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Modeling of Cracked Beams by the Experimental Design Method

Abstract

The understanding of phenomena, no matter their nature is based on the experimental results found. In the most cases, this requires an important number of tests in order to put a reliable and useful observation served into solving the technical problems subsequently. This paper is based on independent and variables combination resulting from experimentation in a mathematical formulation. Indeed, mathematical modeling gives us the advantage to optimize and predict the right choices without passing each case by the experiment. In this work we plan to apply the experimental design method on the experimental results found by Deokar, A (2011), concerning the effect of the size and position of a crack on the measured frequency of a beam console, and validating the mathematical model to predict other frequencies

Keywords

Parameters, experimental design method, modeling, frequency, crack.

M. Serier [a]
N. Benamara [a]
A. Megueni [a]
K. Refassi [a]

[a] Mechanics of Structures and Solids Laboratory. Faculty of Technolgy, University of Sidi-Bel-Abbes, Bp 89, cité Ben M'hidi sidi- Bel-Abbes 22000-Algeria

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1 INTRODUCTION

Several scientific works has been conducted on the experimental design method each in its field of application and its objective, Bounazef M, Goupy J and Castro. L (2009, 1925, 2004). Other authors Aît Yala (2009) have implemented experimental design using numerical results. Experimental design were used for the first time by Fisher R (1925), in the field of agriculture where the experimental parameters are numerous and significant which leads to mathematical modeling and therefore to optimize the model sought. In mechanical terms, for example, Serier et al (2013a) used experimental design method for optimizing the machining parameters in order to increase the life of the tool. These same authors (2013b) performed a work in the field of vibration where they showed the importance of the geometrical properties, the speed of rotation of a shaft (tree) on the opening and closing mechanism of crack in a rotating machine. It is clear that the phenomenon of cracking beams has negative consequences on the large industrial constructions, Thomas, M (1995). The work of Deokar, A (2011) has focused on the experimental investigation for the detection of a crack on a cantilever used a criteria based on natural frequencies. Our work consisted on the use of experimental design method for modeling and prediction of frequencies.

2 EXPERIMENTAL RESULTS FOR EACH FREQUENCY MODE

In the tables below, the experimental results of the variation of frequency ratio as a function of position and depth of modes 1, 2 and 3 are presented. These results came from the work of Deokar (2011).

Physical values of parameters				Coded values of parameters				Experimental results
N	Crack depth	Crack location	Frequency ratio first mode	a_0	X_1	X_2	I_{12}	y
01	2	25	0,9887	1	-1	-1	1	0,9887
02	2	100	0,9956	1	-1	-0,14	0,14	0,9956
03	2	200	0,9996	1	-1	+1	-1	0,9996
04	6	25	0,91	1	-0,2	-1	0,2	0,91
05	6	100	0,9626	1	-0,2	-0,14	0,028	0,9626
06	6	200	0,9966	1	-0,2	+1	-0,2	0,9966
07	12	25	0,6465	1	+1	-1	-12	0,6465
08	12	100	0,8081	1	+1	-0,14	-1,68	0,8081
09	12	200	0,9766	1	+1	+1	12	0,9766

Table 1: Frequency ratio of the various tests of the first mode and experience matrix.

Physical values of parameters				Coded values of parameters				Experimental results
N	Crack depth	Crack location	Frequency ratio second mode	a_0	X_1	X_2	I_{12}	y
01	2	25	0,9951	1	-1	-1	1	0,9951
02	2	100	0,9975	1	-1	-0,14	0,14	0,9975
03	2	200	0,995	1	-1	+1	-1	0,995
04	6	25	0,9641	1	-0,2	-1	0,2	0,9641
05	6	100	0,9663	1	-0,2	-0,14	0,028	0,9663
06	6	200	0,9316	1	-0,2	+1	-0,2	0,9316
07	12	25	0,8903	1	+1	-1	-12	0,8903
08	12	100	0,9039	1	+1	-0,14	-1,68	0,9039
09	12	200	0,7836	1	+1	+1	12	0,7836

Table 2: Frequency ratio of the various tests of the second mode and experience matrix.

Physical values of parameters				Coded values of parameters				Experimental results
N	Crack depth	Crack location	Frequency ratio third mode	a_0	X_1	X_2	I_{12}	y
01	2	25	0,9986	1	-1	-1	1	0,9986
02	2	100	0,9946	1	-1	-0,14	0,14	0,9946
03	2	200	0,9927	1	-1	+1	-1	0,9927
04	6	25	0,9888	1	-0,2	-1	0,2	0,9888
05	6	100	0,9569	1	-0,2	-0,14	0,028	0,9569
06	6	200	0,9441	1	-0,2	+1	-0,2	0,9441
07	12	25	0,9574	1	+1	-1	-12	0,9574
08	12	100	0,8279	1	+1	-0,14	-1,68	0,8279
09	12	200	0,7995	1	+1	+1	1	0,7995

Table 3: Frequency ratio of the various tests of the third mode and experience matrix.

3 CALCULATIONS OF THE EFFECTS OF FACTORS

Each factor xi is affected (acted on) the behavior of the beam and it's defined by the effect of a_i. It is possible that the factors interact with each other, this is the case in our work, and therefore we are left with three factors, instead of two, including the average of these latter. In other words any response y_i depends on the action of all factors together xi. Analytically, and the relationship between the response factor can exist only when a certain proportionality exists between them. This leads us to write:

$$y_{(n1)} = x_{(np)} \times a_{(p1)} \tag{1}$$

The Solve of system of equations is based on the least squares method, and the solution is noted a. This solution is given by the following formula derived from the theory of matrix calculation.

$$(x^t\, x)^{-1}.x^t y = a \tag{2}$$

Therefore

Coefficients	Mode 1	Mode 2	Mode3
a_0	0,92	0,93	0,93
a_1	-0,090	-0,070	-0,070
a_2	0,076	-0,026	-0,036
I_{12}	0,081	-0,028	-0,037

Table 4: Coefficients that make up the mathematical model

The models are thus written in the following forms

Mode 1: $$y_1 = 0.92 - 0.090(X_1) + 0.076(X_2) + 0.081(X_1X_2)$$

Mode 2: $$y_2 = 0.93 - 0.070(X_1) - 0.026(X_2) - 0.028(X_1X_2)$$

Mode 3: $$y_3 = 0.93 - 0.070(X_1) - 0.036(X_2) - 0.037(X_1X_2)$$

4 ANALYSES WITH ONE VARIABLE FACTOR

4.1 Effect of Each Factor for the Three Modes

The figures below represent the effect of the two most principals factors (the crack depth and position of the crack) for each mode of vibration. We note that the ratio of the frequency is proportional to the crack depth; the slope is negative regardless of the active mode. This confirms that the propagation of a crack causes a decrease in the ratio of the frequency. In the case of the effect of crack position on the frequency, we observed a change of the upward frequency in the first mode. In other words, the more crack is remote from the recess the more important is the frequency, however in the case of other modes (2 and 3) an opposite change is noticed with slopes more or less important. This is due to the values of the amplitudes of modes 2 and 3.

Mode1

Mode2

Mode3

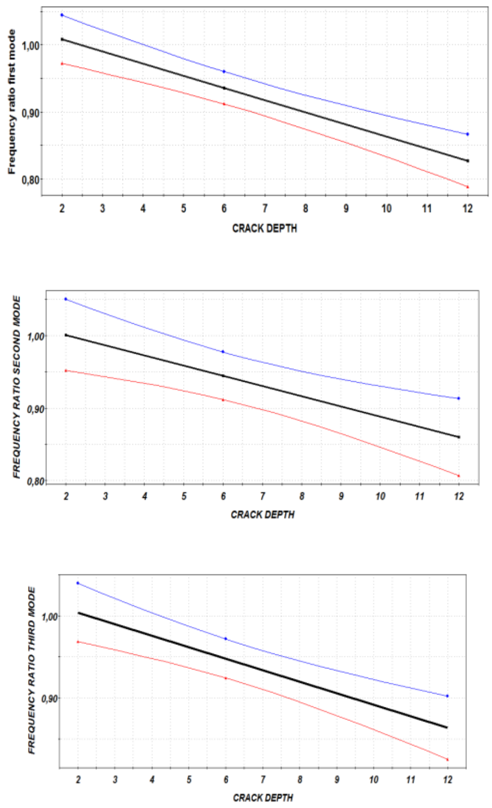

Figure 1: Effect of the depth of the crack on the frequency for the three modes.

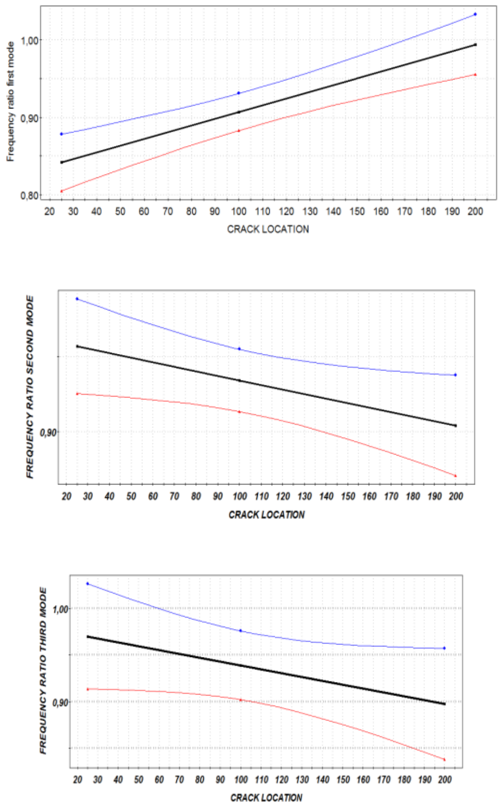

Figure 2: Effect of the position of the crack on the frequency for the three modes.

5 INTERACTION ANALYSE

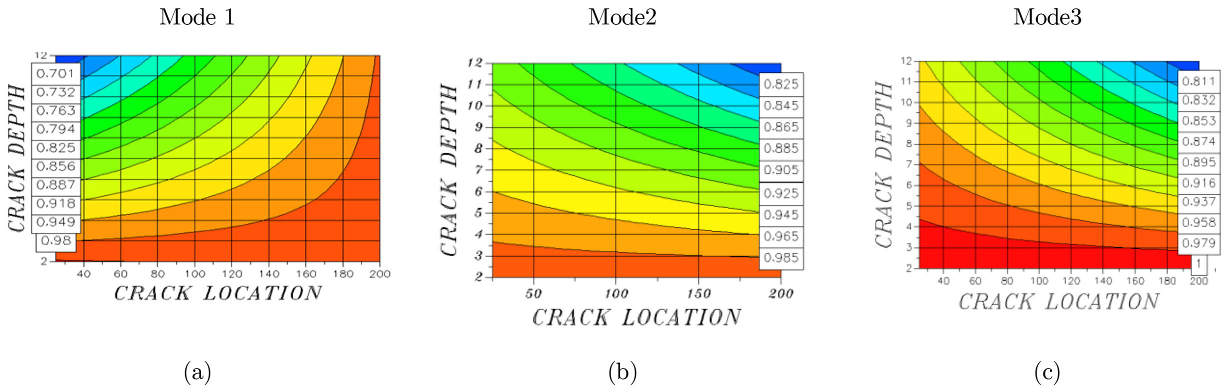

Figure 3: Effect of the interaction of two factors on the frequencies for the three modes.

The analyze of the interaction between the depth and crack location was done by the help of a representation of iso-courbes

Mode 1	Crack depth (2mm)	Crack depth(12mm)
Crack location (25mm)	0,98	0,701
Crack location (200mm)	0,99	0,94

Table 5: The results of combined effects (mode1).

The figure 3a represents the effects of the two factors versus the ratio of frequency in mode 1. We observed that there is just one near the abscissa of embedding and at low depths frequency ratio reached the value of 0.98. This ratio decreases to great depths. For abscissa near the edge of the beam, the depth of the crack has not effect. In the second and third mode analysis of iso-curves (fig.3b, 3c) shows that the ratio reaches its maximum when the two factors are the lowest values (tableaux 6 and 7)

Mode 2	Crack depth (2mm)	Crack depth(12mm)
Crack location (25mm)	0,99	0,78
Crack location (200mm)	0,98	0,90

Table 6: The results of combined effects (mode 2).

Mode 3	Crack depth (2mm)	Crack depth(12mm)
Crack location (25mm)	1,00	0,93
Crack location (200mm)	0,99	0,79

Table 7: The results of combined effects (mode 3).

6 ANALYZE WITH THREE FACTORS

The mathematic models already established, they allowed us to calculate the predicted frequencies and the residues for each mode (table 8, 9 and 10).

$y_1 = 0.92 - 0.090(X_1) + 0.076(X_2) + 0.081(X_1 X_2)$	Y_{pre}	Y_{obse}	Residus= $Y_{obse} - Y_{pre}$
$y_{11} = 0.92 - 0.090(-1) + 0.076(-1) + 0.081(1)$	1,02	0,9887	-0,0263
$y_{12} = 0.92 - 0.090(-1) + 0.076(-0.14) + 0.081(0.14)$	1,01	0,9956	-0,0151
$y_{13} = 0.92 - 0.090(-1) + 0.076(1) + 0.081(-1)$	1,01	0,9996	-0,0054
$y_{14} = 0.92 - 0.090(-0.2) + 0.076(1) + 0.081(-0.2)$	0,88	0,91	0,0318
$y_{15} = 0.92 - 0.090(-0.2) + 0.076(-0.14) + 0.081(0.028)$	0,93	0,9626	0,032972
$y_{16} = 0.92 - 0.090(-0.2) + 0.076(1) + 0.081(-0.2)$	1,00	0,9966	-0,0012
$y_{17} = 0.92 - 0.090(1) + 0.076(-1) + 0.081(-1)$	0,67	0,6465	-0,0265
$y_{18} = 0.92 - 0.090(1) + 0.076(-0.14) + 0.081(-0.14)$	0,80	0,8081	0,0081
$y_{19} = 0.92 - 0.090(1) + 0.076(1) + 0.081(1)$	0,99	0,9766	-0,0104

Table 8: Residues for the mode 1.

$y_2 = 0.93 - 0.070(X_1) - 0.026(X_2) - 0.028(X_1 X_2)$	Y_{pre}	Y_{obse}	Residus= $Y_{obse} - Y_{pre}$
$y_{21} = 0.93 - 0.070(-1) - 0.026(-1) - 0.028(1)$	1,00	0,9951	-0,0029
$y_{22} = 0.93 - 0.070(-1) - 0.026(-0.14) - 0.028(0.14)$	1,00	0,9975	-0,00222
$y_{23} = 0.93 - 0.070(-1) - 0.026(-1) - 0.028(-1)$	1,00	0,995	-0,007
$y_{24} = 0.93 - 0.070(-0.2) - 0.026(-1) - 0.028(0.2)$	0,96	0,9641	-0,0003
$y_{25} = 0.93 - 0.070(-0.2) - 0.026(-0.14) - 0.028(0.028)$	0,95	0,9663	0,019444
$y_{26} = 0.93 - 0.070(-0.2) - 0.026(1) - 0.028(-0.2)$	0,92	0,9316	0,008
$y_{27} = 0.93 - 0.070(1) - 0.026(-1) - 0.028(-1)$	0,91	0,8903	-0,0237
$y_{28} = 0.93 - 0.070(1) - 0.026(-0.14) - 0.028(-0.14)$	0,87	0,9039	0,03634
$y_{29} = 0.93 - 0.070(1) - 0.026(1) - 0.028(1)$	0,81	0,7836	-0,0224

Table 9: Residues for the mode 2.

$y_3 = 0.93 - 0.070(X_1) - 0.036(X_2) - 0.037(X_1X_2)$	Y_{pre}	Y_{obse}	Residus $= Y_{obse} - Y_{pre}$
$y_{31} = 0.93 - 0.070(-1) - 0.036(-1) - 0.037(1)$	1,00	0,9986	-0,0004
$y_{32} = 0.93 - 0.070(-1) - 0.036(-0.14) - 0.037(0.14)$	1,00	0,9946	-0,00526
$y_{33} = 0.93 - 0.070(-1) - 0.036(1) - 0.037(-1)$	1,00	0,9927	-0,0083
$y_{34} = 0.93 - 0.070(-0.2) - 0.036(-1) - 0.037(0.2)$	0,97	0,9888	0,0162
$y_{35} = 0.93 - 0.070(-0.2) - 0.036(-0.14) - 0.037(0.028)$	0,95	0,9569	0,008896
$y_{36} = 0.93 - 0.070(-0.2) - 0.036(1) - 0.037(-0.2)$	0,92	0,9441	0,0287
$y_{37} = 0.93 - 0.070(1) - 0.036(-1) - 0.037(-1)$	0,93	0,9574	0,0244
$y_{38} = 0.93 - 0.070(1) - 0.036(-0.14) - 0.037(-0.14)$	0,87	0,8279	-0,04232
$y_{39} = 0.93 - 0.070(1) - 0.036(1) - 0.037(1)$	0,79	0,7995	0,0125

Table 10: Residues for the mode 3.

With the help of the statistics calculations, we were able to define the effects the most significant of the factors and their gaps of confidence (trust), all with calculating the residues ei. The residues are the difference between the experimental value and predicted value by the mathematic model (figure4) and they are linked by the linear regression.

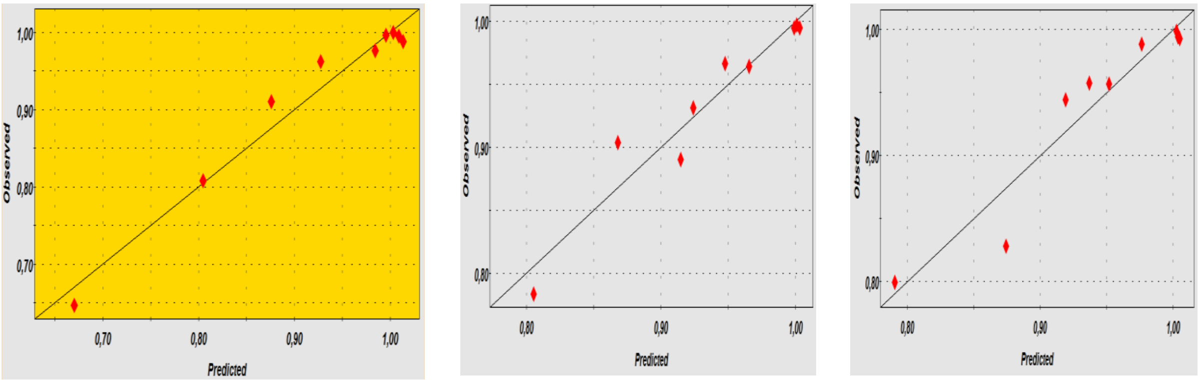

Figure 4: Distribution of the experimental points from the mathematical model for each mode.

7 REALIZATION OF THE TEST OF EFFECTS SIGNIFICANCE

The test used is the Student test << t>>. An effect is said significant (that is to say that the interaction or the variable associated with it has an influence on the response), if, for a given risk significantly different from 0. So we test the hypothesis

Hypothesis:

$$H_i = << a_i \neq 0 >>$$

For this, we calculate

$$t_i = \frac{|a_i|}{s_i} \tag{3}$$

Student table is then used to v= n – p degrees of freedom (n is the number of experiments and p is the number of effects including the constant). The risk of a first species is selected (usually 1% or 5%) and is read in the table of the Student t value, using the part of the table related to a bilateral test.

Mode 1:

$$s^2 = \frac{1}{n-p}\sum e_i^2 \tag{4}$$

$$s^2 = \frac{1}{9-4}\sum(-0.0263)^2 + (-0.0151)^2 + (-0.0054)^2 + (0.0318)^2 + (0.032972)^2 + (-0.0012)^2 + (-0.0265)^2 + (-0.0081)^2 + (-0.0104)^2$$

$$s^2 = \frac{1}{5}(0.0039)$$

$$s^2 = (0.00078)$$

The variance calculated for each effect is then:

$$s^2 = \frac{1}{9}(0.00078)$$

$$s^2 = 0.000086$$
$$s_i = 0.0093$$

$$\begin{cases} \alpha = 0.01(1\%) \\ \upsilon = 9-4=5 \end{cases} \leftrightarrow t_i = 4.032 \tag{5}$$

Therefore: $t_i \times s_i = 0.037$

The effect of crack depth	:	|-0.090|> 0.037	→	significatif
The effect of crack location		|+0.076|> 0.037	→	significatif
The effect of general average		|+0.920|> 0.037	→	significatif
The effect of the interaction	:	|-0.810|< 0.037	→	significatif

Confidence interval of model effects of the first mode

$$[a_i - t(\alpha, \nu)\, s_i \,;\, a_i + t(\alpha, \nu)\, s_i] \tag{6}$$

$a_i-(t(\alpha,\nu) * s_i)$		a_i	$a_i + (t(\alpha,\nu) * s_i)$
-0,127	Crack depth	-0,09	-0,053
0,039	Crack location	0,076	0,113
0,044	Interaction	0,081	0,118

Table 11: Confidence interval of model effects of the first mode.

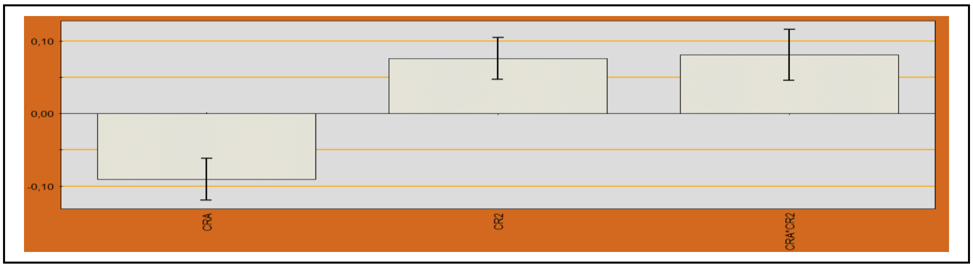

Figure 5: Confidence interval of model effects of the first mode.

Mode 2:

$$s^2 = \frac{1}{n-p}\sum e_i^2$$

$$s^2 = \frac{1}{9-4}\sum(-0.000008)^2 + (-0.000004)^2 + (-0.00004)^2 + (0.00000009)^2 + (0.0003)^2 + (-0.00006)^2 + (-0.0005)^2 + (-0.0013)^2 + (-0.0005)^2$$

$$s^2 = \frac{1}{5}(0.0028)$$

$$s^2 = (0.00057)$$

The variance calculated for each effect is then:

$$s^2 = \frac{1}{9}(0.00057)$$

$$s^2 = 0.000060$$

$$s_i = 0.0077$$

$$\begin{cases} \alpha = 0.01(1\%) \\ \upsilon = 9-4=5 \end{cases} \leftrightarrow t_i = 4.032$$

Therefore: $t_i \times s_i = 0.030$

Then:

The effect of crack depth $|+0.070| > 0.032$ $\longrightarrow$ significatif

The effect of crack location $|-0.026| < 0.032$ $\longrightarrow$ non significatif

The effect of general average $|+0.930| > 0.032$ $\longrightarrow$ significatif

The effect of the interaction $|-0.028| < 0.032$ $\longrightarrow$ non significatif

Confidence interval of model effects of the first mode

$$[a_i - t(\alpha, \nu)\, s_i \; ; \; a_i + t(\alpha, \nu)\, s_i]$$

$a_i-(t(\alpha,\nu) * s_i)$		a_i	$a_i + (t(\alpha,\nu) * s_i)$
-0,098	Crack depth	-0,07	-0,038
-0,05824	Crack location	-0,026	0,006
-0,06024	Interaction	-0,028	0,004

Table 12: Confidence interval of model effects of the second mode.

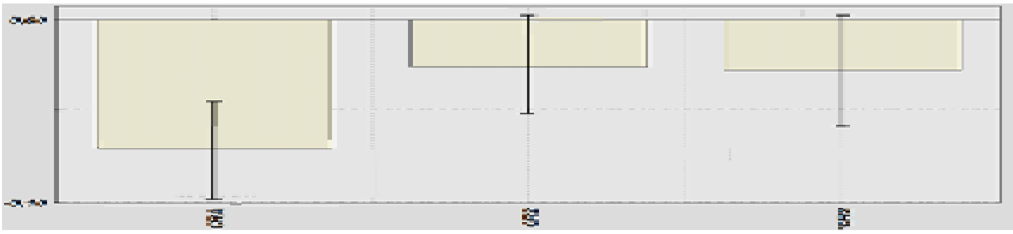

Figure 6: confidence interval of model effects of the second mode.

Mode 3:

$$s^2 = \frac{1}{n-p}\sum e_i^2$$

$$s^2 = \frac{1}{9-4}\sum(-0.0004)^2 + (-0.00002)^2 + (-0.000068)^2 + (0.00026)^2 + (0.00007)^2 + (-0.00082)^2 + (-0.00059)^2 + (-0.0017)^2 + (-0.00015)^2$$

$$s^2 = \frac{1}{5}(0.0038)$$

$$s^2 = (0.00076)$$

The variance calculated for each effect is then:

$$s^2 = \frac{1}{9}(0.00076)$$

$$s^2 = 0.000084$$

$$s_i = 0.0092$$

$$\begin{cases} \alpha = 0.01(1\%) \\ \upsilon = 9 - 4 = 5 \end{cases} \leftrightarrow t_i = 4.032$$

Therefore: $t_i \times s_i = 0.0370$

Then:

The effect of crack depth $|-0.070| > 0.037$ → significatif

The effect of crack location $|-0.036| < 0.037$ → non significatif

The effect of general average $|+0.930| > 0.037$ → significatif

The effect of the interaction $|-0.037| > 0.032$ → significatif

Confidence interval of model effects of the first mode

$$[a_i - t\ (\alpha,\nu)\ s_i\ ;\ a_i + t\ (\alpha,\nu)\ s_i]$$

a_i- $(t(\alpha,\nu)\ {*}s_i)$		a_i	$a_i + (t\ (\alpha,\nu)\ {*}s_i)$
-0,107	Crack depth	-0,07	-0,033
-0,073	Crack location	-0,036	0,001
-0,074	Crack depth	-0,037	0

Table 13: Confidence interval of model effects of the third mode.

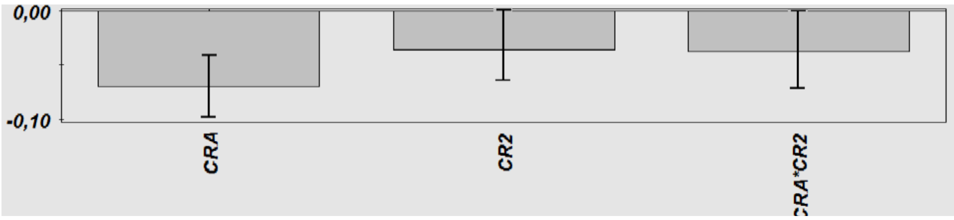

Figure 7: Confidence interval of model effects of the third mode.

8 CONCLUSIONS

Using the method of experimental design allows to extract the maximum information at a reasonable cost and minimum time. In our case, we have:

1) Modeled the vibration behavior of a cracked beam console
2) Plotted the iso-curves for different modes of vibration

These curves allow the choice according to the needs of the operating state (condition) of the beam.

It further notes that:

The depth of the crack remains significantly regardless of the current mode vibration, while this is not the case for the location of the crack which is significant only in the first mode.
The interaction between the two factors of an unexplained way varies from one mode to another; in the first mode of interaction is significant, it is only slightly in the third mode when it no longer is in the second mode

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