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Uncertainty analysis of flexible rotors considering fuzzy parameters and fuzzy-random parameters

Abstract

The components of flexible rotors are subjected to uncertainties. The main sources of uncertainties include the variation of mechanical properties. This contribution aims at analyzing the dynamics of flexible rotors under uncertain parameters modeled as fuzzy and fuzzy random variables. The uncertainty analysis encompasses the modeling of uncertain parameters and the numerical simulation of the corresponding flexible rotor model by using an approach based on fuzzy dynamic analysis. The numerical simulation is accomplished by mapping the fuzzy parameters of the deterministic flexible rotor model. Thereby, the flexible rotor is modeled by using both the Fuzzy Finite Element Method and the Fuzzy Stochastic Finite Element Method. Numerical simulations illustrate the methodology conveyed in terms of orbits and frequency response functions subject to uncertain parameters.

Keywords

Roto dynamics; uncertainty analysis; Fuzzy-random analysis.

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1 INTRODUCTION

Nowadays, the industrial applications require mechanical systems working with optimal performance subject to specific operational conditions, which demands: high reliability, robustness against environmental conditions and low operating requirements. Consequently, it is necessary to develop reliable numerical models that take into account uncertain parameters, and allow the prediction of the dynamic behavior of the system under realistic conditions.

The components of flexible rotors are realistically subjected to uncertainties. The main sources of uncertainties include the variation of mechanical properties, such as damping, stiffness, and geometrical imperfections (Lalanne and Ferraris, 1998). The dynamic behavior of rotors is then affected by the uncertain parameters of the system. Consequently, the quantification of parametric uncertainties is necessary to predict how the dynamic response varies, by means of numerical models of industrial applications and research testbeds (Chowdhury and Adhikari, 2012). Ritto et al. (2011) considered parametric uncertainties to develop a methodology to optimize the performance of the flexible rotor and, so, they demonstrated that parametric uncertainties in the dynamic model represent a complex problem of engineering design. As mentioned above, the components of flexible rotors that are typically subjected to uncertainty are the stiffness of the shaft (Young's Modulus), bearings (stiffness and damping coefficients) together with the dimensional tolerances of shafts and disks. Therefore, uncertain parameters should be taken into account in the numerical model in order to obtain reliable predictions in the computational simulations.

Uncertainty analysis of flexible rotors has been studied by applying stochastic approaches based on the *Stochastic Finite Elements Method* (Ghanem and Spanos, 1991). Didier et al. (2012) quantified the uncertainty effects in the response of flexible rotors based on the Polynomial Chaos Theory. Koroishi et al. (2012) represented the uncertainties in the rotor parameters by using Gaussian homogeneous stochastic fields discretized by Karhunen-Loève expansion; the dynamic response of the system with uncertainties was characterized through Latin Hypercube Sampling and Monte Carlo Simulation.

The fuzzy theory permits to model the uncertainty as an alternative to the stochastic methods. By means of fuzzy theory it is possible to describe incomplete and inaccurate information. The fuzzy sets theory was initially formulated by Zadeh (1965) to characterize the vague aspect of the information. Thereafter, Zadeh developed the theory of possibility based on fuzzy sets that can be compared to the theory of possibilities to deal with the uncertainty of information (Zadeh, 1978). The theory of fuzzy sets and the theory of possibilities are connected, so that it is possible to get along with the uncertainty and imprecision of the information sets by applying the fuzzy sets theory. Then, the uncertainties are modeled by means of fuzzy sets theory for the cases in which the stochastic process that describes the random variables is unknown. In this context, Moens and Hanss (2011) presented a literature review of the non-probabilistic approaches for the analysis of parametric uncertainty; they applied these methods to analyze structures modeled by the finite element method with uncertain parameters. The two principal approaches presented in this contribution to model the uncertainties are the interval analysis and the fuzzy approach. These two procedures require solving interval problems to compute the uncertain structural response. Several research works in this area attempt to implement alternative methods to simulate mechanical systems with fuzzy uncertain parameters (Lara-Molina et al., 2014b; Waltz and Hanss, 2013; Farkas et al., 2008; Klimke, 2006). Specifically, Lara-Molina et al. (2014a) represented the uncertainties in the rotor parameters by using fuzzy variables and performing a fuzzy dynamic analysis.

In agreement with the fuzzy approach, the present work proposes the application of a straightforward method to simulate the dynamic response of flexible rotors with uncertain parameters by performing a fuzzy dynamic analysis. For this purpose, the fuzzy uncertain parameters are mapped onto the model with the aid of α-level optimization Möller and Beer (2004).

The remainder of this paper has five sections. In section 2, the flexible rotor model based on the finite element method is shown. Section 3 presents the methodology to analyze the response of structures with fuzzy parameters and fuzzy random parameters by using the *Fuzzy Finite Element Method* and *Fuzzy Stochastic Finite Element Method*, respectively. Section 4 presents the numerical simulations of flexible rotor model in which the time and frequency responses were characterized by fuzzy and fuzzy random functions. Finally, the conclusions and the proposals of future work are outlined.

2 MODELING OF FLEXIBLE ROTORS

2.1 Theoretical background on the deterministic finite element modeling of a rotor system

The dynamic response of the considered mechanical system can be represented by using principles of variational mechanics, namely the Hamilton`s principle. For this aim, the strain energy of the shaft and the kinetic energies of the shaft and discs are calculated. An extension of Hamilton`s principle makes possible to include the effect of energy dissipation. The parameters of the bearings are included in the model by using the principle of the virtual work. For computational purposes, the finite element method is used to discretize the structure so that the energies calculated are concentrated at the nodal points. Shape functions are used to connect the nodal points (Morais et al., 2010). In this model, 4 degrees of freedom per node are taken into account, namely two displacements (u and w) and the cross-section rotations about axes x and z (denoted by $\theta = \partial w / \partial y$ and $\psi = \partial u / \partial y$, respectively). Fig. 1 shows the finite element used to model the rotor.

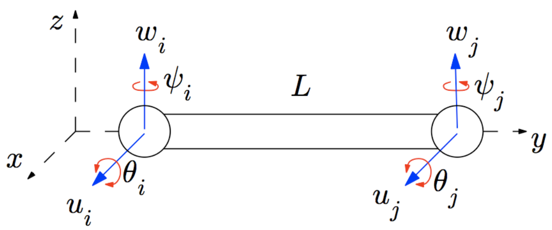


Figure 1: Illustration of the shaft finite element (Simões et al., 2007).

According to Lalanne and Ferraris (1998) the model depicted in Fig. 1 can be represented mathematically by the following set of differential equations:

$$\mathbf{M}\ddot{\mathbf{q}}(t) + (\mathbf{C} + \Omega_S \mathbf{G})\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{F}(t) \tag{1}$$

where $\mathbf{M} = (\mathbf{M}_S + \mathbf{M}_D) \in R^{N\times N}$ and $\mathbf{K} = (\mathbf{K}_S + \mathbf{K}_B) \in R^{N\times N}$ are, respectively, the mass and stiffness matrices, $\mathbf{C} = (\mathbf{C}_B + \mathbf{C}_P) \in R^{N\times N}$ is the damping matrix formed by the contributions of the viscous damping matrix, $\mathbf{C}_B$, and the inherent proportional damping matrix, $\mathbf{C}_P = \alpha_M \mathbf{M} + \beta_K \mathbf{K}$ and $\mathbf{G} = (\mathbf{G}_S + \mathbf{G}_D) \in R^{N\times N}$ designates the gyroscopic matrix formed by the gyroscopic contributions of the rigid discs and the shaft. $\mathbf{q}(t) \in R^N$ and $\mathbf{F}(t) \in R^N$ are, respectively, the vectors of the amplitudes of the harmonic generalized displacements and external loads, Ω_S is the angular speed of the shaft, and α_M and β_K represent, respectively, the proportional coefficients of mass and stiffness.

Lima et al. (2010) used the receptance or Frequency Response Function (FRF) matrix to analyze the deterministic system, whose components are complex functions of the excitation frequency that establish linear relations between the amplitudes of the harmonic responses and the amplitudes of the excitation forces and moments, as expressed by Eq. (2):

$$\mathbf{Q}(\omega,\Omega_S) = \hat{\mathbf{H}}(\omega,\Omega_S)^{-1}\mathbf{F}(\omega) = \left[-\omega^2\mathbf{M} + i\omega(\mathbf{C} + \Omega_s\mathbf{G}) + \mathbf{K}\right]^{-1}\mathbf{F}(\omega) \tag{2}$$

where $\mathbf{Q}(\omega,\Omega_S) \in R^N$ and $\mathbf{F}(\omega) \in R^N$ are, respectively, the vectors of the amplitudes of the harmonic generalized displacements and external loads.

2.2 Parameterization of the deterministic FE model

At this point it is important to consider that, in order to study the system behavior when uncertainties are to be taken into account, the uncertain responses have to be computed with respect to a set of uncertain geometrical and/or physical parameters associated with the flexible rotor. In general, such uncertain variables intervene in a rather complicated manner in the finite element matrices. Hence, for evaluating the variability of the responses associated with these uncertainties, it becomes interesting to perform a parameterization of the FE model, which is understood as a means of making the design parameters factored-out of the elementary matrices. At the expense of lengthy algebraic manipulations, this procedure makes it possible to introduce not only the uncertainties into the flexible rotor model, but also to perform a sensitivity analysis in a straightforward way, leading to significant cost savings in iterative robust optimization and/or model updating processes. After manipulations, those parameters of interest can be factored-out of the elementary matrices as indicated below:

$$\text{Shaft} \quad = \begin{cases} \mathbf{M}_S^{(e)} = \rho_S A_S \bar{\mathbf{M}}_S{}^{(e)} \\ \mathbf{K}_S^{(e)} = E_S I_S \bar{\mathbf{K}}_S^{(e)} \\ \mathbf{G}_S^{(e)} = \rho_S I_S \bar{\mathbf{G}}_S^{(e)} \end{cases} \tag{3}$$

$$\text{Bearing} \quad = \begin{cases} \mathbf{K}_B^{(e)} = k_{xx}\bar{\mathbf{K}}_B^{(e)} + k_{zz}\bar{\mathbf{K}}_B^{(e)} \\ \mathbf{C}_B^{(e)} = d_{xx}\bar{\mathbf{C}}_B^{(e)} + d_{zz}\bar{\mathbf{C}}_B^{(e)} \end{cases} \tag{4}$$

where ρ_S , A_S , I_S and E_S represent, respectively, the mass density, the cross-section area, the inertia and the Young's modulus of the shaft. d_{xx} , d_{zz} and k_{xx} , k_{zz} designate, respectively, the damping and stiffness coefficients of the bearings. It is worth mentioning that the matrices appearing in the right hand side of Eqs. (3) and (4) are those from which the design parameters of interest have been factored-out.

3 FUZZY AND FUZZY STOCHASTIC ANALYSIS

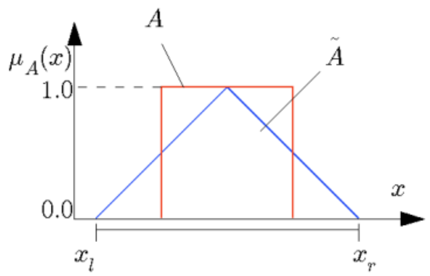
In several cases, some parameters of the systems cannot be accurately estimated due to small variations around its nominal value. In these cases, these parameters can be modeled by means of fuzzy variables. As mentioned above, the fuzzy set theory was initially formulated by Zadeh (1965) to represent vague or ambiguous information. Thereby, it is possible to represent inaccurate or uncertain parameters by using fuzzy variables, especially when the stochastic process which models the uncertain parameter cannot be determined by a simple observation.

In this section a brief introduction to two different numerical methods is presented, with the purpose of quantifying the uncertain structural response. First, the fuzzy analysis computes the uncertain structural response when the uncertain parameters are modeled as fuzzy variables that represent the uncertain parameters. The methodology to analyze the fuzzy uncertainties is based on $\alpha-$ level optimization, previously presented by Möller et al. (2000); thus, an optimization problem is solved to obtain the fuzzy output of the system by means of $\alpha-$level optimization.

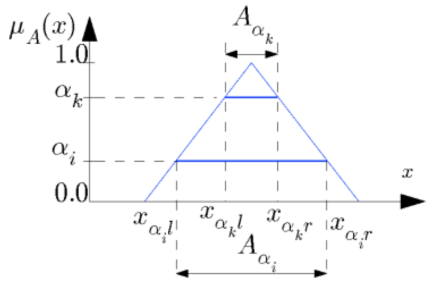
In the other hand, the fuzzy stochastic analysis computes the uncertain structural response of the system when the parameters are modeled such as fuzzy-random variables (Möller et al., 2009). These types of variables quantify uncertainties produced by variable and random sources. The fuzzy stochastic analysis algorithm encompasses a fuzzy analysis and a stochastic analysis to cope with the fuzziness and randomness characteristics of the uncertainties. It is worth mentioning that both aforementioned analyses are relying on the deterministic model of the system.

3.1 Fuzzy Variables

Let $\mathbf{X}$ be an universal set of objects whose generic elements are denoted by x. The subset A (where, $A \in \mathbf{X}$) is defined by the classical membership function $\mu_A : \mathbf{X} \rightarrow \{0,1\}$ (see Fig. 2(a)). Furthermore, a fuzzy set $\tilde{A}$ is defined by means of the membership function $\mu_A : \mathbf{X} \rightarrow [0,1]$, where $[0,1]$ is a continuous interval. The membership function indicates the degree of compatibility of the element x to $\tilde{A}$. The closer the value of $\mu_A(x)$ to "1", the more x belongs to $\tilde{A}$.



(a) Fuzzy set



(b) $\alpha-$levels

Figure 2: Fuzzy set and $\alpha-$ level representation.

For numerical purposes, Möller and Beer (2004) represented the fuzzy variables as intervals weighted by a membership function using the $\alpha-$level representation. Thus, the fuzzy set is completely defined by:

$$\tilde{A} = \left\{ \left(x, \mu_A(x) \right) \middle| x \in \mathbf{X} \right\}, \text{where} \quad 0 \leq \mu_A \leq 1 \tag{5}$$

For computational purposes, the fuzzy set $\tilde{A}$ can be represented by means of subsets that are denominated $\alpha-$levels. These subsets, which correspond to real and continuous intervals, are defined by A_{α_K} (see Fig. 2(b)), thus:

$$A_{\alpha_K} = \left\{ x \in \mathbf{X}, \mu_A(x) \geq \alpha_K \right\} \tag{6}$$

The $\alpha-$level subsets of $\tilde{A}$ have the property:

$$A_{\alpha_K} \subseteq A_{\alpha i} \forall \alpha_i, \alpha_K \in (0,1] \qquad \text{whit} \quad \alpha_i \leq \alpha_k \tag{7}$$

If the fuzzy set is convex (in the unidimensional case), each $\alpha-$level subset A_{α_K} corresponds to the interval $\left[x_{\alpha kl}, x_{\alpha kr} \right]$ where:

$$\begin{aligned} x_{\alpha_K l} &= \min\left[x \in \mathbf{X}, \mu_A(x) \geq \alpha_K \right] \\ x_{\alpha_K r} &= \max\left[x \in \mathbf{X}, \mu_A(x) \geq \alpha_K \right] \end{aligned} \tag{8}$$

3.2 Fuzzy Random Variables

The fuzzy random variables were defined previously by (Kwakernaak, 1978; Puri and Ralescu, 1986). According to the probability theory, the space of random elementary events Ω is presented. A fuzzy realization of the form $\underline{\tilde{x}}(\omega) = \tilde{x} \subseteq \underline{\mathbf{X}}$ is assigned to each elementary event $\omega \in \Omega$. Accordingly, a fuzzy random variable $\underline{\tilde{X}}$ is the fuzzy result of the uncertain mapping

$$\underline{\tilde{X}} : \Omega \tilde{\rightarrow} \mathbf{F}(\underline{\mathbf{X}}) \tag{9}$$

where $\mathbf{F}(\mathbf{X})$ is the set of all fuzzy numbers on $\mathbb{R}^n$. Each real random variable $\underline{X}$ on $\underline{\mathbf{X}}$ is contained in $\underline{\tilde{X}}$.

Based on this definition, a fuzzy random variable $\underline{\tilde{X}}$ can be described by a fuzzy cumulative distribution function $\tilde{\mathrm{F}}\left(\underline{x}\right)$. Considering an original value $\underline{x}_j$, the fuzzy probability distribution function $\tilde{\mathrm{F}}\left(\underline{x}_j\right)$ can be represented as the set of probability distribution functions with membership value $\mu\left(F\left(\underline{x}_j\right)\right) = \alpha$ (see Fig. 3). $\tilde{\mathrm{F}}\left(\underline{x}_j\right)$ is associated to the fuzzy bunch parameter $\underline{\tilde{s}}$ in order to parameterize the fuzziness of $\tilde{\mathrm{F}}\left(\underline{x}\right)$ (Möller and Beer 2005), thus:

$$\tilde{F}\left(\underline{x}\right) = F\left(\underline{\tilde{s}}, \underline{x}\right) \tag{10}$$

Thereby, for numerical computation, the $\alpha-$discretization is properly used.

$$F\left(\underline{\tilde{s}}, \underline{x}\right) = \left\{ \left(F_\alpha(\underline{x}), \mu(F_\alpha(\underline{x})) \right) \middle| F_\alpha(x) = \left[F_{\min,\alpha}(\underline{x}); F_{\max,\alpha}(\underline{x}) \right]; \mu\left(F_\alpha(\underline{x})\right) = \alpha \forall \alpha \in (0,1] \right\} \tag{11}$$

with

$$\begin{aligned} F_{\min,\alpha}(\underline{x}) &= \min\left[F(\underline{s}, \underline{x}) \middle| \underline{s} \in \underline{S}_\alpha \right] \\ F_{\max,\alpha}(\underline{x}) &= \max\left[F(\underline{s}, \underline{x}) \middle| \underline{s} \in \underline{S}_\alpha \right] \end{aligned}$$

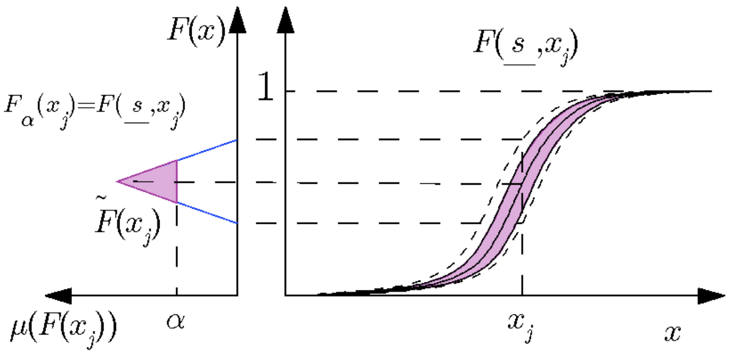


Figure 3: $\alpha-$ discretization of $F\left(\underline{\tilde{s}},\underline{x}\right)$, adapted from (Möller and Beer, 2004).

For example, the fuzzy bunch parameters $\tilde{s}_1$ and $\tilde{s}_2$ can be used to represent the fuzzy mean and fuzzy standard deviation of a fuzzy normal probability distribution as in eq. (12)

$$F\left(\underline{\tilde{s}},\underline{x}\right)=\frac{1}{2}\left[1+erf\left(\frac{x-\tilde{s}_1}{\tilde{s}_2\sqrt{2}}\right)\right] \tag{12}$$

3.3 Dynamic Models with Fuzzy Parameters

In the context of this work, the model describes the dynamic behavior of the rotor by means of a set of differential equations. The relationship between the inputs $\underline{x}$ and outputs $\underline{z}$ of an specific dynamic model M is characterized by f, which represents the set of differential equations of the model in the Eq. (13).

$$M:\underline{z}(\tau)=f(\underline{x}) \tag{13}$$

Therefore, the function f maps the inputs $\underline{x}$ onto the outputs $\underline{z}(\tau)$, thus $\underline{x}\rightarrow\underline{z}(\tau)$, where τ is the independent variable of the dynamic response that may represent time, frequency or spacial coordinates.

When considering the inputs of the model as fuzzy variables $\underline{\tilde{x}}$ or fuzzy functions $\underline{\tilde{x}}\left(\tau\right)$, the dynamic response of the system corresponds to the resulting fuzzy functions $\underline{\tilde{z}}\left(\tau\right)$. These fuzzy functions result of the mapping, thus $\underline{\tilde{x}}\rightarrow\underline{\tilde{z}}\left(\tau\right)$.

3.4 Fuzzy Dynamic Analysis

The fuzzy dynamic analysis is an appropriate method to map a fuzzy input vector $\underline{\tilde{x}}$ onto the fuzzy output $\underline{\tilde{z}}\left(\tau\right)$ of a numerical model using the deterministic model given by Eq. (13). In structural analysis, the combination of uncertainties modeled as fuzzy variables with the deterministic model based on Finite Element Method is denominated *Fuzzy Finite Element Method*. The fuzzy dynamic analysis is composed of two stages shown in Fig. 4.

In the first stage, for computational purposes, the input vector that corresponds to the fuzzy parameter is discretized by means of $\alpha-$level representation, presented in Eq. (6) and Fig. 2(b).

Thus each element of the fuzzy parameters vector $\underline{\tilde{x}} = \left(\tilde{x}_1,\ldots,\tilde{x}_n\right)$ is considered as an interval $X_{i\alpha_k} = \left[x_{i\alpha_k l}, x_{i\alpha_k r}\right]$, where $\alpha_k \in (0,1]$. Consequently, the sub-space $\underline{X}_{\alpha k}$ is defined so that $\underline{X}_{\alpha_k} = \left(X_{1\alpha_k},\cdots,X_{n\alpha_k}\right)$, where $\underline{X}_{\alpha_k} \in \mathbb{R}^n$.

The second stage is related to solving an optimization problem. This optimization problem consists in finding the maximum and minimum value of the output, at each evaluated value τ, for the mapping model $M : \underline{z} = f(\underline{x})$, thus:

$$z_{\alpha_K r} = \max_{x \in \underline{X}\alpha_K} f(x) \qquad z_{\alpha_K l} = \min_{x \in \underline{X}\alpha_K} f(x) \tag{14}$$

$z_{\alpha_k r}$ and $z_{\alpha_k l}$ correspond to the upper and lower bounds of the interval $z_{\alpha_k} = \left[z_{\alpha_k r}, z_{\alpha_k l}\right]$ in the $\alpha-$level α_k . The set of discretized intervals $\left[z_{\alpha_k r}, z_{\alpha_k l}\right]$ for $\alpha_k \in (0,1]$ composes the whole fuzzy resulting variable $\underline{\tilde{z}}$.

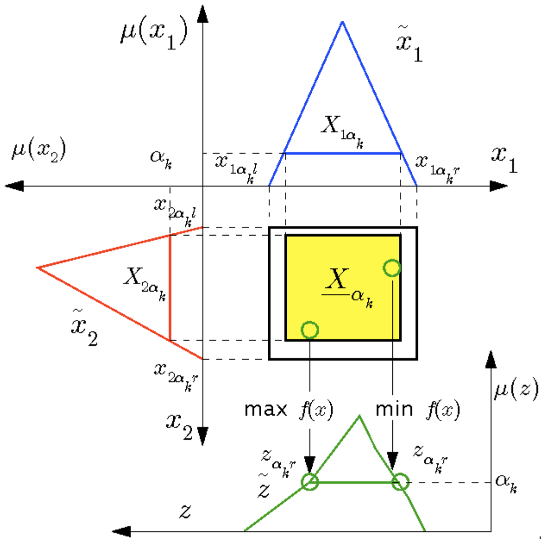


Figure 4: $\alpha-$ Level optimization.

The fuzzy analysis of a transient time-domain system demands the solution of a large number of optimization problems regarding all $\alpha-$level of interest for each considered time step. Each upper and lower bounds of the system analysis at a given time instant is obtained by using the Differential Evolution optimization algorithm (Price et al., 2005). The output value of the transient analysis at the evaluated time-step constitutes the objective function. The inputs to this function are the uncertain parameters described previously as fuzzy or fuzzy random variables.

3.5 Fuzzy Stochastic Analysis

In structural analysis, the combination of uncertainties modeled by using fuzzy randomness with the deterministic model based on Finite Element Method is denominated *Fuzzy Stochastic Finite Element Method*.

Fuzzy stochastic analysis is a straightforward computational method for processing uncertain data modeled by fuzzy random functions or variables. The aim of fuzzy stochastic analysis is to map fuzzy random parameters $\underline{\tilde{X}}$ onto the structural fuzzy random response $\underline{\tilde{Z}}(\tau)$, thereby, the following problem is to be solved for a crisp mapping model

$$\underline{\tilde{X}} \rightarrow \underline{\tilde{Z}}(\tau) \tag{15}$$

The diagram of Fig. 5 shows the fuzzy stochastic analysis algorithm. The basic concepts and definitions of the fuzzy stochastic analysis are presented with more details by Möller and Beer (2004).

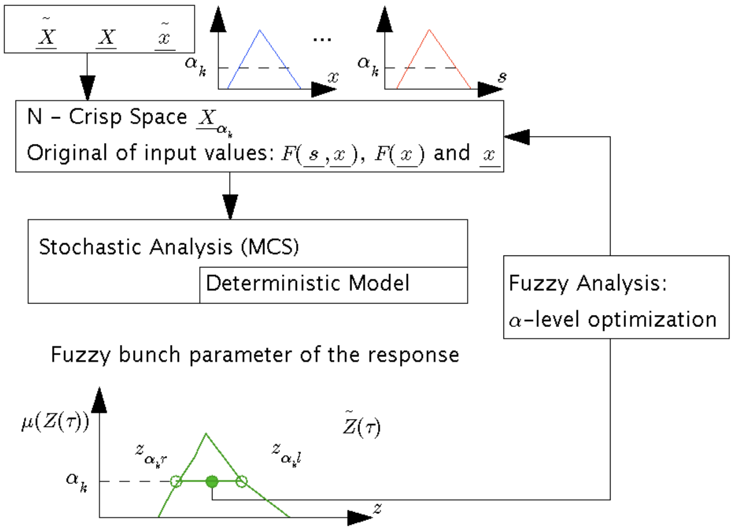


Figure 5: Fuzzy stochastic analysis, adapted from (Möller and Beer 2004).

The uncertainty model determines the type of the uncertain parameters. In this method an uncertain parameter can be modeled as a real random variable, fuzzy variable or fuzzy random variable and/or functions; e.g. a normally distributed fuzzy random variable with fuzzy mean and fuzzy standard deviation. Fuzzy variables and/or real random variables are special cases of fuzzy random variables. Moreover, the Karhunen Loève expansion can be used to represent a real random field.

The fuzzy random parameters are modeled as fuzzy random variables, which represent a generalized model i.e., high order uncertainty representation. The fuzziness of the fuzzy random variables $\underline{\tilde{X}}$ is described by means of fuzzy bunch parameters. Thereby, the fuzzy bunch parameters $\underline{\tilde{s}}$ of Eq. (10) are discretized by using the $\alpha-$level representation; thus, the $\alpha-$level sets of Eq. (6) are obtained for the corresponding intervals of each determined $\alpha-$level.

The fuzzy stochastic analysis algorithm encompasses a fuzzy analysis and a stochastic analysis to deal effectively with the fuzziness and randomness characteristics of the uncertainties (see Fig. 5). Both analyses are based on the deterministic model.

The purpose of fuzzy analysis is to map fuzzy input variables and fuzzy bunch parameters of the fuzzy random variables onto fuzzy random response; thus, the resulting fuzzy bunch parameters of the fuzzy random response are obtained. The $\alpha-$level optimization as stated by Möller et al. (2000)

is applied in the fuzzy analysis. An extended formulation of the fuzzy analysis based on the α-optimization was previously presented in Sec. 3.4.

In the stochastic analysis each element of fuzzy set $\underline{\tilde{s}}$ determines an original real random variable of every fuzzy random variable and an original deterministic value of every fuzzy variable. These original variables are mapped onto the original function $Z(\tau)$ of fuzzy random result values $\tilde{Z}(\tau)$ by a stochastic analysis. The mapping model is to be established by a stochastic analysis and it is based on the Finite Element approach, thus leading to the Stochastic Finite Element Method. Accordingly, the Monte Carlo Simulation is an appropriated method to perform the stochastic analysis. The result of a Monte Carlo sampling is an original function $Z(\tau)$ of the fuzzy random response $\tilde{Z}(\tau)$ at the membership $\alpha-$level $\mu(Z(\tau)) = \alpha_k$. The optimization problem of the α-level optimization is solved so that the assigned elements $z_{\alpha_k} = \left[z_{\alpha_k r}, z_{\alpha_k l}\right]$ at the specific membership level α_k (e.g. fuzzy mean or fuzzy variance) of the fuzzy random results are found.

4 NUMERICAL SIMULATIONS

The proposed methodology was numerically applied to analyze the dynamic behavior of a rotor system composed of a horizontal flexible steel shaft, modeled by 20 Euler-Bernoulli's beam elements, two rigid steel discs (D_1 and D_2) and three asymmetric bearings (see Fig. 6).

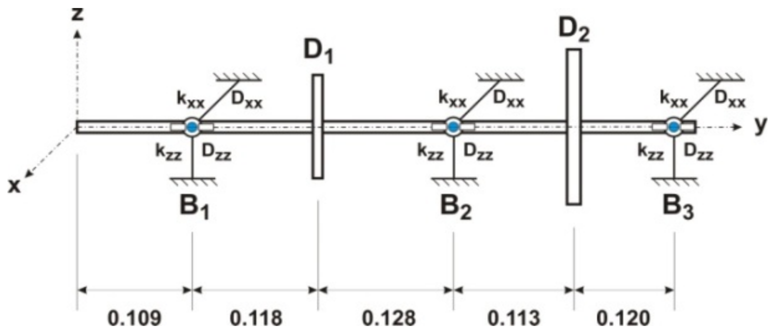


Figure 6: Flexible rotor (Cavalini Jr et al., 2011).

The physic and geometric properties of the shaft, discs and bearings are given in Table 1. The matrix equation of motion of the studied rotor was solved by using a MATLAB/SIMULINK® code. In all analyses performed in this contribution, the model considered only the first six vibration modes of the rotor, measured along the x and z directions at the positions of the bearings.

The parameters used in the Differential Evolution Algorithm to solve the optimization problem in the fuzzy analysis are described as follows: population size is 10 per uncertain variable, 100 generations, crossover probability rate is 0.8, perturbation rate is 0.8 and the strategy for the mutation mechanism is DE/rand/1/bin. These parameters were derived from previous contributions (Price et al., 2005). The two objective functions for the optimization problem are the following: the norm $\left|\hat{\mathbf{H}}(\omega, \Omega_S)\right|$ and the generalized displacements $\mathbf{q}(t)$ to compute both the FRF and the orbits; the corresponding expressions are given by Eq. (2) and (1). The uncertain parameters were considered by means of fuzzy triangular numbers, which is the simplest representation to describe a fuzzy variable.

Elements	Properties	Values
Shaft	Length [m]	0.588
	Diameter [m]	0.010
	Young's Modulus [Pa]	2.0×10^{11}
	Density [kg/m^3]	7800
D_1	Thickness [m]	0.005
	Diameter [m]	0.100
	Density [kg/m^3]	7800
D_2	Thickness [m]	0.010
	Diameter [m]	0.150
	Density [kg/m^3]	7800
B_1, B_2, B_3	k_{xx} [N/m]	49×10^3
	k_{zz} [N/m]	60×10^3
	d_{xx} [Ns/m]	5.0
	d_{zz} [Ns/m]	7.0
Proportional damping $\mathbf{C}_P = \alpha_M \mathbf{M} + \beta_K \mathbf{K}$	α_M	1.0×10^{-1}
	β_K	1.0×10^{-5}

Table 1: Physic and geometric properties of the rotor elements.

Two case studies are considered to analyze the uncertain dynamic behavior of the flexible rotor. In the first case, the uncertain parameters are modeled as fuzzy variables to perform the fuzzy modeling. In the second case, the uncertain parameters are modeled as fuzzy random variables, thus the fuzzy stochastic modeling is performed.

4.1 Fuzzy Analysis

In this contribution, the uncertainties are considered in the Young's Modulus E_S of the shaft and in the parameters (stiffness and damping) of the bearings (B_1, B_2 and B_3). The uncertain parameters are modeled by using fuzzy triangular numbers, thus: $\tilde{a} = a\left(1 - p / 100 \;\; / \;\; 1 \;\; / \;\; 1 + p / 100\right)$ where a represents the nominal value of the parameter and p stands for the maximum percentage of amplitude in $\alpha = 0$. To investigate the influence of uncertainties on the flexible rotor, three uncertainty scenarios were considered. Table 2 presents the percentages of variation of the triangular fuzzy variables corresponding to the uncertainty scenarios that are taken into consideration in the numerical simulations.

Scenarios	Shaft	Bearings			
	E_S	k_{xx}	k_{zz}	d_{xx}	k_{zz}
(a)	15%	—	—	—	—
(b)	—	5%	5%	5%	5%
(c)	15%	5%	5%	5%	5%

Table 2: Uncertainty scenarios for fuzzy modeling.

The first scenario was dedicated to investigate the influence of uncertainties only in the shaft (Young's Modulus E_S). The second one took into account only uncertainties in the bearings. Finally, the third scenario has considered uncertainties both in the shaft and the bearings. In these three uncertain scenarios, the Frequency Response Functions (FRFs) along both the directions x and z and the orbits of rotor were analyzed. Fig. 7 shows the Frequency Response Function (FRF). The fuzzy structural response of the rotor was computed for three different values where $\alpha = 1.0$, $\alpha = 0.5$, $\alpha = 0.0$.

The variation of the Young's Modulus E_S of the shaft produced a variation in the third natural frequency and also small changes in the amplitude of the first two natural frequencies.

For observing the system's behavior in the time domain, the orbit of the disc 1 is shown in Fig. 8. For the three uncertain scenarios, the rotor is operated at 600 rpm. The choice of this rotation speed is justified by considering that this rotation is well above the first two natural frequencies, so that it is guaranteed that the rotor will not operate in the proximity of a critical speed for any fuzzy model. Figure 8 shows that the uncertainties influence the amplitude of displacement; this influence can be observed in the small variation of the lower and upper envelopes of the orbits.

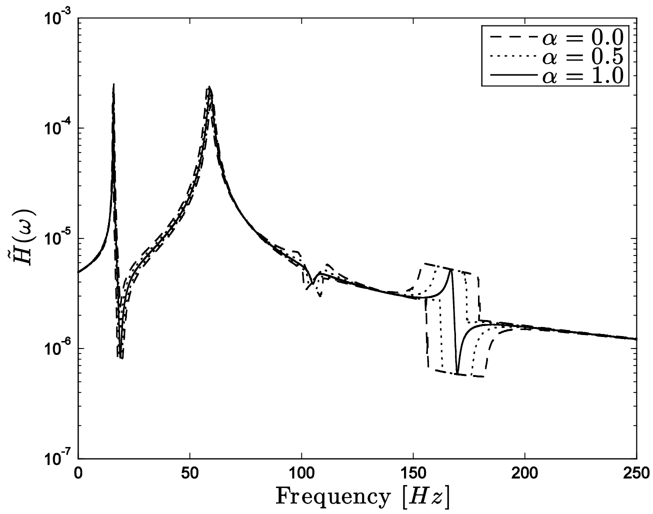


Figure 7: FRFs, case (a).

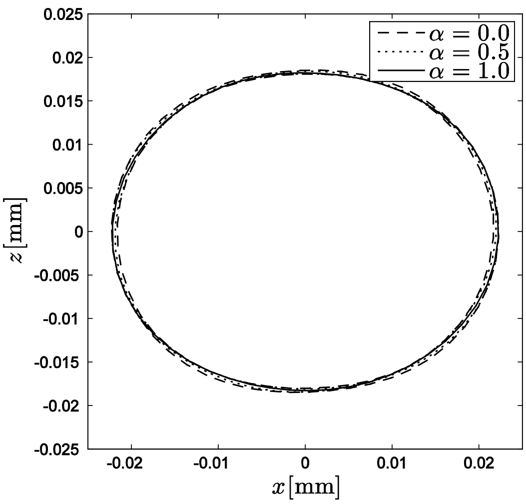


Figure 8: Envelope of the orbits, case (a).

For the scenario (b), the uncertainties are introduced only in the parameters of the bearing, namely the stiffness and damping. The FRFs and orbits are shown, respectively, in Figs. 9 and 10. The FRFs (see Fig. 9) shows that the uncertainties of the bearings result in a small variation of the amplitude at the two natural frequencies. The main difference between the FRFs of scenario (a) and the FRFs of scenario (b) occurs in the natural frequency near 165 Hz. For the scenario (b) there is a very small variation of the third natural frequency.

Analyzing the orbit shown in Fig. 10, for the second scenario, and comparing with the orbit shown in Fig. 8, it is possible to see that the uncertainty applied in the first scenario results in a smaller change in the orbits (see again the orbits shown in Fig. 8). However, the uncertainties in the stiffness and damping coefficients of the bearing result in important variations on the displacement of the rotor.

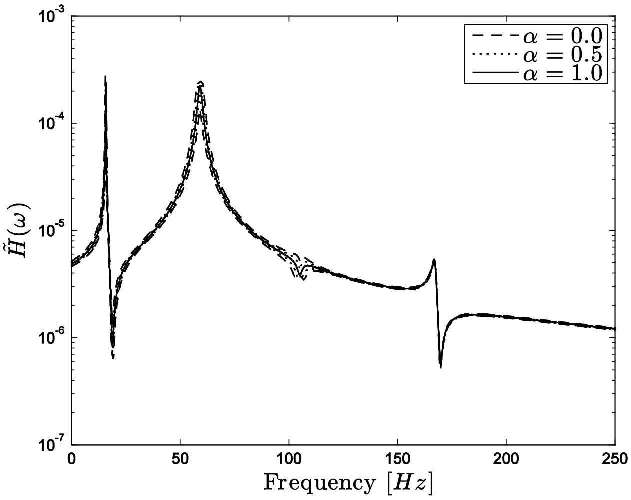


Figure 9: FRFs, case (b).

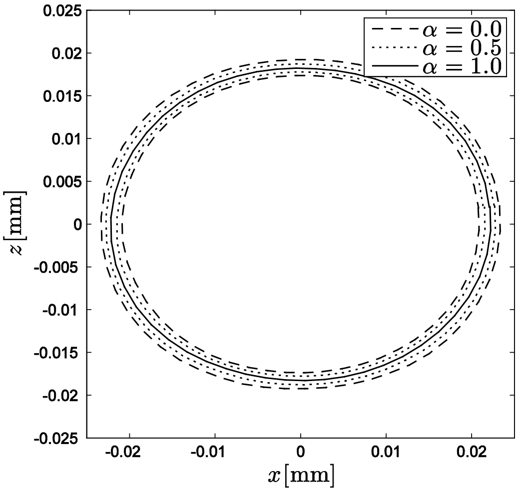


Figure 10: Envelope of the orbits, case (b).

Finally, the scenario (c) considers the uncertainties in the shaft (Young's Modulus) and bearings (stiffness and damping coefficients), simultaneously. Figs. 11 and 12 show, respectively, the FRFs and the orbits. The results presented by the scenario (c) permit to observe the combined influence of the uncertainty parameters on the dynamic behavior of the system. The result of this combination of uncertainty parameters leads to an important variation between the lower and upper curves of the envelope.

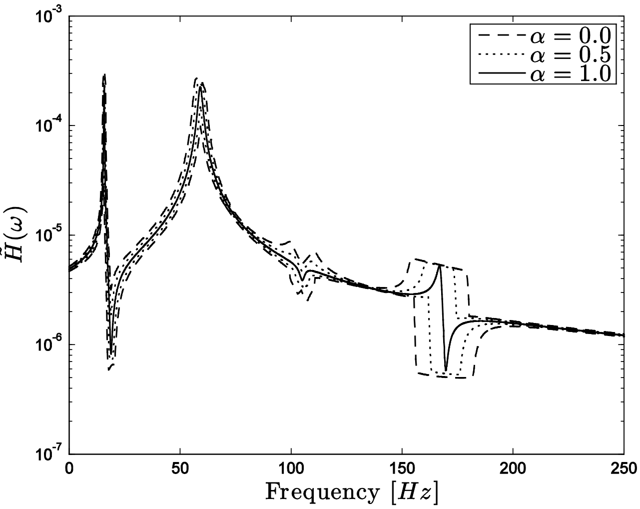


Figure 11: FRFs, case (c).

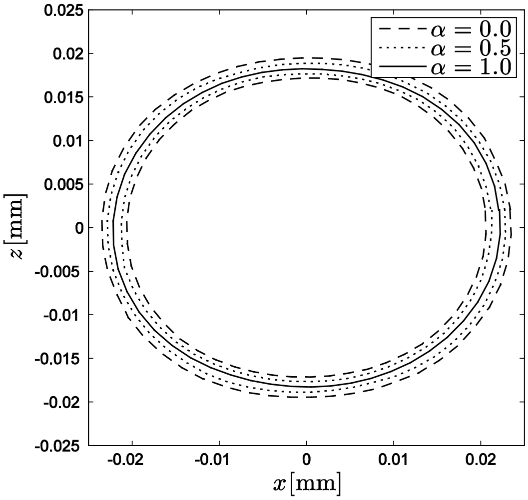


Figure 12: Envelope of the orbits, case (c).

In terms of FRFs, the influence of the uncertain E_S is characterized by a variation at the third natural frequency shown in Fig. 11, while the uncertainties in the bearings influence the amplitudes at the first two natural frequencies. Analogously, in terms of the orbits, a similar influence is observed.

In this scenario, the uncertainties in the stiffness of the shaft and damping coefficients of the bearings produce a combined effect in the envelop of the orbit, as illustrated in Fig. 12.

4.2 Fuzzy Stochastic Analysis

The fuzzy stochastic analysis of sec. 3.5 allows evaluating uncertain parameters modeled as fuzzy variables, real random variables and fuzzy random variables. Nevertheless, in order to demonstrate the performance of this method the parameters of the bearings are modeled as fuzzy-random variables with a fuzzy mean. This analysis aims at determining the orbits of the flexible rotor when the stiffness and viscous damping of the bearings are modeled as fuzzy random variables. Thereby, the fuzzy stochastic analysis method of sec. 3.5 is applied. The fuzzy random variables are defined to be normally distributed with a fuzzy mean $\tilde{m} = m\left(1 - p\,/\,100 \quad / \quad 1 \quad / \quad 1 + p\,/\,100\right)$ where p is the maximum percentage of the amplitude for the case in which $\alpha = 0$ and m holds for the nominal value of each parameter. Additionally, the uncertainty of the Young's Modulus of the shaft E_S is taken into account. Therefore, the stochastic model of the flexible rotor and specifically the stiffness of the shaft was modeled as a Gaussian random field by means of the Karhunen-Loève decomposition such as implemented by Koroishi et al. (2012). Therefore, two uncertain scenarios are analyzed as described in Table 3: in the first scenario, the influence of the bearings is analyzed; in the second uncertain scenario, the uncertainty introduced in the bearings and shaft is taken into account simultaneously.

Scenarios	Shaft	Bearings			
	E_S	k_{xx}	k_{zz}	d_{xx}	d_{zz}
(a)	—	5%	5%	5%	5%
(b)	5%	5%	5%	5%	5%

Table 3: Uncertainty scenarios for fuzzy stochastic modeling.

The results presented by the scenario (a) in Fig. 13 shows how fuzzy random damping and stiffness of the bearings affect the orbit of the shaft. The results show the fuzzy mean evaluated for $\alpha = 0.0$ and $\alpha = 1.0$. Additionally, the envelope of the uncertain orbit is bounded by the outer and the inner curves. The results show the influence in the amplitude of the displacements taking into account uncertainties that consider fuzziness and randomness characteristics. Consequently, the fuzzy random uncertainties introduced in the coefficients of the bearings produce a large variation in the envelopes of the orbit with respect to fuzzy modeling in which the uncertainties were considered only as fuzzy variables.

The results of the scenario (b) (see Fig. 14) show the combined influence of the uncertain fuzzy random coefficients of the bearings and the uncertain stiffness of the shaft. The fuzzy mean has not significant changed as compared with the previous uncertainty scenario. Nevertheless, the uncertain stiffness of the shaft produces a large variation in the inner curve of the envelope with respect to the previous uncertain scenario. As seen, this method allows evaluating simultaneously uncertain parameters of the rotor modeled as fuzzy random variables and Gaussian random fields.

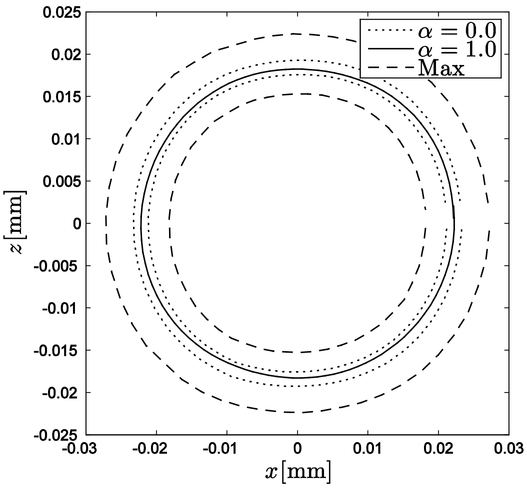


Figure 13: Envelope of the orbits, case (a).

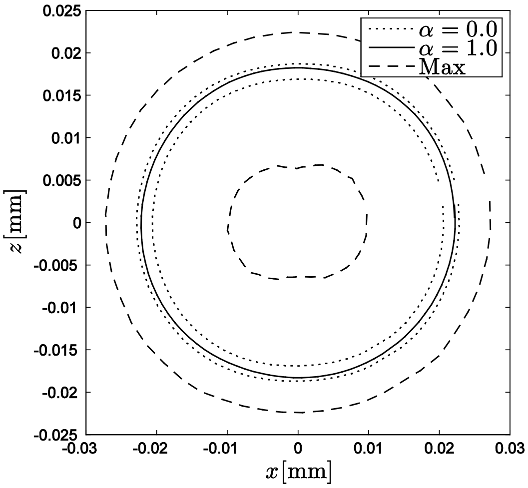


Figure 14: Envelope of the orbits, case (b).

5 CONCLUSIONS

In this contribution the uncertainty analysis of a flexible rotor by using *Fuzzy Finite Element Method* and *Fuzzy Stochastic Finite Element Method* was proposed and implemented. The uncertainties in the parameters that characterize the rotor system are introduced directly through a parametric approach and modeled as fuzzy variables and fuzzy random variables. The orbits and frequency response functions of the flexible rotor with uncertain parameters are simulated by using fuzzy stochastic dynamic analysis method. The numerical results indicate the degree of influence of the fuzzy uncertain variables on the dynamic behavior of the rotor. The strategy used in this work demonstrated to be straightforward for design and analysis of rotating systems. The selection of the uncertain fuzzy variables (stiffness, modulus of elasticity) was based on a previous knowledge regarding their sensitivities with respect to the frequency response functions. It is worth mentioning that these parameters are directly associated with the dynamic behavior of the rotor as represented by the orbits for the various uncertain scenarios considered. Additionally, the results show that the performance of the rotor, in terms of orbits and frequency response functions, are similar to the stochastic approach accomplished by Koroishi et al. (2012) in previous studies.

The uncertain parameters were properly described as fuzzy variables and fuzzy random variables. Fuzzy variables are a simple representation of uncertainties in which it is not necessary to know the probability distribution function of the stochastic variables. Fuzzy random variables permit to model high order uncertainties, which characterize fuzziness and randomness aspects of the uncertainty. However, the main disadvantage of the fuzzy analysis and fuzzy stochastic analysis is the necessity of solving the α-level optimization problem, which often demands high computational effort.

Further work will include an experimental verification aiming at evaluating the effect of variable parameters modeled as fuzzy variables on the dynamic behavior of rotating machines.

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