_L} \tag{10}$$

There are 4 extra modulus in micro-polar theory α, β, γ and k .If these modulus are set equal to zero, the classic continuum media is obtained.

3 MODEL DESCRIPTION AND ASSUMPTIONS

As shown in figure 3, the proposed model consists of a clamped-clamped micro-beam with a rectangular cross section bounded between two fixed layers. The distance between the micro-beam and the parallel layers is filled with air. It is supposed that the beam width/fluid gap ratio is large. Also it is assumed that the deflection of the micro-beam is in the small deflection regime and the strain is negligible.

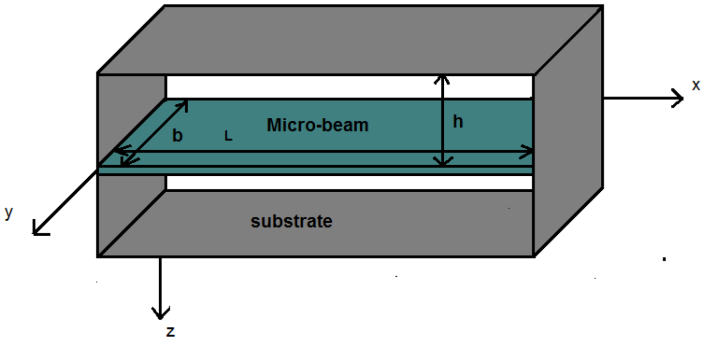

Figure 3: Shematic of proposed model for studying squeeze film damping.

By adding a term representing the force acting on the micro-beam owing to the pressure of the squeeze gas film, the equation of motion governing the transverse deflection of the beam is as follow (Li et al., 2007; Asghar et al., 2010; Arbind et al., 2014, Sedighi et al, 2014)

$$(\mathrm{EI})_{\mathrm{eq}} \frac{\partial^4 w(x,t)}{\partial x^4} + \rho \mathrm{b} \mathrm{h}_{\mathrm{b}} \frac{\partial^2 w(x,t)}{\partial t^2} + \left[\frac{\mathrm{Ebh_b}}{2\mathrm{L_b}} \int_0^{\mathrm{L_b}} \left(\frac{\partial w}{\partial x}\right)^2 dx\right] \times \frac{\partial^2 w}{\partial x^2} = -2\int_0^{\mathrm{b}} (p(x,y,t) - \mathrm{p_a})\, dy \tag{11}$$

Where $(EI)_{eq} = (EI + Gbh_b(lb)^2)$, $x\,,y$ are the position along the length and width of the micro-beam. $w(x,t)$ is the defletion of the micro-beam at the position x and the time t. L_b , b , h_b, ρ , G and lb are the length, width, thickness , density , shear modulus and the length scale parameter of the micro-beam material, respectively. $p(x,y,t)$ is the absolute pressure in the gap at the position x,y and and the time t and p_a is the ambient pressure. The boundary conditions of the micro-beam are::

$$w(0,t) = w(L_b,t) = 0 \qquad \frac{\partial w(0,t)}{\partial x} = \frac{\partial w(L_b,t)}{\partial x} = 0 \tag{12}$$

The pressure $p(x,y,t)$ is governed by the non linear Reynolds equation of the compressible micro-polar fluid as: (Naduvinamani and Mrali, 2007)

$$\frac{\partial}{\partial x}\left\{\left[(h_f^3 + 12l^2 h_f - 6Nlh_f^2 coth(\frac{Nh_f}{2l})\right] p\frac{\partial p}{\partial x}\right\} + \frac{\partial}{\partial y}\left\{\left[(h_f^3 + 12l^2 h_f - 6Nlh_f^2 coth(\frac{Nh_f}{2l})\right] p\frac{\partial p}{\partial y}\right\} = 12\eta_{eff}\left(p\frac{\partial h_f}{\partial t} + h_f\frac{\partial p}{\partial t}\right) \tag{13}$$

where h_f is the variable distance between the micro-beam and the parallel substrates and η_{eff} is the effective viscosity of the gas in the gap, which accounts for the rarefied gas effect through its dependence on the Knudsen number. Here we use the model of Veijola et al. (1995) for μ_{eff} . Coupling Parameter N, $0 \le N \le 1$ and material parameter l are micro-polar fluid properties that distinguish it from a Newtonian fluid. In the limiting case of $l \to 0$, Equation (13) reduces to the Reynolds equation of a Newtonian fluid. Boundary conditions of equation (13) are:

$$p(x,0,t) = p(x,b,t) = p_a \qquad \frac{\partial p}{\partial x}(0,y,t) = \frac{\partial p}{\partial x}(L_b,y,t) = 0 \tag{14}$$

By considering the following non-dimensional variables:

$$X = \frac{x}{L_b}\ ,\ Y = \frac{y}{b}\ ,\quad W = \frac{w}{g_0},\quad T = \frac{t}{s}\ ,\quad P = \frac{p}{p_a},\quad L = \frac{l}{g_0},\ \ H = \frac{h_f}{g_0} = 1 - W \tag{15}$$

where g_0 is the initial gap, $s = \sqrt{\frac{\rho b h_b L_b^4}{(EI)_{eq}}} = , \lambda = 4.73\ ,\ \omega_1 = \frac{\lambda^2}{L_b^2}\sqrt{\frac{(EI)_{eq}}{\rho b h_b}}$ is the first natural frequency of the clamped-clamped micro-beam, the Equations (11) and (13) are rewritten in the non-dimensional form as:

$$\frac{\partial^4 W}{\partial X^4} + \frac{\partial^2 W}{\partial T^2} + \alpha\left[\int_0^1 \left(\frac{\partial W}{\partial X}\right)^2 dX\right] \times \frac{\partial^2 W}{\partial X^2} = -2P_{non}\int_0^1 (P-1)dY \tag{16}$$

$$\frac{\partial}{\partial X}\left\{\left[(H^3 + 12L^2H - 6NLH^2 \coth\left(\frac{NH}{2L}\right))\right] P\frac{\partial P}{\partial X}\right\}$$

$$+\beta^2\frac{\partial}{\partial Y}\left\{\left[(H^3 + 12L^2H - 6NLH^2 \coth(\frac{NH}{2L})\right] P\frac{\partial P}{\partial Y}\right\} = \sigma\left(P\frac{\partial H}{\partial T} + H\frac{\partial P}{\partial T}\right) \tag{17}$$

where $\alpha = \frac{Ebhg_0^2}{2(EI)_{eq}}$ $p_{non} = \frac{bp_a {L_b}^4}{(EI)_{eq} g_0}$, $\beta = \frac{L_b}{b}$ is the aspect ratio of the cross section of the micro-beam and $\sigma = \frac{12\mu_{eff}L_b^2}{s g_0^2 p_a}$ is the squeeze number which represents a measurement of the compressibility of the fluid in the gap. For higher values of σ due to higher oscillation frequencies, the fluid is trapped in the gap and acts like a spring. For lower values of σ due to lower oscillation frequencies the fluid is nearly incompressible. The corresponding non-dimensional boundary conditions of Equations (16) and (17) are:

$$W(0,t) = W(1,t) = 0\ \ ,\qquad \frac{\partial W(0,t)}{\partial X} = \frac{\partial W(1,t)}{\partial X} = 0 \tag{18}$$

$$P(X,0,T) = P(X,1,T) = 1\ \ ,\qquad \frac{\partial P}{\partial X}(0,Y,T) = \ \frac{\partial P}{\partial X}(1,Y,T) = 0 \tag{19}$$

Due to non-linearity of Equation (17), solving this equation requires linearizing. As mentioned above, due to small oscillation of the micro-beam, the variation of the pressure from the ambient pressure in every point in the domain is also small. $P(X,Y,T)$ is given by :

$$P(X,Y,T) = \frac{p}{p_a} = 1 + P_d \tag{20}$$

Substituting Equation (20) into Equation (17) and linearizing it around p_a , the following equation is obtained:

$$\left\{\left[(12L^2 - 6NLcoth(\frac{N}{2L})\right]\frac{\partial^2 P_d}{\partial X^2} + \beta^2\left[(12L^2 - 6NLcoth(\frac{N}{2L})\right]\frac{\partial^2 P_d}{\partial Y^2}\right\} = \sigma\left\{\frac{\partial P_d}{\partial T} - \frac{\partial W}{\partial T}\right\} \tag{21}$$

With the following boundary conditions:

$$P_d(X,0,T) = P_d(X,1,T) = 0, \qquad \frac{\partial P_d}{\partial X}(0,Y,T) = \frac{\partial P_d}{\partial X}(1,Y,T) = 0 \tag{22}$$

It should be noted that, for vibration amplitude of approximate less than 0.5 μm , the values of non-linear term that are dropped in linearizing process of the Reynolds equation is less than 10 % of the linear terms. So we can claim that our linearizing process is valid for vibration amplitude of approximate less than 0.5 μm.

4 NUMERICAL SOLUTIONS

In the field of numerical analysis, Galerkin method is a means for converting a partial differential equation to a problem of linear or nonlinear system of ordinary differential equations, which may then be projected to a lower dimensional system. It relies on the weak formulation of an equation and works in principle by restricting the possible solutions as well as the test functions to a smaller space than the original one. These small systems are easier to solve than the original problem, but their solution is only an approximation to the original solution. In Galerkin method unknown function is expressed as a linear combination of a set of prescribed basis or shape functions. The overall quality of a Galerkin approximation depends on number and type of the shape functions. In this work a Galerkin based reduced order model are applied to solve the squeeze film damping coupled equations (16) and (21).

In this work, we seek approximate solutions for $W(X,T)$ and $P_d(X,Y,T)$ in the form of:

$$W(X,T) \cong \sum_{k=1}^{r} q_k(T)\psi_k(X) \tag{23}$$

$$P_d(X,Y,T) \cong \sum_{i=1}^{n}\sum_{j=1}^{m} u_{ij}(T)\varphi_i(X)\phi_j(Y) \tag{24}$$

where we approximated the deflection and pressure changes of the system with linear combinations of finite number of suitable shape functions.

Substituting Equations (23) and (24) into Equations (16) and (21) leads to the following equations:

$$\mathfrak{R}\big(W(X,T)\big) = \sum_{k=1}^{r} \ddot{q}_k(T)\psi_k(X) + \sum_{k=1}^{r} q_k(T)\psi_k^{\mathrm{IV}}(X) - \alpha\sum_{k=1}^{r}\psi_k^{\mathrm{II}}(X)q_k(T)\int_0^1 (\psi_k^{\mathrm{II}}(X))^2\, dX \;\;) \tag{25}$$

$$+2P_{non}\sum_{i=1}^{n}\sum_{j=1}^{m}u_{ij}(T)\varphi_i(X)\int_0^1\phi_j(Y)dY$$

$$\Im\big(P_d(X,Y,T)\big)=[(1+12L^2-6\,\mathrm{NLcoth}\left(\frac{N}{2L}\right))]\sum_{i=1}^{n}\sum_{j=1}^{m}u_{ij}(T)\varphi_i''(X)\phi_j(Y)$$

$$+\beta^2\left[(1+12L^2-6\,\mathrm{NLcoth}\left(\frac{N}{2L}\right))\right]\sum_{i=1}^{n}\sum_{j=1}^{m}u_{ij}(T)\varphi_i(X)\phi_j''(Y)-\sigma\sum_{i=1}^{n}\sum_{j=1}^{m}\dot{u_{ij}}(T)\varphi_i(X)\phi_j(Y)$$

$$+\sigma\sum_{k=1}^{r}\dot{q}_k(T)\psi_k(X) \tag{26}$$

By using the Galerkin method, following reduced order models can be obtained:

$$\sum_{k=1}^{r}M_{fk}\ddot{q_k}+\sum_{k=1}^{r}K_{fk}\,q_k+2\mathrm{D}_1\sum_{i=1}^{n}\sum_{j=1}^{m}E_{fi}^{(1)}E_j^{(2)}u_{ij}=0\quad \mathrm{f}=1,\dots\mathrm{r} \tag{27}$$

$$\mathrm{D}_2\sum_{i=1}^{n}\sum_{j=1}^{m}G_{qi}^{(1)}G_{gj}^{(2)}u_{ij}+\beta^2\,\mathrm{D}_2\sum_{i=1}^{n}\sum_{j=1}^{m}G_{qi}^{(3)}G_{gj}^{(4)}u_{ij}-\mathrm{D}_3\sum_{i=1}^{n}\sum_{j=1}^{m}C_{qi}^{(1)}C_{gj}^{(2)}\dot{u}_{ij}+\mathrm{D}_3\sum_{k=1}^{p}C_{qk}^{(3)}\dot{C_g^{(4)}}\dot{q}_k=0 \tag{28}$$

$$\mathrm{q}=1,\dots\mathrm{n}\,,\qquad \mathrm{g}=1,\dots\mathrm{m}$$

With the following coefficients:

$$M_{fk}=\int_0^1\psi_f(X)\psi_k(X)dX,\ K_{fk}=\int_0^1\psi_f(X)\psi_k^{IV}(X)dX-\alpha\int_0^1[\psi_f(X)\psi_k^{\mathrm{II}}(X)(\int_0^1(\psi_k^{\mathrm{II}}(X))^2\,dX]dX\,,$$

$$E_{fi}^{(1)}=\int_0^1\psi_f(X)\varphi_i(X)dX,\quad E_j^{(2)}=\int_0^1\phi_j(Y)dY,\ G_{qi}^{(1)}=\int_0^1\varphi_q(X)\varphi_i''(X)dX,$$

$$G_{gj}^{(2)}=\int_0^1\phi_g(Y)\phi_j(Y)dY, G_{qi}^{(3)}=\int_0^1\varphi_q(X)\varphi_i(X)dX,\ G_{gj}^{(4)}=\int_0^1\phi_g(Y)\phi_j''(Y)dY, \tag{29}$$

$$C_{qi}^{(1)}=\int_0^1\varphi_q(X)\varphi_i(X)dX\,,\ C_{qk}^{(3)}=\int_0^1\varphi_q(X)\psi_k(X)dX,\ C_g^{(4)}=\int_0^1\phi_g(Y)dY$$

$$\mathrm{D}_1=\mathrm{p}_{\mathrm{non}},\mathrm{D}_2=\left(1+12\mathrm{L}^2-6\,\mathrm{NLcoth}\left(\frac{\mathrm{N}}{2\mathrm{L}}\right)\right),\mathrm{D}_3=\sigma \tag{30}$$

Applying proper shape functions in Equations.(23) and (24) which satisfy the accompanying boundary conditions (18) and (22) and integrating the coupled Equations (27) and (28) simultaneously, in which $q_k=\bar{q}_k e^{\omega_k T}$ and $u_{ij}=\bar{u}_{ij}e^{\omega_{ij}T}$, the complex frequencies for the first mode vibration of the beam are achieved. Shape functions are considered as following:

$$\psi_k(X)=sin^2(k\pi X),\ \ \varphi_i(X)=sin^2(i\pi X),\ \ \phi_j(Y)=\sin(j\pi Y) \tag{31}$$

So, According to complex frequency approach, quality factor can be calculated as:

$$quality\ factor = \frac{1}{2}\left|\frac{Re(\omega)}{Im(\omega)}\right|$$

5 NUMERICAL RESULTS

The material properties of the micro-beam resonator studied in this paper are given in table 1

Parameters	Values
Density (kg/m^3)	2700
Young's modulus (GPa)	70
Shear modulus (GPa)	26
Length (μm)	300
Width (μm)	20
Thickness (μm)	2
Initial gap (μm)	1.5

Table 1: The material and geometrical properties of the micro-beam resonator.

For the micro-beam with mentioned properties, the numerically obtained values of quality factor exhibit good convergence when r value is taken as 6. The converged results for a cantilever micro-beam resonator based on classical theory are validated with the analytical, numerical and experimental results available in open literature. For design properties that are used for the experimental study by Pandey and Pratap (2007) the obtained results in this work are in good agreement with the analytical, numerical and experimental results. Table 2 shows comparison of damping ratio obtained by different methods.

Experimental (Pandey and Pratap 2007)	FE model (Pandey and Pratap 2007)	Analytical (Pandey and Pratap 2007)	FE model (Chaterjee and Pohit 2009)	Semi-analytical model (Chaterjee and Pohit 2010)	This work
0.415 ± 0.002	0.45	0.422	0.4475	0.4484	0.4122

Table 2: Comparison of damping ratio (ξ) obtained by different experimental and analytical methods.

In order to investigation the effect of the micro-polar parameters of air on complex frequencies of the resonator, the values of quality factor in the classic micro-beam resonator with the properties listed in Table.1, for different values of non-dimension length scale (L) and coupling parameter

(N) of air are obtained and shown in figure 4. Assuming the values of micro-polar parameters of water obtained experimentally in (Kucaba-Pietal, 2008), we can consider coupling parameter of air being in the range of $0 \leq N \leq 0.4$ Results show that the values of quality factor decreases by increasing the values of L and N.

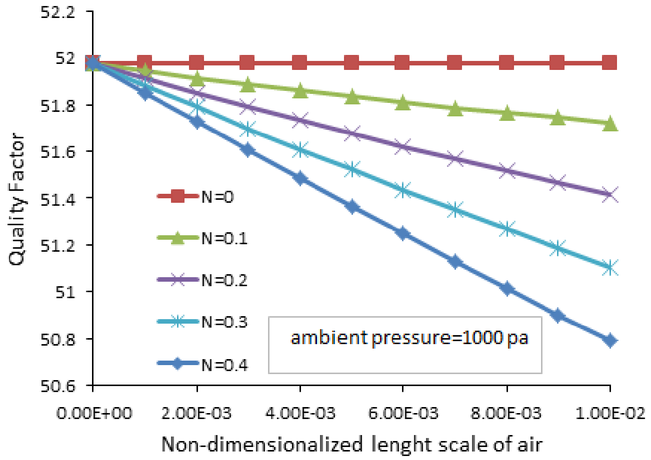

Figure 4: Calculated quality factor for different values of micro-polar parameters of air.

Then, the values of quality factor versus different values of non-dimension length scale of the micro-beam and coupling parameter of air for $p_a = 1000\,pa$ and $L = 0.01$ are obtained and shown in figure 5. Results show that higher values of non-dimensionalized length scale of the micro-beam and lower values of coupling parameter of air result in higher values of quality factor.

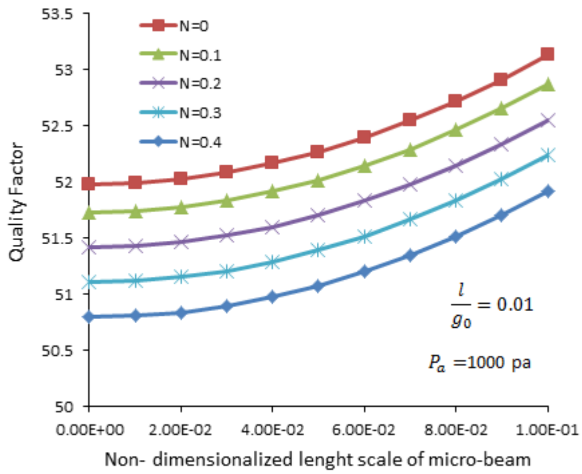

Figure 5: Calculated quality factors versus non-dimensional length scale parameter of the micro-beam for different values of coupling parameter of air.

Considering $N = 0.4$, determined quality factor of the resonator based on classic and micro-polar theory for different values of $\beta = \frac{L_b}{b}$ are shown in figure 6. Morever the results show that by increasing β, the difference between the obtained values of the quality factor increases. Results also show that the minimum difference is observed in the case of the classic micro-beam.

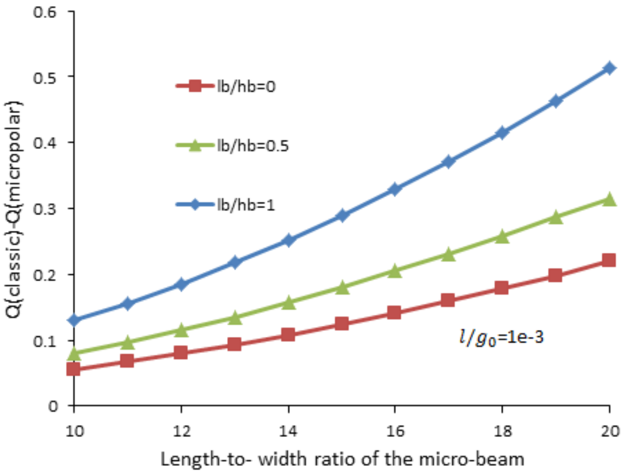

Figure 6: Difference between the obtained values of quality factor based on classic and micro-polar theory.

Quality factor of the micro-beam based on classic and modified couple stress theory versus different values of $\beta = \frac{L_b}{b}$ arecalculated and shown in Fig.7. Results show that higher values of β results in higher differences. Furthermore it can be observed that the maximum differences occur in the case of the classic fluid field.

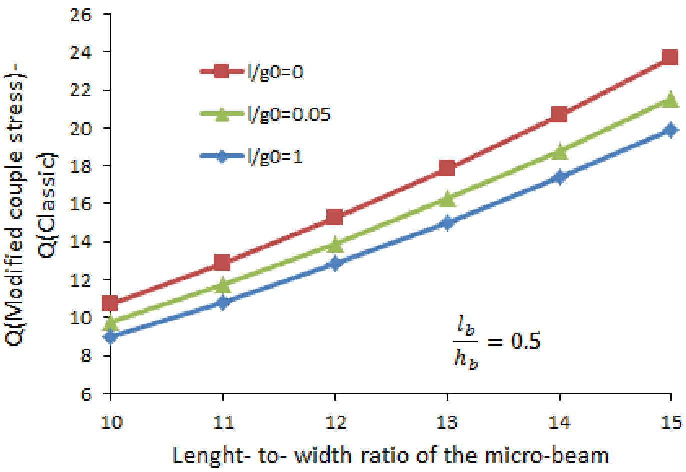

Figure 7: Difference between the obtained values of quality factor based on classic and modified couple stress theory

In order to investigation the effect of squeeze number and non-dimensionalized pressure on the obtained values of quality factor based on classic and micro-polar theory, the differences are calculated for variety of squeeze numbers and non-dimensionalized pressure and are shown in figure

8. It can be noticed that the maximum differences are observed in lower values of squeeze number and non-dimension pressure.

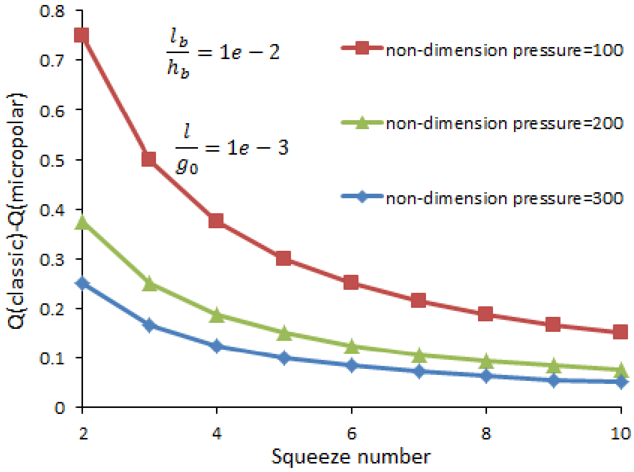

Figure 8: Difference between the obtained valuesof quality factor based on classic and micro-polar theory versus different values of squeeze number

4 CONCLUSIONS

In this paper, squeeze film damping in the micro-beam resonator was studied. Governing equations of motion for the micro-beam and fluid field respectively based on modified couple stress theory and micropolar theory were solved simultaneously applying Galerkin discretization and complex frequency approach. By considering coupling parameter of air being in the range of $0 \ll N \leq 0.4$. quality factor values were calculated for different values of non-dimensionalized micro-polar parameters of air. It was shown that increasing the values of non-dimensionalizes micro-polar parameters causes the quality factor of the resonator to decrease. The effect of non-dimensionalized length scale parameter of the micro-beam on the quality factor of the resonator was also studied. It was demonstrated that applying micro-polar theory underestimates the values of quality factor that are obtained based on classic theory while applying modified couple stress theory overestimates them. The effect of length-to-width ratio of the micro-beam on the difference obtained values of quality factor based on classic and micro-polar theory was studied. Results showed that the classic micro-beam with lower values of length-to-width ratio results in lower differences between classic and micro-polar theory results. The effect of length-to-width ratio of the micro-beam on the obtained values of quality factor based on classic and modified couple stress theory showed that higher values of length-to-width ratio of the micro-beam in the case of classic fluid field results in higher differences. Effect of squeeze number and non-dimensionalized pressure on the values of quality factor was studied and higher differences were observed for the lower values of squeeze number and non-dimensionalized pressure. The obtained results in this paper can be useful for MEMS community in designing MEMS resonators.

References

Nayfeh, H., Younis, M. I. (2004). A new approach to the modeling and simulation of flexible microstructures under the effect of squeeze-film damping, Journal of Micromechanics and Microengineering 14: 170–181.

Younis, M. I. (2004). Modeling and simulation of microelectromechanical systems in multi-physics field, Ph.D. Thesis, Virginia Polytechnic Institute and State University, United States.

Newell, W. E. (1968). Miniaturization of tuning forks, Science 161: 1320–1326.

Zook, J. D., Burns, D. W., Guckel, H., Sniegowski, J. J., Engelstad, R. L and Feng. Z. (1992). Characteristics of polysilicon resonant microbeams, Sensors and Actuators 35: 290–294.

Christian, R. G. (1966). The theory of oscillating-vane vacuum gauges, Vacuum 16: 175–178.

Legtenberg, R., Tilmans, H. A. (1994). Electrostatically driven vacuum-encapsulated polysilicon resonators. Part I. Design and fabrication, Sensors and Actuators 45: 57–66.

Starr, J. B. (1994). Squeeze-film damping in solid-state accelerometers, in Proceeding. IEEE Solid-State Sensors and Actuators Workshop: 44–47.

Blech, J. J. (1983). On isothermal squeeze films, Journal of Lubrication. Technology 105: pp. 615–620.

Kadar, Z., Kindt, W., Bossche, A., Mollinger, J. (1996). Quality factor of torsional resonators in the low-pressure region, Sens. Actuators 53: 299–303.

Li, B., Wu, H., Zhu, C., Liu, J. (1999). The theoretical analysis on damping characteristics of resonant microbeam in vacuum, Sensors and Actuators 77: 191–194.

Bao, M., Yang, H., Yin, H., Sun, Y. (2002). Energy transfer model for squeeze-film air damping in low vacuum, Journal of Micromechanics and Microengineering 12: 341–346.

Yang, Y. J., Gretillat, M. A., Senturia, S. D. (1997). Effect of air damping on the dynamics of nonuniform deformations of microstructures, in 1997 IEEE. International Conference of Solid State Sensors and Actuators 2:1093–1096.

Hung, E. S., Senturia, S. D. (1999). Generating efficient dynamical models for microelectromechanical systems from a few finite-element simulations runs, Journal of Microelectromechal Systems 8: 280–289. Pandey, A. K., and Pratap, R. (2007). Effect of flexural modes on squeeze film damping in MEMS cantilever resonators, Journal of Micromechanics and Microengineering 17: 2475-2484.

Younis, M. I., Nayfeh, A. H. (2007). Simulation of squeeze-film damping of microplates actuated by large electrostatic load, ASME. Journal of Computational and Nonlinear Dynamics 2: 101-112.

Chatrejee, S., Pohit, G. (2009). A large deflection model for the pull-in analysis of electrostatically actuated micro cantilever beams, Journal of Sound and Vibration 322: 969-986.

Chatrejee, S., Pohit, G. (2010). Squeeze- film characteristics of cantilever micro-resonators for higher modes of flexural vibration, International Journal of Engineering. Science and Technologhy 2: 187-199.

Khatami, F., Rezazadeh, G. (2009). Dynamic response of a torsional micromirror to electrostatic force and mechanical shock, Microsystem Technologies 15: 535–545.

Kucaba-Pietal, A., 2004, Microchannels flow modelling with the micropolar fluid theory, Bulletin of the Polish Academy of Science Technical science 52: 209-213

Eringen, A.C. (1966). Theory of micro-polar fluids, Journal of Applied Mathematics and Mechanics 16: 1-18.

Eringen, A. C. (1972). Theory of thermo micro-polar fluids, Journal of Mathematic Analyses and Applications 38: 480-496.

Srinivasacharya, D., Ramana Murthy, J. V., Venugopalam, D. (2001). Unsteady stokes flow of Micropolarfluid between two parallel porous plates, International Journal of Engineering Science 39: 1557–1563.

Kucaba-Pietal, A. (2008). Applicability of the micropolar fluid theory in solving microfluidics problems, in Proceeding 1st European Conference on Microfluidics, Bologna.

Deo, S., Shukla, P. (2012). Creeping flow of micro-polar fluid past a fluid sphere with non-zero spin boundary conditions, International Journal of Engineering Technologhy 2: 67-76.

Cosserat, E., Cosserat, F. (1909). Theory of deformable bodies, Hermann, Paris.

Mindlin, R. D., Tiersten, H. F. (1962). Effect of couple stresses in linear elasticity, Archive for Rational Mechanics and analysis 11: 415-448.

Mindlin, R. D. (1965). Stress functions for a cosserat continuum," International Journal of Solids and Structures 1: 265-271.

Eringen, A. C. (1968). Theory of micropolar elasticity," in Fracture 1: 621-729.

Li, P., Hu, R., Fang, Y. (2007). A new model for squeeze-film damping of electrically actuated microbeams under the effect of a static deflection, Journal of Micromechanics and Microengineering 17: 1242-1251.

Asghari, M., Rahaeifard, M., Kahrobaiyan, M.H., Ahmadian, M. T. (2010). The modified couple stress functionally graded Timoshenko beam formulation, Materials and Design 31: 2324-2329.

Arbind, A., Reddy, J. N and Srinivasa, A. R. (2014). Modified couple stress-based third-order theory for nonlinear analysis of functionally grades beams, Latin American Journal of Solids and Structures 11: 459-487.

Sedighi, H. M., Changizian, M and Noghrehabadi, A. (2014). Dynamic pull-in instability of geometrically nonlinear actuated micro-beams based on the modified couple stress theory, Latin American Journal of Solids and Structures 11: 810-825.

Naduvinamani, N.B., Mrali, G. B. (2007). Dynamic Reynolds equation for micropolar fluid lubrication of porous slider bearings, Journal of Marine Science and Technolghy 16: 182-190.

Veijola, T., Kuisma, T., Lahdenper¨a, T and Ryh¨anen, T. (1995). Equivalent-circuit model of the squeezed gas film in a silicon accelerometer, Journal of Sensors and Actuators 48: 239–48.