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Two-Dimensional Fractional Order Generalized Thermoelastic Porous Material

Abstract

In the work, a two-dimensional problem of a porous material is considered within the context of the fractional order generalized thermoelasticity theory with one relaxation time. The medium is assumed initially quiescent for a thermoelastic half space whose surface is traction free and has a constant heat flux. The normal mode analysis and eigenvalue approach techniques are used to solve the resulting non-dimensional coupled equations. The effect of the fractional order of the temperature, displacement components, the stress components, changes in volume fraction field and temperature distribution have been depicted graphically.

Keywords

Fractional derivative; Porous material; Normal mode analysis; Eigenvalue approach.

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1 INTRODUCTION

Porous materials make their appearance in a wide variety of settings, natural and artificial and in diverse technological applications. As a consequence a number of problems arise dealing, among others, with statics and strength, fluid flow and heat conduction, and the dynamics of such materials. In connection with the latter, we note that problems of this kind are encountered in the prediction of behavior of sound-absorbing materials and in the area of exploration geophysics, the steadily growing literature bearing witness to the importance of the subject Pecker and Deresiewiez (1973).

The problem of a fluid-saturated porous material has been studied for many years. A short list of papers pertinent to the present study includes Biot(1941, 1956), Gassmann (1951), Biot and Willis (1957), Biot (1962), Deresiewicz and Skalak (1963), Mandl (1964), Nur and Byerlee (1971), Brown and Korringa (1975), Rice and Cleary (1976), Burridge and Keller (1981), Zimmerman et al. (1986,1994), Berryman and Milton (1991), Thompson and Willis (1991)], Pride et al. (1992), Berryman and Wang (1995), Tuncay and Corapcioglu (1995), Alexander and Cheng (1991), Charlez, P. A., and Heugas, O. (1992), Abousleiman et al. (1998), Ghassemi and Diek (2002), Tod (2003).

Eringen (1970) and Nowacki (1966)developed the linear theory of micropolar thermoelasticity which are known as micropolar coupled thermoelasticity to include thermal effects. Goodman and Cowin (1972) established a continuum theory for granular materials, whose matrix material (or skeletal) is elastic and interstices are voids and they introduced the concept of distributed body, which represents a continuum model for granular materials (sand, grain, powder, etc) as well as porous materials (rock, soil, sponge, pressed powder, cork, etc.). Nunziato and Cowin (1979), developed the non-linear theory of elastic materials with void, underlying the basic concept that the bulk density of the material is written as the product of two fields, the density field of the matrix material and the volume fraction field (the ratio of volume occupied by grains to the bulk volume at a point of the material) Kumar and Gupta (2010)]. Othman (2007) studied the effect of rotation and relaxation time on a thermal shock problem for a half-space in generalized thermo-viscoelasticity and Othman and Singh (2005) studied the effect of rotation on generalized micropolar thermoelasticity for a half-space under five theories. Youssef (2007)constructed theory of generalized porothermoelasticity which describe the behavior of thermoelastic porous medium in the context of the theory of generalized thermoelasticity with one relaxation time (Lord-Shulman). The energy and the entropy equations have been derived also in general co-ordinates. The uniqueness of the solution for the complete system of the equations of the theorem has been proved by Kumar et al. (2013) and he discussed the plane deformation due to thermal source in fractional order thermoelastic media, while Abbas and Kumar (2014) studied the deformation due to thermal source in micropolar generalized thermoelastic half- space by finite element method.

Recently, a new formula of heat conduction has been considered in the context of the fractional integral operator definition by Youssef (2010). This new consideration generated the fractional order generalized thermoelasticity which was cited by Youssef who approved the uniqueness of its solutions.

Youssef solved one dimensional problem in the context of the fractional order generalized thermoelasticity and discussed the effects of the fractional order parameter on all the studied fields and with Al-Leheabi i(2010). Youssef (2012) solved two-dimensional thermal shock problem of fractional order generalized thermoelasticity with thermal shock. Povstenko (2005) solved a problem of fractional heat conduction equation and associated thermal stress. The counterparts of our problem in the contexts of the thermoelasticity theories have been considered by using analytical and numerical methods Abbas et al. (2002, 2008, 2009, 2011, 2012).

In this paper, a two-dimensional problem of a porous material will be considered within the context of the fractional order generalized thermoelasticity theory with one relaxation time. The medium will be assumed initially quiescent for a thermoelastic half space whose surface is traction free and has a constant heat flux. The normal mode analysis and eigenvalue approach techniques will be used to solve the resulting non-dimensional coupled equations. The effect of the fractional order of the temperature, displacement components, the stress and components, changes in volume fraction field distribution will be depicted graphically.

2 GOVERNING EQUATIONS

For homogeneous, linear and thermally elastic medium with voids and temperature dependent mechanical properties, the basic equations in the context of the Lord and Shulman (1997) model and Cowin and Nunziato (1983) in absence of body forces and heat source are given by Kumar and Devi (2011).

The equations of motion Kumar and Devi (2011):

$$\sigma_{ij,j} = \rho \ddot{u}_i, \quad i,j = x,y,z \quad , \tag{1}$$

and

$$\left(\beta \phi_{,i}\right)_{,i} - be - \xi_1 \phi - w_0 \frac{\partial \phi}{\partial t} + mT = \rho \psi \frac{\partial^2 \phi}{\partial t^2}, \quad i,j = x,y,z \tag{2}$$

The generalized heat conduction equation Youssef (2010) and Kumar and Devi (2011):

$$\left(K\, I^{\alpha-1} T_{,i}\right)_{,i} = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right)\left(\rho c_e T + mT_o \phi + \gamma T_o e\right), \quad i,j = x,y,z\,, \tag{3}$$

where the fractional integral operator defined as follows Youssef (2010):

$$I^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(\tau)\, d\tau, \qquad \left\{ \begin{array}{ll} 0 < \alpha < 1 & weak\ conductivity \\ \alpha = 1 & normal\ conductivity \\ 1 < \alpha \le 2 & strong\ conductivity \end{array} \right\}, \tag{4}$$

and $\Gamma(\alpha)$ is the Gamma function.

The constitutive equations

$$\sigma_{ij} = 2\mu e_{ij} + \left[\lambda e + b\phi - \gamma\left(T - T_o\right)\right]\delta_{ij}, \qquad i,j = x,y,z\,, \tag{5}$$

The cubical dilatation

$$e = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z},$$

where ρ is the mass density, T the temperature change of a material particle, T_o the reference uniform temperature of the body, u_i the displacement vector components, e_{ij} the strain tensor; σ_{ij} the stress tensor, c_E the specific heat at constant strain, γ the thermal elastic coupling tensor in which $\gamma=(3\lambda+2\mu)\alpha_t$, α_t is the coefficient of linear thermal expansion, K is a material constant thermal conductivity, λ,μ are elastic parameters, β, b, ξ_1, m, w_0, and ψ are the material constants due to presence of voids and ϕ is the change in volume fraction field of voids.

Formulation and solution of the problem

We consider an isotropic, homogenous and elastic body with voids in two-dimensional fills the region $0\le x<\infty$, $-\infty<z<\infty$ subjected to a time-dependent heat source and traction free on the surface $x=0$. The governing equations will be written in the context of Lord and Shulman model when the body has no heat sources or anybody forces, and we will use the Cartesian co-ordinates (x,y,z) and the components of the displacement $u_i=(u,0,w)$ to write them as follows:
The equations of motion are in the forms

$$\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{xz}}{\partial z}=\rho\frac{\partial^2 u}{\partial t^2}, \tag{7}$$

$$\frac{\partial\sigma_{zx}}{\partial x}+\frac{\partial\sigma_{zz}}{\partial z}=\rho\frac{\partial^2 w}{\partial t^2}\ , \tag{8}$$

and

$$\beta\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial z^2}\right)-be-\xi_1\phi-w_1\frac{\partial\phi}{\partial t}+mT=\rho\psi\frac{\partial^2\phi}{\partial t^2}, \tag{9}$$

The heat conduction equation

$$I^{\alpha-1}K\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}\right)=\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)\left(\rho c_e T+mT_o\phi+\gamma T_o e\right), \tag{10}$$

The heat flux equation in x-direction

$$q(x,z,t)+\tau_o\dot{q}(x,z,t)=-K\,I^{\alpha-1}\frac{\partial T(x,z,t)}{\partial x},\quad 0<\alpha\le 2 \tag{11}$$

The constitutive relations are

$$\sigma_{xx}=2\mu\frac{\partial u}{\partial x}+\lambda e+b\phi-\gamma(T-T_o), \tag{12}$$

$$\sigma_{zz}=2\mu\frac{\partial w}{\partial z}+\lambda e+b\phi-\gamma(T-T_o), \tag{13}$$

and

$$\sigma_{xz}=\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right). \tag{14}$$

The cubical dilatation

$$e=\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\,. \tag{15}$$

For our convenience, the following non-dimensional variables and notations are used:

$$(x',y',z')=\frac{\eta}{c}(x,y,z),\ (u',v',w')=\frac{\rho c\eta}{\gamma T_o}(u,v,w),t'=\eta t,T'=\frac{T-T_o}{T_o},\sigma'_{ij}=\frac{\sigma_{ij}}{\gamma T_o},\phi'=\frac{\rho c^2}{\gamma T_o}\phi,\ q'=\frac{c}{T_o\eta K}q,$$

where $c^2=\dfrac{\lambda+2\mu}{\rho},\eta=\dfrac{c^2}{K}$.

In terms of the non-dimensional quantities defined above, the governing equations will be reduce to (dropping the dashed for convenience)

$$\frac{\partial^2 u}{\partial x^2}+b_1\frac{\partial^2 u}{\partial z^2}+\frac{\partial^2 w}{\partial x\partial z}+b_3\frac{\partial\phi}{\partial x}-\frac{\partial T}{\partial x}=\frac{\partial^2 u}{\partial t^2}, \tag{16}$$

$$\frac{\partial^2 w}{\partial z^2}+b_1\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 u}{\partial z\partial x}+b_3\frac{\partial\phi}{\partial z}-\frac{\partial T}{\partial z}=\frac{\partial^2 w}{\partial t^2}. \tag{17}$$

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial z^2}-b_4 e-b_5\phi-b_6\frac{\partial\phi}{\partial t}+b_7 T=b_8\frac{\partial^2\phi}{\partial t^2}, \tag{18}$$

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}=\frac{\partial^\alpha}{\partial t^\alpha}\left(1+\tau_0\frac{\partial}{\partial t}\right)\left(T+b_9\phi+b_{10}e\right), \tag{19}$$

$$\sigma_{xx}=2b_1\frac{\partial u}{\partial x}+\left(2b_2-1\right)e+b_3\phi-T, \tag{20}$$

$$\sigma_{zz}=2b_1\frac{\partial w}{\partial z}+\left(2b_2-1\right)e+b_3\phi-T, \tag{21}$$

$$\sigma_{xz}=b_1\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right), \tag{22}$$

$$q\left(x,z,t\right)+\tau_o\dot{q}\left(x,z,t\right)=-I^{\alpha-1}\frac{\partial T\left(x,z,t\right)}{\partial x},\quad 0<\alpha\le 2 \tag{23}$$

where

$$b_1=\frac{\mu}{\lambda+2\mu},b_2=1-b_1,b_3=\frac{b}{\lambda+2\mu},b_4=\frac{bc^2}{\beta\eta^2},b_5=\frac{\xi_1 c^2}{\beta\eta^2},b_6=\frac{w_1 c^2}{\beta\eta},b_7=\frac{m\rho c^4}{\beta\eta^2\gamma},b_8=\frac{\rho\psi c^2}{\beta},$$

$$b_9 = \frac{mT_o\gamma}{\rho K\eta}, \text{ and } b_{10} = \frac{T_o\gamma_o^2}{\rho K\eta}.$$

3 NORMAL MODE ANALYSIS

The solution of considered physical variables can be decomposed in terms of normal mode as following form

$$\left(u, w, \phi, T, \sigma_{ij}, q\right)(x,z,t) = \left(u^*, w^*, \phi^*, T^*, \sigma_{ij}^*, q^*\right)(x)e^{(\omega t + ibz)}, \tag{24}$$

where ω is a complex constant, $i=\sqrt{-1}$, b is the wave numbers in the z-directions. Using equation (24), equations (16)-(23) become respectively:

$$\frac{d^2u^*}{dx^2} = Au + B\frac{dw^*}{dx} + C\frac{d\phi^*}{dx} + \frac{dT^*}{dx}, \tag{25}$$

$$\frac{d^2w^*}{dx^2} = Dw^* + E\phi^* + FT^* + G\frac{du^*}{dx}, \tag{26}$$

$$\frac{d^2\phi^*}{dx^2} = Hw^* + M\phi^* + NT^* + P\frac{du^*}{dx}, \tag{27}$$

$$\frac{d^2T^*}{dx^2} = Qw^* + R\phi^* + ST^* + Z\frac{du^*}{dx}, \tag{28}$$

$$\frac{\partial T^*}{\partial x} = -Lq^*(x), \tag{29}$$

where

$$A = b_1b^2 + \omega^2,\ B = -ib,\ C = -b_3,\ D = \frac{1}{b_1}\left(b^2 + \omega^2\right),\ E = -\frac{ibb_3}{b_1}, F = \frac{ib}{b_1}, G = -\frac{ibb_2}{b_1},$$

$$H = ibb_4,\ M = b_8\omega^2 + b^2 + b_5 + \omega b_6, N = -b_7,\ P = b_4,\ Q = Libb_{10},\ R = Lb_9,\ S = b^2 + L,\ Z = Lb_{10},$$

$$L = \left(1 + \tau_0\omega\right)e^{-\omega t}t^{-\alpha}\sum_{n=1}^{\infty}\frac{(\omega t)^n}{\Gamma\left[n+1-\alpha\right]}.$$

Equations (25)-(28) can be written in a vector-matrix differential equation as follows

$$\frac{d\vec{V}}{dx} = W\vec{V}, \tag{30}$$

where

$$\vec{V} = \left[u^* \ w^* \ \phi^* \ T^* \ \frac{du^*}{dx} \ \frac{dw^*}{dx} \ \frac{d\phi^*}{dx} \ \frac{dT^*}{dx} \right]^T , \tag{31}$$

and

$$W = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ A & 0 & 0 & 0 & 0 & B & C & 1 \\ 0 & D & E & F & G & 0 & 0 & 0 \\ 0 & H & M & N & P & 0 & 0 & 0 \\ 0 & Q & R & S & Z & 0 & 0 & 0 \end{bmatrix}. \tag{32}$$

4 SOLUTION OF THE VECTOR-MATRIX DIFFERENTIAL EQUATION

Let us now proceed to solve equation (30) by the eigenvalue approach proposed by Das et al. (2009). The characteristic equation of the matrix W takes the form

$$\lambda^8 - F_1\lambda^6 + F_2\lambda^4 - F_3\lambda^2 + F_4 = 0, \tag{33}$$

where

$$F_1 = A + D + BG + M + CP + S + Z,$$

$$F_2 = -(\mathrm{E} + CG)H + BGM - B\mathrm{E}P - FQ - GQ - NR - PR + BGS + \\ MS + CPS + A(D + M + S) + (-BF + M - CN)Z + D(M + CP + S + Z),$$

$$F_3 = -GMQ + \mathrm{E}NQ + CGNQ + \mathrm{E}PQ + GHR - DNR - BGNR - DPR - \\ A(\mathrm{E}H - DM + FQ + NR) - \mathrm{E}HS - CGHS + DMS + BGMS + \\ A(D + M)S + CDPS - B\mathrm{E}PS - \mathrm{E}HZ + DMZ - CDNZ + BeNZ + \\ F(-CPQ + HR + BPR + CHZ - M(Q + BZ)),$$

$$F_4 = -AFMQ + A\mathrm{E}NQ + AFHR - ADNR - A\mathrm{E}HS + ADMS.$$

The roots of the characteristic equation (33) which are also the eigenvalues of matrix W in the form

$$\lambda = \pm\lambda_1, \pm\lambda_2, \pm\lambda_3, \pm\lambda_4 . \tag{34}$$

The eigenvector

$$\vec{X} = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8]^T, \tag{35}$$

which are corresponding to eigenvalue λ can be calculated as

$$\begin{aligned} x_1 = &-\left(FR + \mathrm{E}\left(\lambda^2 - S\right)\right)\left(NQ + H\left(\lambda^2 - S\right)\right) + \\ &\left(-FQ + \left(D - \lambda^2\right)\left(-\lambda^2 + S\right)\right)\left(\lambda^4 - NR + MS - \lambda^2\left(M + S\right)\right), \end{aligned} \tag{36}$$

$$x_2 = \lambda\left(\lambda^2 - S\right)\begin{pmatrix} G\left(\lambda^4 - NR + MS - \lambda^2\left(M + S\right)\right) + \\ F\left(PR + \lambda^2 Z - MZ\right) + \mathrm{E}\left(\lambda^2 P - PS + NZ\right) \end{pmatrix}, \tag{37}$$

$$x_3 = \lambda\left(\lambda^2 - S\right)\begin{pmatrix} G\left(NQ + H\left(\lambda^2 - S\right)\right) + P\left(-FQ - \left(D - \lambda^2\right)\left(\lambda^2 - S\right)\right) + \\ \left(FH + \left(-D + \lambda^2\right)N\right)Z \end{pmatrix}, \tag{38}$$

$$x_4 = \lambda\left(\lambda^2 - S\right)\begin{pmatrix} G\left(\lambda^2 Q - MQ + HR\right) + \mathrm{E}\left(PQ - HZ\right) - \\ \left(D - \lambda^2\right)\left(PR + \lambda^2 Z - MZ\right) \end{pmatrix}, \tag{39}$$

$$x_5 = \lambda x_1, \quad x_6 = \lambda x_2, \quad x_7 = \lambda x_3, \qquad x_8 = \lambda x_4. \tag{40}$$

From equations (36)-(40) we can easily calculate the eigenvector $\vec{X}_j$, corresponding to eigenvalue $\lambda_j, j = 1,2,3,4,5,6,7,8.$

For further reference, we shall use the following notations:

$$\begin{aligned} &\vec{X}_1 = \left[\vec{X}\right]_{\lambda=-\lambda_1}, \quad \vec{X}_2 = \left[\vec{X}\right]_{\lambda=-\lambda_2}, \quad \vec{X}_3 = \left[\vec{X}\right]_{\lambda=-\lambda_3}, \quad \vec{X}_4 = \left[\vec{X}\right]_{\lambda=-\lambda_4}, \\ &\vec{X}_5 = \left[\vec{X}\right]_{\lambda=\lambda_1}, \quad \vec{X}_6 = \left[\vec{X}\right]_{\lambda=\lambda_2}, \quad \vec{X}_7 = \left[\vec{X}\right]_{\lambda=\lambda_3}, \quad \vec{X}_8 = \left[\vec{X}\right]_{\lambda=\lambda_4}. \end{aligned} \tag{41}$$

The solution of equation (30) can be given by:

$$\vec{V} = \sum_{j=1}^{8} A_j \vec{X}_j e^{\lambda_i x} = A_1 \vec{X}_1 e^{-\lambda_1 x} + A_2 \vec{X}_2 e^{-\lambda_2 x} + A_3 \vec{X}_3 e^{-\lambda_3 x} + A_4 \vec{X}_4 e^{-\lambda_4 x}, \tag{42}$$

where the terms containing exponentials of growing nature in the space variable x have been discarded due to the regularity condition of the solution at infinity, A_1, A_2, A_3 and A_4 are constants to be determined from the boundary condition of the problem. Thus, the field variables can be written for $x \geq 0,\ t > 0,\ -\infty \leq z \leq \infty$, as:

$$u(x,z,t) = e^{(\omega t + ibz)} \sum_{j=1}^{4} A_j x_5^j e^{-\lambda_j x}, \tag{43}$$

$$w(x,z,t) = e^{(\omega t + ibz)} \sum_{j=1}^{4} A_j x_6^j e^{-\lambda_j x}, \tag{44}$$

$$\phi(x,z,t) = e^{(\omega t + ibz)} \sum_{j=1}^{4} A_j x_7^j e^{-\lambda_j x}, \tag{45}$$

$$T(x,z,t) = e^{(\omega t + ibz)} \sum_{j=1}^{4} A_j x_8^j e^{-\lambda_j x}, \tag{46}$$

$$\sigma_{xx}(x,z,t) = e^{(\omega t + ibz)} \sum_{j=1}^{4} \left(-2b_1 \lambda_j x_5^j + (1-2b_1)\left(-\lambda_j x_5^j + ibx_6^j\right) + b_3 x_7^j - x_8^j \right) A_j e^{-\lambda_j x}, \tag{47}$$

$$\sigma_{zz}(x,z,t) = e^{(\omega t + ibz)} \sum_{j=1}^{4} \left(2b_1 ibx_6^j + (1-2b_1)\left(-\lambda_j x_5^j + ibx_6^j\right) + b_3 x_7^j - x_8^j \right) A_j e^{-\lambda_j x}, \tag{48}$$

$$\sigma_{xz}(x,z,t) = b_1 e^{(\omega t + ibz)} \sum_{j=1}^{4} \left(ibx_5^j - \lambda_j x_6^j \right) A_j e^{-\lambda_j x}, \tag{49}$$

To complete the solution we have to know the constants A_1, A_2, A_3 and A_4, so we will use the following boundary conditions.

5 APPLICATION

We will consider that the bounding plane of the medium $x = 0$ traction free and has a constant heat flux with constant strength.

Thus, the appropriate boundary conditions are

$$\sigma_{xx}(0,z,t) = \sigma_{xz}(0,z,t) = 0 , \tag{50}$$

$$\frac{\partial \phi}{\partial x}(0,z,t) = 0 , \tag{51}$$

and

$$q(0,z,t) = q_o , \tag{52}$$

which gives

$$\frac{\partial T^*(0,z,t)}{\partial x} = -Lq_o , \tag{53}$$

where q_o is the strength of the heat flux and it is constant

From the boundary conditions (50), (51) and (53), we obtain

$$\begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} & H_{13} & H_{14} \\ H_{21} & H_{22} & H_{23} & H_{24} \\ H_{31} & H_{23} & H_{33} & H_{34} \\ H_{41} & H_{42} & H_{43} & H_{44} \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ 0 \\ -Lq_o \end{pmatrix}, \tag{54}$$

where the element of matrix H_{rs} are given by:

$$H_{11} = -2b_1\lambda_1 x_5^1 + (1-2b_1)\left(-\lambda_1 x_5^1 + ibx_6^1\right) + b_3 x_7^1 - x_8^1,$$

$$H_{12} = -2b_1\lambda_2 x_5^2 + (1-2b_1)\left(-\lambda_2 x_5^2 + ibx_6^2\right) + b_3 x_7^2 - x_8^2,$$

$$H_{13} = -2b_1\lambda_3 x_5^3 + (1-2b_1)\left(-\lambda_3 x_5^3 + ibx_6^3\right) + b_3 x_7^3 - x_8^3,$$

$$H_{14} = -2b_1\lambda_4 x_5^4 + (1-2b_1)\left(-\lambda_4 x_5^4 + ibx_6^4\right) + b_3 x_7^4 - x_8^4,$$

$$H_{=21} = ibx_5^1 - \lambda_1 x_6^1, \quad H_{22} = ibx_5^2 - \lambda_2 x_6^2, \quad H_{23} = ibx_5^3 - \lambda_3 x_6^3, \quad H_{24} = ibx_5^4 - \lambda_4 x_6^4,$$

$$H_{31} = -\lambda_1 x_7^1, \quad H_{32} = -\lambda_2 x_7^2, \quad H_{33} = -\lambda_3 x_7^3, \quad H_{34} = -\lambda_4 x_7^4,$$

$$H_{41} = -\lambda_1 x_8^1, \quad H_{42} = -\lambda_2 x_8^2, \quad H_{43} = -\lambda_3 x_8^3, \quad H_{44} = -\lambda_4 x_8^4.$$

6 NUMERICAL RESULTS AND DISCUSSIONS

Following Kumar and Devi (2011), magnesium material was chosen for purposes of numerical evaluations. The physical data are given as

$$\begin{aligned}
&\lambda_o = 2.17\times10^{10}\,(\mathrm{N})(\mathrm{m}^{-2});\ \mu_o = 3.278\times10^{10}\,(\mathrm{N})(\mathrm{m}^{-2});\ \omega_o = 2; a = 1.2; b = 1.3;\\
&\rho = 1.74\times10^{3}\,(\mathrm{kg})(\mathrm{m})^{-3}; c_E = 1.04\times10^{3}\,(J)(\mathrm{kg})^{-1}(K)^{-1}; T_o = 298(K);\\
&\gamma_o = 2.68\times10^{6}\,(\mathrm{N})(\mathrm{m}^{-2})(K)^{-1}; k_o^* = 1.7\times10^{2}\,(W)(m^{-1})(K)^{-1}; r^* = 100; \nu = 50;\\
&\psi_o = 1.753\times10^{-15}\,(\mathrm{m}^2); \omega_{01} = 0.0787\times10^{-3}\,(\mathrm{N})(\mathrm{m})^{-2}(s)^{-1}; \xi_{1o} = 1.475\times10^{10}\,(\mathrm{N})(\mathrm{m}^{-2});\\
&b_o = 1.13849\times10^{10}\,(\mathrm{N})(\mathrm{m}^{-2}); m_o = 2\times10^{6}\,(\mathrm{N})(\mathrm{m}^{-2})(K)^{-1}; \alpha_o = 3.688\times10^{-5}\,(\mathrm{N}).
\end{aligned}$$

Figures 1-8 represent the temperature distribution, displacement u distribution, displacement w distribution, the change in volume fraction field of voids distribution, the strain distribution, the stress σ_{xx} distribution, the stress σ_{xz} distribution and the stress σ_{zz} distribution respectively at constant time $t = 2.5$ and constant $z = 0.5$ with different values of the fractional parameter $\alpha = 0.5, 1.0, 1.5$ which express for the weak thermal conductivity, normal thermal conductivity and super thermal conductivity respectively.

In figure 1, the fractional order parameter α has a significant effect on the temperature distribution, where increasing on α causes increasing on T and the rate of change of T with respect to x also increases when α increases which is compatible with the definition of the thermal conductivity.

In figures 2 and 3, the fractional order parameter α has a significant effect on the displacement u and w distributions, where increasing on α causes increasing on the absolute values of u and w, and the rate of change of them with respect to x also increase when α increases which is compatible with the definition of the thermal conductivity.

Figure 4 shows the variation of change in volume fraction field respect to x with different value of the fractional order parameter α. It is seen that, the volume fraction starts with its maximum value at the origin and decreases until attaining zero. The fractional order parameter α has a significant effect on the change in volume fraction field of voids distribution ϕ, where decreases with the decrease in the value of fractional parameter α.

In figure 5, the fractional order parameter α has a significant effect on strain distribution e, where increasing on α causes increasing on e, and the rate of change of e with respect to x also increases when α increases which is compatible with the definition of the thermal conductivity.

In figures 6-8, the fractional order parameter α has significant effects on all components of the stress distribution, where increasing on α causes increasing the absolute values of the stresses, and the rate of change of them with respect to x also increase when α increases which is compatible with the definition of the thermal conductivity.

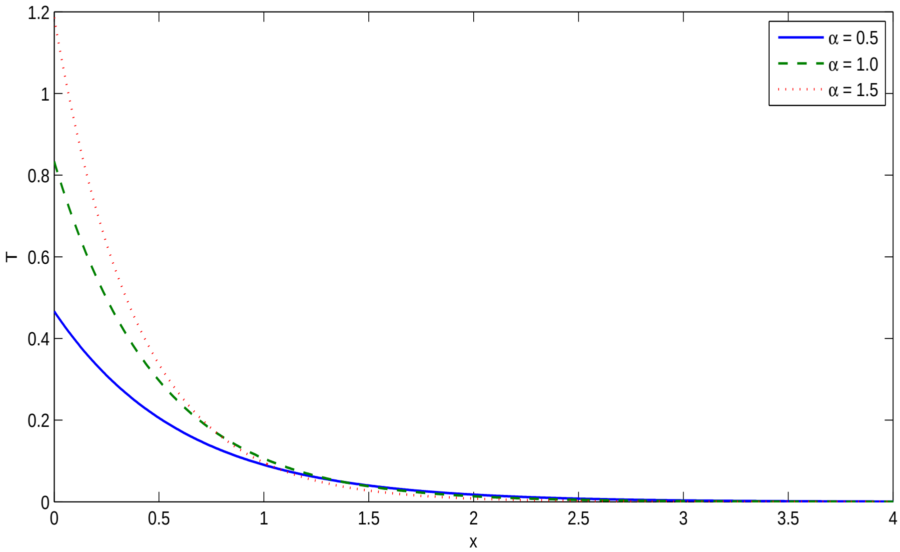

Figure 1: The temperature distribution with different value of the fractional parameter.

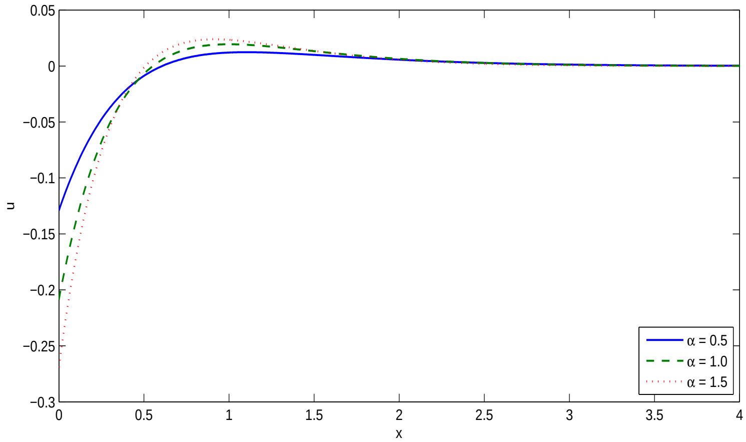

Figure 2: The displacement u distribution with different value of the fractional parameter.

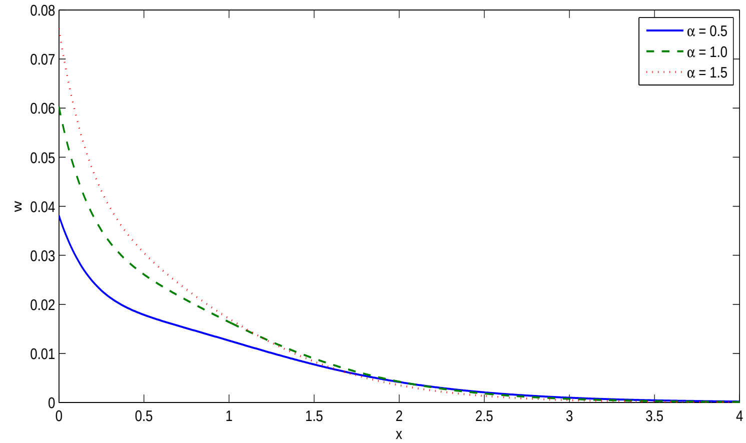

Figure 3: The displacement w distribution with different value of the fractional parameter.

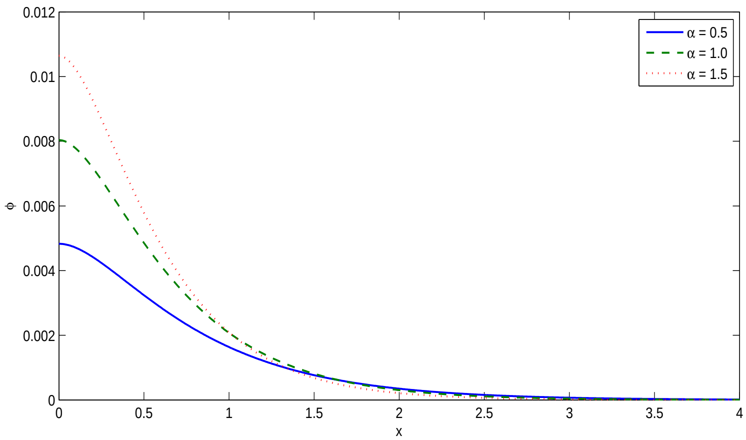

Figure 4: The change in volume fraction field of voids distribution with different value of the fractional parameter.

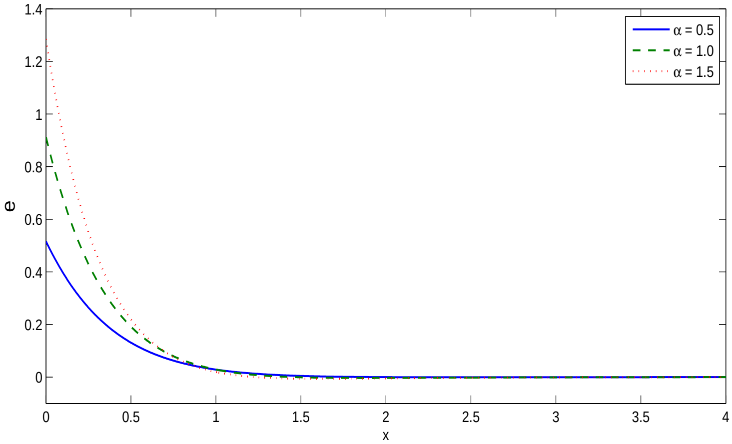

Figure 5: The strain distribution with different value of the fractional parameter.

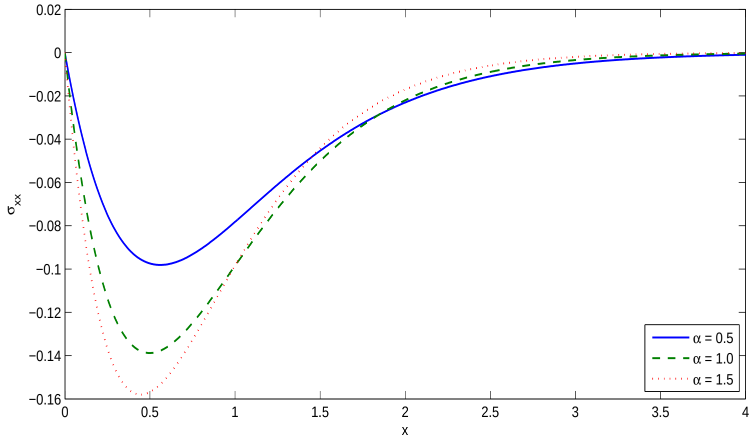

Figure 6: The stress σ_{xx} distribution with different value of the fractional parameter.

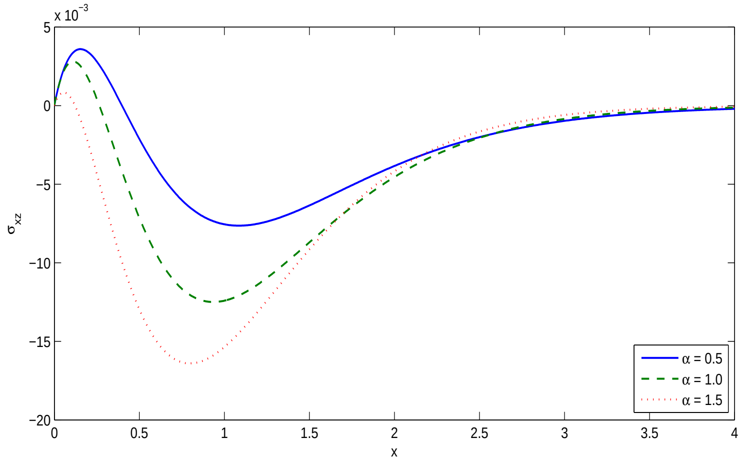

Figure 7: The stress σ_{xz} distribution with different value of the fractional parameter.

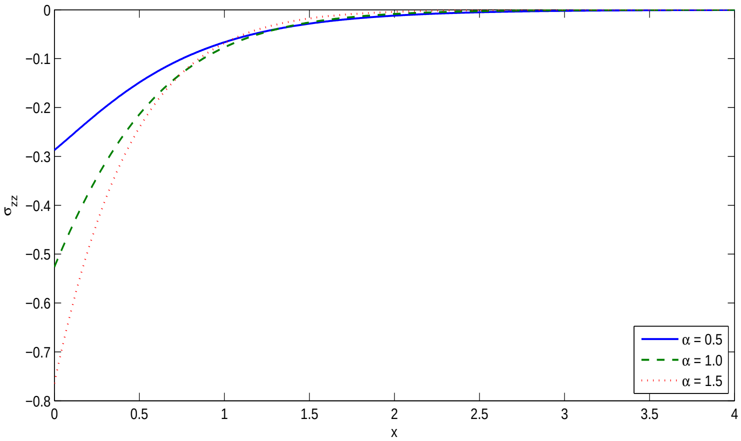

Figure 8: The stress σ_{zz} distribution with different value of the fractional parameter.

7 CONCLUSION

In this work, the effect of the fractional order of the temperature, displacement components, the stress components, changes in volume fraction field and temperature distribution have been studying for a two-dimensional problem of a porous material is considered within the context of the fractional order generalized thermoelasticity theory with one relaxation time. We found that, the fractional order parameter has significant effects on all the studied fields and the results supporting the definition of the classification of the thermal conductivity of the materials to three types; weak, normal and super conductivity.

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