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Risk optimization of a steel frame communications tower subject to tornado winds

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Abstract

The structural engineering community in Brazil faces new challenges with the recent occurrence of high intensity tornados. Satellite surveillance data shows that the area covering the south-east of Brazil, Uruguay and some of Argentina is one of the world most tornado-prone areas, second only to the infamous tornado alley in central United States.

The design of structures subject to tornado winds is a typical example of decision making in the presence of uncertainty. Structural design involves finding a good balance between the competing goals of safety and economy. This paper presents a methodology to find the optimum balance between these goals in the presence of uncertainty.

In this paper, reliability-based risk optimization is used to find the optimal safety coefficient that minimizes the total expected cost of a steel frame communications tower, subject to extreme storm and tornado wind loads. The technique is not new, but it is applied to a practical problem of increasing interest to Brazilian structural engineers. The problem is formulated in the partial safety factor format used in current design codes, with an additional partial factor introduced to serve as optimization variable. The expected cost of failure (or risk) is defined as the product of a limit state exceedance probability by a limit state exceedance cost. These costs include costs of repairing, rebuilding, and paying compensation for injury and loss of life. The total expected failure cost is the sum of individual expected costs over all failure modes.

The steel frame communications tower subject of this study has become very common in Brazil due to increasing mobile phone coverage. The study shows that optimum reliability is strongly dependent on the cost (or consequences) of failure. Since failure consequences depend on actual tower location, it turns out that different optimum designs should be used in different locations. Failure consequences are also different for the different parties involved in the design, construction and operation of the tower. Hence, it is important that risk is well understood by the parties involved, so that proper contracts can be made.

The investigation shows that when non-structural terms dominate design costs (e.g., in residential or office buildings) it is not too costly to over-design; this observation is in agreement with the observed practice for non-optimized structural systems. In this situation, it is much easier to loose money by under-design. When structural material cost is a significant part of design cost (e.g. concrete dam or bridge), one is likely to lose significant

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money by over-design. In this situation, a cost-risk-benefit optimization analysis is highly recommended. Finally, the study also shows that under time-varying loads like tornados, the optimum reliability is strongly dependent on the selected design life.

Keywords: structural reliability, risk, optimization, structural cost, tornadoes.

1 Introduction

In a competitive environment, structural systems have to be designed taking into account not just their functionality, but their expected construction and operation costs, or their capacity to generate profits. Costs and profits are directly dependent on the risk that the construction and operation of a given facility offer to the user, to employees, to the general public or to the environment.

Risk, or expected cost of failure, is given in monetary units as the product of a failure cost by a failure probability. Failure probabilities, in their turn, are directly affected by the level of safety adopted in the design, construction and operation of a given facility. This includes the safety coefficients adopted in design, safety and quality assurance measures adopted during construction and the levels of inspection and maintenance practiced during operation.

In structural engineering design, economy and safety are competing goals. Generally, more safety involves greater costs and more economy implies less safety. Hence, designing a structural system involves a tradeoff between safety and economy. In common engineering practice, this tradeoff is addressed subjectively. In codified design the issue is decided by the code committee, which defines the safety coefficients to be adopted. Deterministic structural optimization addresses the economic part of the problem by aiming at reducing mass or material use, but largely neglects the safety issue. Reliability Based Design Optimization (RBDO) addresses the safety issue by imposing restrictions in terms of failure probabilities, but it does not account for the cost of failure. Deterministic structural optimization and RBDO can both be used to achieve mechanical structural efficiency, but they do not address the safety-economy tradeoff.

By including the (expected) cost of failure in the economic balance, Reliability Based Risk Optimization (RBRO) allows the optimum tradeoff point between safety and economy to be found. RBRO aims at finding the optimum level of safety to be achieved in a given structural system in order to minimize the total expected cost or maximize the expected profit. It is as a tool for decision making in the presence of uncertainty. RBRO is complementary to deterministic structural optimization or RBDO in the sense that the most economic design also requires mechanical efficiency.

Reliability Based Risk Optimization has very broad applications. It has been advocated as one of the main improvements to be incorporated in future revisions of current design codes [8]. Modern design codes have already incorporated the notion of reliability as a measure of safety, but so far they have only been calibrated to reproduce the levels of safety of previous code versions [7]. RBRO can be used to define optimum reliability targets for different load combinations [22]. It can be used to find optimum inspection and maintenance strategies for deteriorating structures [4]. This is particularly important for the live extension of existing facilities, which are beyond or approaching their original design lives.

In this article, RBRO is used to find the optimal safety coefficient to be used in the design of a steel frame communication tower, subject to extreme storm and tornado winds, in order to minimize its total expected cost. The problem is formulated in very general terms. The technique used herein is not new, but it is used to solve a practical problem of increasing interest to the Brazilian structural engineering community.

2 Formulation

2.1 Limit States

Let $\mathbf{X}$ and $\mathbf{z}$ be vectors of structural system parameters. Vector $\mathbf{X}$ includes geometric characteristics, resistance properties of materials or structural members, and loads. Some of these parameters are random in nature; others cannot be defined deterministically due to several sources of uncertainty. Typically, resistances are modeled as random variables and loads as random processes. Vector $\mathbf{z}$ contains all the deterministic structural system parameters like partial safety factors, parameters of the inspection and maintenance programs, etc.

The existence of randomness and uncertainty implies risk, that is, the possibility of undesirable structural responses. The boundary between desirable and undesirable structural response is formulated in terms of limit state functions $g(\mathbf{z}, \mathbf{x}) = 0$ such that:

$$\begin{array}{ll} D_f = \{\mathbf{z}, \mathbf{x} | g(\mathbf{z}, \mathbf{x}) \leq 0\} & \text{is the failure domain} \\ D_f = \{\mathbf{z}, \mathbf{x} | g(\mathbf{z}, \mathbf{x}) > 0\} & \text{is the safety domain} \end{array} \tag{1}$$

Each limit state describes one possible failure mode of the structure, either in terms of performance (serviceability) or ultimate capacity of the structure. The probability of undesirable structural response or probability of failure is given by:

$$P_f(\mathbf{z}, \mathbf{X}) = P[g(\mathbf{z}, \mathbf{X}) \leq 0] \tag{2}$$

where $P[.]$ stands for *probability*. The probabilities of failure for individual limit states and for system behavior are evaluated using traditional structural reliability methods such as FORM and SORM, as described in references [15] and [1].

2.2 Objective (cost) functions

The total expected cost of a structural system subject to risk of failure can be decomposed in:

a) initial or construction cost;

b) cost of operation;

c) cost of inspections and maintenance;

d) expected cost of failure.

The expected cost of failure, or failure risk, is given by the product of a failure cost by the failure probability:

$$\text{expected cost of failure}(\mathbf{z}, \mathbf{X}, P_f) = \text{failure cost}(\mathbf{z}).P_f(\mathbf{z}, \mathbf{X}) \tag{3}$$

Failure costs include the costs of repairing or replacing damaged structural members, removing a collapsed structure, rebuilding it, cost of unavailability, cost of compensation for injury or death of employees or general users, penalties for environmental damage, etc. All failure consequences have to be expressed in terms of monetary units, which can be a problem when dealing with human injury, human death or environmental damage. Measuring such failure consequences in terms of compensation payoff costs allows the problem to be formulated without really addressing the question.

For each structural member or structural system failure mode, there is a corresponding failure cost term. The total expected cost of a structural system becomes:

$$\begin{aligned} \text{total expected cost}(\mathbf{z}, \mathbf{X}, P_f) = {} & \text{initial cost}(\mathbf{z}) \\ & + \text{ operation cost}(\mathbf{z}) \\ & + \text{ inspection and maintenance cost}(\mathbf{z}) \\ & + \sum_{\text{failure modes}} \text{failure cost}(\mathbf{z}).P_f(\mathbf{z}, \mathbf{X}) \end{aligned} \tag{4}$$

The initial or construction cost increases with the safety coefficients used in design and with the practiced level of quality assurance. More safety in operation involves more safety equipment, more redundancy and more conservatism in structural operation. Inspection cost depends on intervals, quality of equipment and choice of inspection method. Maintenance costs increase with smaller intervals and level of repair. Increasing the level of safety, however, generally reduces expected costs of failure by reducing failure probabilities.

Any change in $\mathbf{z}$ that affects cost terms is likely to affect the expected cost of failure. Changes in $\mathbf{z}$ which reduce costs may result in increased expected costs of failure. Reduction in expected failure costs can be achieved by targeted changes in $\mathbf{z}$, which generally increase costs. This compromise between safety and cost is typical of structural systems.

The reduction of total expected costs can hence be formulated as an unconstrained optimization problem:

$$\text{minimize: total expected cost}(\mathbf{z}, \mathbf{X}, P_f) \tag{5}$$

When social or non-monetary consequences of failure are involved (like death or environmental damage), an acceptable limit on failure probabilities can be imposed, leading to a constrained optimization problem:

$$\begin{aligned} &\text{minimize: total expected cost}(\mathbf{z}, \mathbf{X}, P_f) \\ &\text{subject to: } P_f(\mathbf{z}, \mathbf{X}) < P_f^{\text{admissible}} \end{aligned} \tag{6}$$

The revenue to be obtained with a structural system is generally independent of $\mathbf{z}$ or $\mathbf{X}$, that is, it only depends on the facility been build. Hence, an alternative optimization problem can be formulated as:

$$\text{maximize: expected profit}(\mathbf{z}, \mathbf{X}, P_f) = \text{revenue} - \text{total expected cost}(\mathbf{z}, \mathbf{X}, P_f) \tag{7}$$

2.3 Consequence classes

Measuring failure consequences in monetary terms is not always easy. To facilitate this task, a distinction of failure consequences in classes has been proposed [13]. With ρ being the rate between total costs (construction plus direct failure costs) and construction costs, the following classes have been devised:

Class 1 - *Minor consequences*: ρ is less than about 2. Risk to life, given a failure, is small or negligible. Economical consequences of failure are also small. Example: silos and agricultural facilities.

Class 2 - *Moderate consequences*: ρ is between 2 and 5. Risk to life, given a failure, is moderate and economic consequences are considerable. Example: office buildings, industrial buildings, residential buildings.

Class 3 - *Large consequences*: ρ is between 5 and 10. Risk to life, given a failure, is high and economical consequences of failure are significant. Example: hospitals, main bridges, high rise buildings.

When ρ is greater than 10 failure consequences should be considered extreme and a full cost-benefit analysis is highly recommended. The conclusion could be that the structure should not be build at all [13].

2.4 Partial safety factor format

In modern structural design codes, safety margins are created by means of partial safety factors for resistances and loads. In the ANSI/AISC code, these factors are applied on nominal load effects and characteristic member resistances, at member level. The design resistance and design load effect are, respectively:

$$\begin{aligned} R_D &= \phi_R R_n \\ S_D &= \sum_{i=1}^{n} \gamma_i (S_n)_i \end{aligned} \tag{8}$$

were R_n is the nominal member resistance and S_n are nominal load effects. The design equation is simply:

$$R_D = S_D \tag{9}$$

Most design codes exhibit no explicit formulation to account for failure consequences. In order to change the safety margin, and to be able to account for failure consequences, an additional partial factor λ_k is introduced:

$$\lambda_k = \frac{R_D}{S_D} = \frac{\phi_R R_n}{\sum\limits_{i=1}^{n} \gamma_i (S_n)_i} \tag{10}$$

When partial factor λ_k equals one the original design situation is recovered. Partial factor λ_k is used as optimization variable in the problem to follow, allowing the optimum safety margin to be found.

3 Computational program

A computational program for the optimization of structural risk was developed to solve the problem just formulated [20]. The program is composed of three independent modules: an optimization module, a structural reliability module and a commercial finite element program, as shown in Figure 1.

The structural reliability module (StRAnD) was developed at the Department of Structural Engineering, University of São Paulo [2]. It was coded in FORTRAN using the object oriented approach. Time invariant reliability methods include FORM, SORM, simple and importance sampling Monte Carlo simulation. Time variant reliability problems can also be solved.

The mechanical part of the problem is solved in the commercial finite element program ANSYS. Coupling of the reliability with the mechanical (finite element) module is described in detail in ref. [3]. The optimization module (RiskOPT) was also developed in-house [20]. Objective functions are written in terms of the optimization variable λ_k. Available methods for linear search are *quadratic interpolation* and *regula falsi*. Both methods are used to find the step which minimizes the objective function in a particular direction. More details can be found in references [9, 14, 19]. Figure 1 shows the main steps in the solution of a risk optimization problem, and the interaction between the three computational modules.

4 Tornadoes in South-America

In recent years, the occurrence of intense tornados has been reported in the south-east of Brazil, as shown in Table 1. Whether these (recent) occurrences are due to climate change or not, the fact is that the structural engineering community in Brazil faces the challenge of designing against these highly uncertain events. Tornado intensity is measured following the Fujita scale

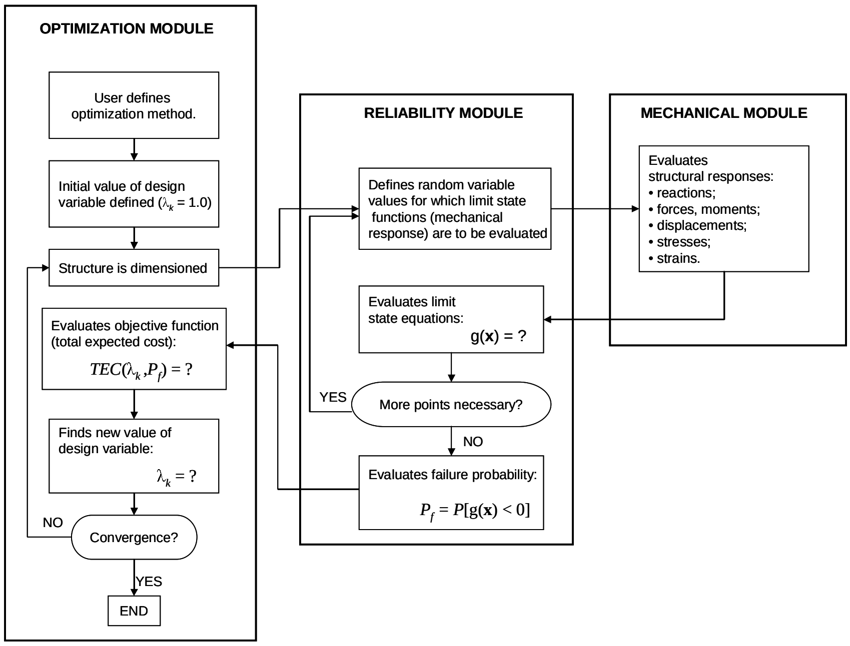

Figure 1: Flowchart of problem solution and interactivity of the computational modules.

[10], according to the level of damage they cause, as shown in Table 2. Actual tornado data in South America, including frequency of occurrence, intensity, path length and diameter is still scarce. However, satellite surveillance by the NOAA - National Weather Prediction Center, United States - shows that tornado threat in the region covering the south-eastern states of Brazil, Uruguay and some of Argentina is very high [21]. In a worldwide comparison, tornado threat in this region is second only to the central region of North America.

The scarcity of tornado data in Brazil makes tackling of the problem very difficult. However, it is possible to gain an idea of the size of the tornado problem in Brazil by making a comparison of satellite surveillance data for the region with similar data for the central part of North America, where abundant data exists. Color graduation in the tornado threat map shown in ref. [21] suggests that tornado threat in the south-east of Brazil is equivalent to the threat in the south-east of the state of Oklahoma. Data on frequency of occurrence, intensity and affected area for this state, as well as information collected from references [6, 11, 12, 16, 18] was used to compose the “order of magnitude” tornado data table shown in Table 3. This table provides information on the magnitude of the tornado problem in the south-east of Brazil, and should be a stimulus for more research and data collection on tornado occurrence in the region. Since the table does not represent real measured data, it should not be used as a reference for actual decision making. Very recent results [5] apud. ref. [17] actually suggests that the mean occurrence rates in Table 3 may be over-estimated.

Table 1: Major tornados observed in the south-east of Brazil.

Localization	**Date**	**Category**
Itu, SP	30/9/1991	F4
Aguas Claras, Viamão, RS	11/10/2000	F3
Region of Campinas, SP	4/5/2001	F3
Palmital, SP	25/5/2004	F3
Indaiatuba, SP	4/5/2005	F3
Muitos Capões, RS	29/8/2005	F3

Table 2: Fujita scale for tornados.

Class	Wind speeds (km/h)	Observed damage
F_0	65 - 117	Minor
F_1	117 - 180	Weak
F_2	182 - 252	Strong
F_3	253 - 333	Severe
F_4	334 - 419	Devastating
F_5	420 - 511	Incredible
F_6	Above 511	Unthinkable

Table 3: Estimated (order-of-magnitude) tornado data for the south-east of Brazil*.

Tornado	mean (m/s)	s.dev. (m/s)	mean occurrence rate (υ)
F_1	42	5.04	$1.300 \ . \ 10^{-3}$
F_2	60	7.20	$1.167 \ . \ 10^{-3}$
F_3	81	9.72	$3.500 \ . \ 10^{-4}$
F_4	105	12.60	$3.300 \ . \ 10^{-5}$
F_5	130	15.60	$3.300 \ . \ 10^{-6}$

*This is not measured data, do not use as reference.

5 Application example: steel frame communications tower subject to tornado loads

This example presents a case study of an actual steel frame communications tower subject to extreme storm and tornado wind loads. The tower supports telephone antennas as shown in Figure 2. This tower is very common in Brazil, and is largely used for mobile phone signal transmission. CAD models and the reference loading were supplied by the constructor.

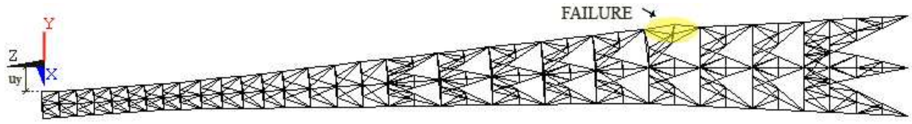

Figure 2: Tower collapse in finite element model.

Other than antenna and self-weight vertical loads, the main action on the tower is wind pressure. The tower has a triangular shape, hence design code provisions require wind effects to be verified for three main wind directions. For simplicity, only one wind direction is considered in this study. Tangential and up-lift drag forces are applied to the nodes, according to the elevation. The analysis is performed for extreme storm winds and for tornado winds.

5.1 Problem data

The three random variables considered to have the largest contribution to failure probabilities are included in the analysis. These variables are the steels elasticity module and yield strength, and the wind velocity. Wind velocity changes for each of the situations analyzed (50 year extreme storm wind and 5 tornado levels). The parameters and distributions of these random variables are presented in Table 4. Statistics and the choice of a log-normal distribution for elasticity module and yield strength are based on international references [7, 13].

Table 4: Random variable data for tower example.

R.V.	Name	distribution		mean	s.dev.	unit
X_1	Elasticity module (E)	log-normal		21000	1050	kN/cm^2
X_2	Yield strengh (S_y)	log-normal		25	1.25	kN/cm^2
X_3	Basic wind speed (W_0)	Gumbel for maxima	W_{50}	30	3.60	m/s
			F_1	42	5.04	m/s
			F_2	60	7.20	m/s
			F_3	81	9.72	m/s
			F_4	105	12.60	m/s
			F_5	130	15.60	m/s

Tornado occurrence statistics shown in Table 4 are based on Table 3. Extreme storm and tornado wind speeds are modeled with type I (Gumbel) distributions, following ref. [15]. The coefficient of variation of wind speeds was adopted as 12% [15]. Mean wind speeds were set as medians of the Fujita scale intervals.

5.2 Limit state function

The ultimate (collapse) limit state for the tower can be written as:

$$g(x) = u_{yCR} - u_{y_1} = 0 \tag{11}$$

were u_{yCR} is the critical displacement at the top of the tower, and u_{y1} is the horizontal displacement of node 1 in Y direction. Figure 3 shows results of the non-linear (physical and geometrical) analysis performed by increasing wind load W_0 from zero to 50 m/s. Results show a critical displacement of 60 cm at the top of the tower. A sensitivity analysis of this critical displacement with the optimization variable λ_k was also performed. Figure 4 shows that the critical wind speed varies largely with λ_k, but the critical displacement remains at about 60 cm regardless of λ_k. This justifies use of equation 11 as ultimate limit state. Service limit states involving, for example, loss of transmission are not considered in the study.

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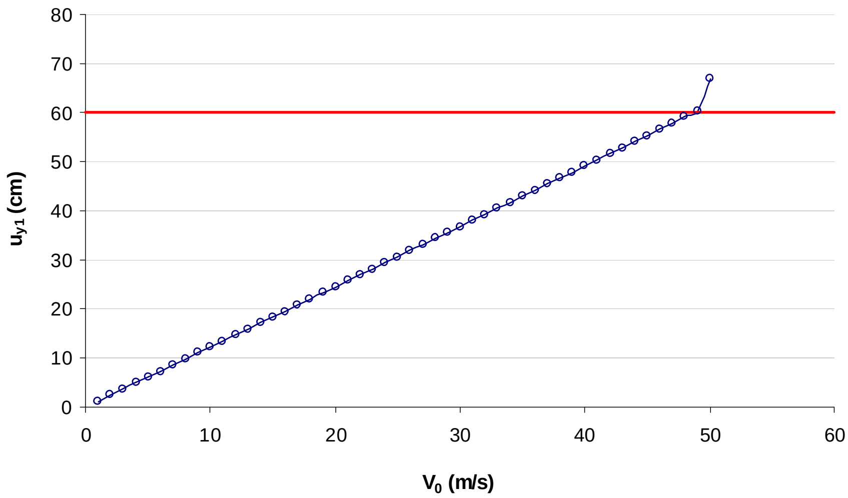

Figure 3: Load (wind speed) versus displacements at the top of the tower.

5.3 Objective function

The original design of this communications tower follows the guidelines of NBR6123 and of the AISC building code. The original design configuration is used as reference, with the optimization variable λ_k being equal to unity. The multiplicative factor λ_k is then applied to all cross-sections, resulting in more or less robust tower designs.

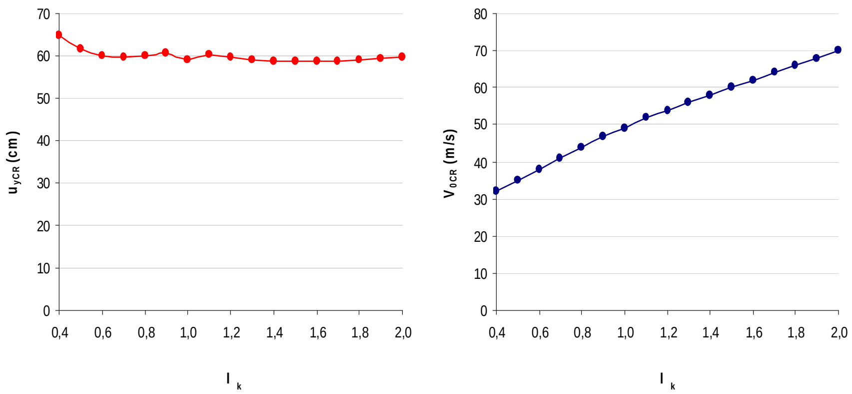

Figure 4: Variability of critical displacements and critical wind speeds as functions of optimization variable λ_k.

The initial or building cost for the tower (CI) is composed of cost of materials ($CM - 95\%$) and cost of workmanship ($CW - 5\%$); these are actual figures supplied by the builder. The cost of materials is composed of a fixed fraction and a fraction that varies proportionally with λ_k:

$$CI(\lambda_k) = CM(\lambda_k) + CW \tag{12}$$

For the original tower ($\lambda_k = 1$) the initial cost is $\$38,737$ monetary units (Brazilian Real). Failure costs are composed by the initial (actually rebuilding) cost plus a fixed part, proportional to the severity of failure consequences. Cost of failure curves were divided in four consequence classes, following the propositions of [13]:

$$\begin{aligned}
\text{Minor consequences}: CF_1(\lambda_k) &= CI(\lambda_k) + 50,000 \\
\text{Moderate consequences}: CF_2(\lambda_k) &= CI(\lambda_k) + 160,000 \\
\text{Large consequences}: CF_3(\lambda_k) &= CI(\lambda_k) + 350,000 \\
\text{Extreme consequences}: CF_4(\lambda_k) &= CI(\lambda_k) + 500,000
\end{aligned} \tag{13}$$

Minor consequences correspond only to the removal of the collapsed tower and replacement by a new one. Moderate consequences include, for example, a penalty for interruption of services. Large consequences include interruption of services and payment of compensation for injury. Extreme consequences are the large consequences plus payment of compensation for loss of one human live.

Two distinct situations were considered in the study: tower subject to 50 year extreme storm wind and tower subject to storm and tornado winds. Hence, a total of eight objective functions were used. Denoting by E the tower collapse event, the i^th objective function for the 50 year extreme storm wind is:

$$CTE_i(\lambda_k) = CI(\lambda_k) + CF_i(\lambda_k).P[E/W_{50}] \tag{14}$$

were i represents one of the four failure consequence classes and $P[.]$ means the probability of the event in brackets.

The occurrence of each type of tornado was modeled as a Poisson process [16], and a 50 year design life was considered. With the estimated rate of occurrence for each class of tornado (Table 3), and following the Poisson process, one has:

$$
\begin{aligned}
P[x \text{ ocurrences in time } t] &= \frac{(\upsilon t)^x}{x!} e^{-\upsilon t} \\
P[\text{ exactly 1 ocurrence in 50 years }] &= 50 \cdot \upsilon \cdot e^{-50\upsilon} \\
P[\text{ at least 1 ocurrence in 50 years }] &= 1 - P[0 \text{ ocurrences in 50 years}] \\
&= 1 - e^{-50\upsilon}
\end{aligned} \tag{15}
$$

Taking the F_1 tornado as an example, the probabilities for this tower are:

$$
\begin{aligned}
P[\text{exactly one } F_1 \text{ tornado in 50 years}] &= 50 \cdot 1.3 \cdot 10^{-3} e^{-50 \cdot 1.3 \cdot 10^{-3}} = 0.0609 \\
P[\text{at least one } F_1 \text{ tornado in 50 years}] &= 1 - e^{-50 \cdot 1.3 \cdot 10^{-3}} = 0.0629
\end{aligned} \tag{16}
$$

Probability of failure of the tower is given by the total probability theorem:

$$
P[E] = P[E/W_{50}] + \sum_{i=1}^{5} P[E/F_i] \cdot P[F_i] \tag{17}
$$

The objective function including tornados is:

$$
CTE_i(\lambda_k) = CI(\lambda_k) + CF_i(\lambda_k) .P[E] \tag{18}
$$

All failure probabilities in the formulation above are also functions of λ_k.

5.4 Results

The objective (cost) functions for the problem are shown in terms of λ_k in Figures 5 and 6, for the 50 year extreme storm wind and for tornado winds, respectively. Optimum values of λ_k and the corresponding optimum reliability indexes β are shown in Table 5.

Figures 5 and 6 show that, as failure become costlier, optimum λ_k (hence optimum reliability) and minimum cost increases. In Figure 5, in particular, it can be observed that higher consequence cost curves increase rapidly for small values of λ_k, but grow slowly as λ_k increases. This is due to the fact that the higher consequence curves are composed of a dominating fixed term, plus a small term that is proportional to λ_k. In other words, the cost of materials (which is proportional to λ_k) in these consequence curves is insignificant. In this situation, over-design does not affect expected costs as much as under-design. In this situation, it is likely to be cheaper to design on the conservative side (over-design), as common structural engineering practice already found out. In Figure 6 it can be observed that the higher consequence curves are nearly flat around the optimum points. This means that designing out of the optimum point does not affect total expected costs too much. However, a comparison of all results in Figures 5 and 6 shows that cost functions are very problem-dependent (and in this case, load dependent), hence a cost optimization study is highly justified when failure consequences are large or extreme.

Table 5 shows that under 50 year extreme wind loads, the current tower design (with λ_k=1) is overly conservative for all failure consequence classes. Currently, the tower is designed for 50 year extreme winds only. However, the locations were the tower is used are subject to tornados, and for large and extreme consequences the tower is under-designed, that is, the most economic design requires cross-section areas that are 40 or 60 percent larger than the actual design. The table also shows that optimum tower reliability varies considerably with failure consequences.

Table 5: Calculated optimum λ_k and β values.

Failure consequences	optimum (λ_k)		optimum reliability index β	
	W_{50}	Tornadoes	W_{50}	Tornadoes
minor	0.60	0.70	1.282	0.999
moderate	0.68	0.80	2.054	1.292
large	0.75	1.50	**2.423**	**1.728**
extreme	0.80	1.67	**2.647**	**1.837**

Optimum reliability indexes shown in Table 5 can be compared with the target reliability indexes used in calibration of the ANSI structural loads code. This code was calibrated for $\beta_{target} = 2.5$ in the dead plus live plus wind load combination, a vaue that compares very favorably to the optimum β obtained for the large and extreme consequence classes, for the storm wind load case. The same code was calibrated for $\beta_{target} = 1.75$ in any load combination involving earthquakes. Tornados are similar to earthquakes in the sense that both are very unlikely, very intense and very uncertain loads. This target β is quite similar to the optimum βs obtained for tornado loads, for the large and extreme consequence classes. This result shows that, despite all the simplifying assumptions made in solving this problem, it still resembles reality very well.

Since both the 50 year extreme storm wind and the probability of tornado occurrences depend on the chosen design live, the optimum λ_k also varies with design life. This is shown in Figure 7. It is noted that greater design lives lead to larger optimum values of λ_k. The increase in optimum λ_k with larger design live is largest for the greater failure consequences.

5.5 Discussion on failure consequence classes

The results shown above were presented for four different classes of failure consequences. Clearly, the significant curve depends on actual location of each tower. For towers located in populated

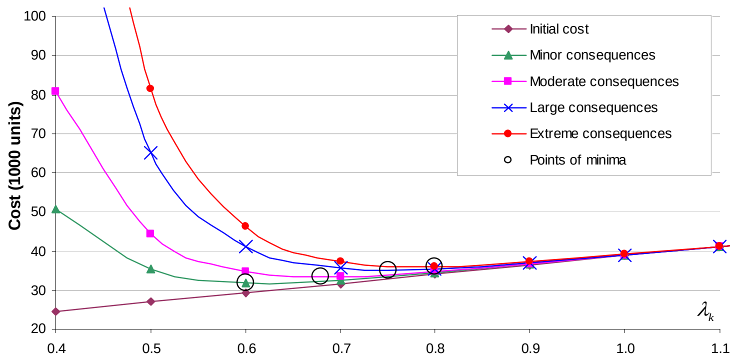

Figure 5: Cost (objective) functions versus λ_k for 50 year extreme storm winds only.

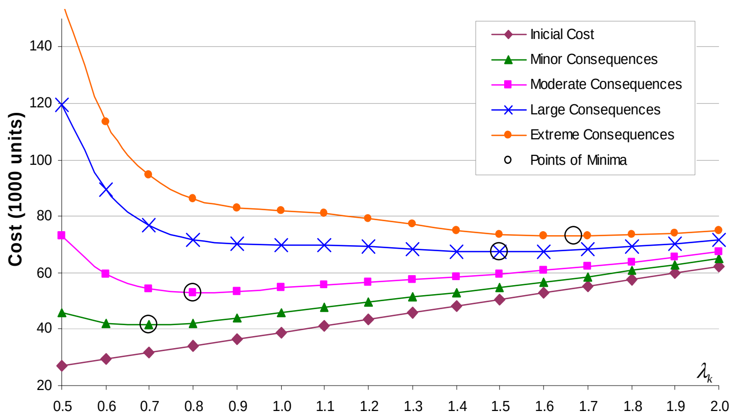

Figure 6: Cost (objective) functions versus λ_k for 50 year extreme storm and tornado winds.

areas, human death and injury due to tower collapse are highly likely, whereas in rural areas this is not the case. Hence, in populated areas the *extreme consequence* curve applies, whereas in rural areas at most the *large consequence* curve is significant.

The significant consequence curve also varies for the different parties involved in the project. The actual construction contract for these towers states that, in the case of loss or collapse of a tower, the contractor (builder) has to replace the lost tower. Hence for the builder the proper cost function is the *minor consequences* of failure, and he could increase his profit margin by reducing member cross sections (optimum $\lambda_k = 0.7$ for tornado winds and minor consequences). This only happens because safety margins or reliability indexes are not specified in the contract. The owner or operator of the towers, on the other hand, will bear the full costs of tower collapse; hence for the owner the proper consequence curve is the *moderate, large* or *extreme*, depending on tower location. In order to avoid such disparity and operate at the optimum (most economic) level of safety, the owner should specify the λ_k to be used in the design of each tower, depending on its location.

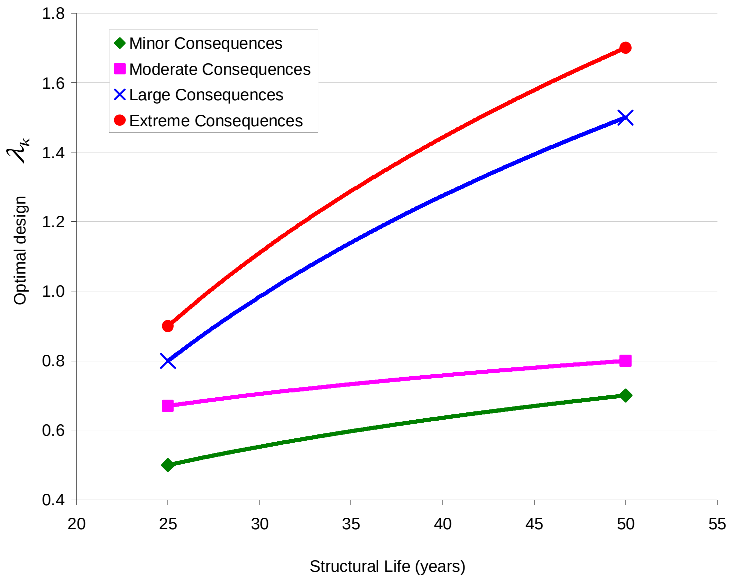

Figure 7: Variation of optimum λ_k with design life, storm and tornado winds.

6 Conclusions

This paper presented a methodology to obtain the optimum balance between safety and economy in the design of structures subject to highly uncertain events like tornado wind loads.

The study shows that the economic performance of structural systems is highly dependent on the consequences of failure. In general, as failure consequence becomes costlier, the optimum level of safety increases. Minimization of the total expected cost of a structure, including the expected costs of failure, is a viable approach to determine optimal safety levels.

In the communication tower problem studied herein, the cost of structural materials in the large and extreme consequence curves is only a small fraction of total failure costs. As a consequence, one is more likely to lose money by under-design than by over-design. This conclusion is in agreement with the actual observed practice for non-optimized structural systems. This is also a characteristic of commercial and residential buildings, where the cost of non-structural items and other large consequences of failure dwarf costs of structural materials. For structures like large concrete dams, where materials represent a larger fraction of total expected costs, the cost for designing outside the optimum point is higher, and a risk optimization analysis as presented herein is highly recommended.

When designing structures for different actions (e.g., 50 year extreme storm and tornado winds) it is not reasonable to require the same level of reliability for all load cases. Load actions showing large uncertainty and high intensity like extreme tornadoes or earthquakes tend to dominate the design. For different actions, different optimum levels of safety should be specified.

For time-dependent loading, optimum safety levels also vary with design live. An increase in design live leads to an increase in optimum safety levels. This is especially true for high consequences of failure. Hence, it is important that design lives closely resemble actual live of the structure.

Total expected cost functions are different for the different parties involved in a structural engineering project. An understanding of risk and failure consequences is fundamental in order for proper contracts to be written. Proper contracts can reduce the gap of interests among the different parties involved. Understanding of risks and proper contracts will benefit the owner of a facility, ensuring that it will be built and operated at the level of safety of his best interest.

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