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Fatigue Equivalent Stress State Approach Validation in Non-conservative Criteria: a Comparative Study

Abstract

This paper is concerned with the fatigue prediction models for estimating the multiaxial fatigue limit. An equivalent loading approach with zero out-of-phase angles intended for fatigue limit evaluation under multiaxial loading is used. Based on experimental data found in literatures, the equivalent stress is validated in Crossland and Sines criteria and predictions compared to the predictions of existing multiaxial fatigue; results over 87 experimental items show that the equivalent stress approach is very efficient.

Keywords

High cycle fatigue, criteria, proportional loading, non-proportional loading, fatigue limit.

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1 INTRODUCTION

Most industrial applications involve high-cycle fatigue problems since they are designed to operate in the elastic region for a long machine life (Papadopoulos, 2001). According to Budynas (2004) there is no universal approach or parameters that can strongly correlate a multiaxial fatigue process with its life, due to the complexity of factors in loading, material and environment.

The engineering approach to solving the multiaxial fatigue problem is to find the equivalent stress (Braccesi et al. 2008; Tsapi and Soh, 2013); however, design engineers are often faced with difficulties in applying these approaches to multiaxial fatigue design of high cycle fatigue components, as in Budynas and Nisbett (2011); Cristofori et al. (2008); Li et al. (2009). One difficulty is that most of the existing multiaxial fatigue criteria can only provide good predictions for proportional (in-phase) loading, like reported by Weber et al (2001); Goncalves et al. (2005); Papuga (2011). Another difficulty involves their implementation for general complex multiaxial fatigue loading that permits arbitrary stress histories such as the variation of the principal stress directions during the stress cycle history (Lambert et al. (2010); Soh et al. (2013); Li et al. (2015)). In this case, it is hard to track the path of each principal stress direction at each moment in time.

Many authors assume that the crack initiation is governed by the second invariant of the deviatoric stress tensor and the hydrostatic stress (Cristofori et al. 2008; Li et al. (2009); Cristofori et al. 2011; Jingran et al. 2015). Among current multiaxial fatigue criteria, stress invariant-based criteria, such as the Crossland (1956) and Sines (1955) models are attractive for engineering design of high cycle fatigue components because it is easy-to-use. Due to their nature, stress invariant based criteria are generally considered to be efficient from a computational point of view and usable in advanced design methodologies, such as computer aided design if accurate and reliable (Cristofori et al. 2008). The decision to improve on the predicting capacity of stress invariant based criteria derives from the requirement of minimum computational times that a virtual simulation instrument must involve. Crossland and Sines stress invariant based criteria can provide good predictions for proportional and non-proportional loads. However, they are not conservative for general complex multiaxial fatigue loading as shown in the works of Weber et al. (2001); Papadopoulos et al. (1997); Goncalves et al. (2005); Papuga (2011).

In this paper, the Tsapi and Soh (2013) equivalent multiaxial stress state with zero out-of-phase angles is reviewed and compared to well known fatigue criteria. The work starts with the presentation of a method of assessing multiaxial fatigue criteria [section 2]. The non-conservative predictions obtained from Crossland and Sines models, using 87 experimental items are presented, and then the procedure for the account of the mobility of principal stress direction in non-conservative multiaxial fatigue criteria is presented [section 3]. Results of validations and comparisons with experimental data are given [section 4]. Conclusions are finally presented [Section 5].

2 THEORETICAL BACKGROUND

2.1 Experimental Data

In the last 70 years many researchers have performed experimental tests on multiaxial fatigue, especially under both in-phase and out-of-phase loading. The experimental data used in this work were obtained from several different sources. The test data used relates above all to harmonic simultaneous loading of smooth specimens in bending and torsion, combinations of rotating bending or push–pull with torsion. The synthesis of the constants of the test data used for the comparisons is reported in Table 1.

Material	References	Number of data	f_{-1} (MPa)	t_{-1} (MPa)
hard steel	Nishihara and Kawamoto (1945)	10	313.9	196.2
34Cr4 steel (400)	Heidenreich et al. (1983)	11	410	251
45MO steel	Lempp (1977)	9	398	260
30NCD16	Froustey and Lasserr (1989)	24	695	415
XC48	Simbürger (1975)	9	463	275
30NCD16	Froustey and Lasserr (1989)	11	690	428
Acier doux	Nishihara and Kawamoto (1945)	8	235	137
34Cr4	Heidenreich and Zenner (1979)	5	415	259

Table 1: Synthesis of the fatigue properties for the considered materials.

2.2 Fatigue Methods of Assessment

Several methods of assessing multiaxial fatigue criteria are used in the literature. Some of the methods are based on the use of the safety factor, while others make use of the fatigue strength error index.

The fatigue strength method consists in defining fatigue strength (E) for the criterion. When the fatigue limit is attained, the fatigue strength yields the value one, for a constant amplitude multiaxial fatigue load. The fatigue strength error index ΔI expressed in percentage ΔI (%) is defined by Goncalves et al. (2005) as:

$$\Delta I = (E - 1) \times 100\% \tag{1}$$

- When the value of ΔI is close to zero, fatigue limits predictions are exact.
- If ΔI is positive predictions are conservative.
- While for a negative ΔI, predictions are said to be non-conservative, the criterion does not predict fracture, although it did occur in the experiment.

By using the values of the fatigue strength error index, it is possible to evaluate the fraction of the total number of experimental fatigue tests for which satisfactory predictions are given by a fatigue criterion.

It is common practice to sum the results of predictions in a histogram, (see Papadopoulos et al. (1997); Goncalves et al. (2005)). This kind of prediction evaluation is sufficiently representative and readily understandable.

3 ACCOUNTING THE MOBILITY OF THE PRINCIPAL STRESS DIRECTIONS

3.1 Assessment of Crossland and Sines Criteria

The Crossland and Sines criteria are among the oldest and best-known fatigue criteria. The fatigue strength of the Crossland criterion proposed in 1956, Crossland (1956), is in the form

$$E_c = \frac{\sqrt{J_{2a}} + \alpha_c \sigma_{H,max}}{\beta_c} \tag{2}$$

While the fatigue strength of the Sines criterion, Sines (1955) is

$$E_s = \frac{\sqrt{J_{2a}} + \alpha_s \sigma_{H,mean}}{\beta_s} \tag{3}$$

Material parameters α_c, βc, α_s, and β_s, are derived from three simple uniaxial tests: the fully reversed bending f_{-1}, the fully reversed torsion limit t_{-1}, and the fully repeated bending limit f_0.

$$\beta_c = t_{-1} \; ; \; \alpha_c = 3\left(\frac{t_{-1}}{f_{-1}} - \frac{1}{\sqrt{3}} \right) \tag{4}$$

$$\beta_s = t_{-1} \; ; \; \alpha_c = 6\frac{t_{-1}}{f_0} - \sqrt{3} \tag{5}$$

In Equations 2 and 3, $\sqrt{J_{2a}}$ is the amplitude of the square root of the second invariant of the alternating deviator stress tensor; $\sigma_{H,max}$ is the maximum hydrostatic stress, $\sigma_{H,mean}$ is the mean hydrostatic stress.

A comparison of predictions through Crossland and Sines criteria over 87 experimental items of Tables 2-9, with 17 of them with fixed principal stress directions, was made and the values of the fatigue strength error index reported in Tables 10-13. With the type of loadings; †: mean stress and out of phase induced mobile principal stress directions; *: out of phase induced mobile principal stress directions; §: mean stress induced mobile principal stress directions.

Test number	$\sigma_{xx,a}$	$\sigma_{xx,m}$	$\tau_{xy,a}$	$\tau_{xy,m}$	$\varphi(°)$
1-1	138.1	0	167.1	0	0
1-2*	140.4	0	169.9	0	30
1-3*	145.7	0	176.3	0	60
1-4*	150.2	0	181.7	0	90
1-5	245.3	0	122.6	0	0
1-6*	249.7	0	124.8	0	30
1-7*	252.4	0	126.2	0	60
1-8*	258.0	0	129.0	0	90
1-9	299.1	0	62.8	0	0
1-10*	304.5	0	63.9	0	90

Table 2: Experimental fatigue data of hard steel (f_{-1}=313.9MPa, t_{-1}=196.2 MPa), from Nishihara and Kawamoto (1945).

Test number	$\sigma_{xx,a}$	$\sigma_{xx,m}$	$\tau_{xy,a}$	$\tau_{xy,m}$	$\varphi(°)$
2-1	314	0	157	0	0
2-2*	315	0	158	0	60
2-3*	316	0	158	0	90
2-4*	315	0	158	0	120
2-5*	224	0	224	0	90
2-6§	316	0	158	158	0
2-7†	314	0	157	157	60
2-8†	315	0	158	158	90
2-9§	279	279	140	0	0
2-10†	284	284	142	0	90
2-11†	212	212	212	0	90

Table 3: Experimental fatigue data of 34Cr4 steel (400)
(f_{-1}=410MPa, t_{-1}=256 MPa, R_m=710 MPa), from Heidenreich et al. (1983).

Test number	$\sigma_{xx,a}$	$\sigma_{xx,m}$	$\tau_{xy,a}$	$\tau_{xy,m}$	$\varphi(°)$
3-1	328	0	157	0	0
3-2*	286	0	137	0	90
3-3	233	0	224	0	0
3-4*	213	0	205	0	90
3-5§	266	0	128	128	0
3-6†	283	0	136	136	90
3-7†	333	0	160	160	120
3-8§	280	280	134	0	0
3-9†	271	271	130	0	90

Table 4: Experimental fatigue data of 45MO steel
(f_{-1}=398MPa, t_{-1}=260MPa, f_0=620MPa), from Lempp (1977).

Test number	$\sigma_{xx,a}$	$\sigma_{xx,m}$	$\tau_{xy,a}$	$\tau_{xy,m}$	$\varphi(°)$
4-1	485	0	280	0	0
4-2*	480	0	277	0	90
4-3§	480	300	277	0	0
4-4†	480	300	277	0	45
4-5†	470	300	271	0	60
4-6†	473	300	273	0	90
4-7§	590	300	148	0	0
4-8†	565	300	141	0	45
4-9†	540	300	135	0	90
4-10§	211	300	365	0	0
4-11§	455	300	263	200	0
4-12†	465	300	269	200	90
4-13§	0	450	395	0	0
4-14§	415	450	240	0	0
4-15†	405	450	234	0	90
4-16§	0	600	350	0	0
4-17§	370	600	214	0	0
4-18†	390	60	225	0	90
4-19	630	300	0	0	0
4-20	550	450	0	0	0
4-21	525	510	0	0	0
4-22	535	600	0	0	0
4-23§	0	300	395	0	0
4-24†	222	300	385	0	90

Table 5: Experimental fatigue data of 30NCD16
(f_{-1}=695MPa, t_{-1}=415MPa, f_0=1040MPa), from Froustey and Lasserr (1989).

Test number	$\sigma_{xx,a}$	$\sigma_{xx,m}$	$\sigma_{xy,a}$	$\sigma_{xy,m}$	$\varphi(°)$
5-1	0	0	261	261	0
5-2	364	0	209	0	0
5-3*	332	0	191	0	30
5-4*	315	0	181	0	60
5-5*	328	0	189	0	90
5-6§	300	300	173	0	0
5-7 †	268	268	154	0	90
5-8§	319	0	183	183	0
5-9 †	294	0	169	169	90

Table 6: Experimental fatigue data of XC48
(f_{-1}= 463MPa, t_{-1}=275MPa, f_0=800MPa), from Simbürger (1975).

Test number	$\sigma_{xx,a}$	$\sigma_{xx,m}$	$\sigma_{xy,a}$	$\sigma_{xy,m}$	$\varphi(°)$
6-1*	474	0	265	0	90
6-2†	220	299	368	0	90
6-3 †	470	299	261	0	90
6-4 †	527	287	129	0	90
6-5†	433	472	240	0	90
6-6†	418	622	234	0	90
6-7§	451	294	250	191	0
6-8†	462	294	250	191	90
6-9†	474	294	265	0	45
6-10†	464	294	259	0	60
6-11†	554	287	135	0	45

Table 7: Experimental fatigue data of 30NCD16
(f_{-1}= 690MPa, t_{-1}= 428MPa, f_0= 1090MPa), from Froustey and Lasserr (1989).

Test number	$\sigma_{xx,a}$	$\sigma_{xx,m}$	$\sigma_{xy,a}$	$\sigma_{xy,m}$	$\varphi(°)$
7-1	100	0	121	0	0
7-2	180	0	90	0	0
7-3	213	0	45	0	0
7-4*	104	0	125	0	60
7-5*	109	0	132	0	90
7-6*	191	0	96	0	60
7-7*	201	0	101	0	90
7-8*	230	0	48	0	90

Table 8: Experimental fatigue data of “Acier doux”
(f_{-1}= 235MPa, t_{-1}= 137MPa, f_0= 342MPa), from Nishihara and Kawamoto (1945).

Test number	$\sigma_{xx,a}$	$\sigma_{xx,m}$	$\sigma_{xy,a}$	$\sigma_{xy,m}$	$\varphi(°)$
8-1§	280	0	140	280	0
8-2†	309	0	155	309	180
8-3§	320	-160	160	160	0
8-4 †	350	-175	175	175	180
8-5	350	-350	275	175	0

Table 9: Experimental fatigue data of 34Cr4
(f_{-1}= 415MPa, t_{-1}= 259MPa, f0= 648MPa), from Heidenreich and Zenner (1979).

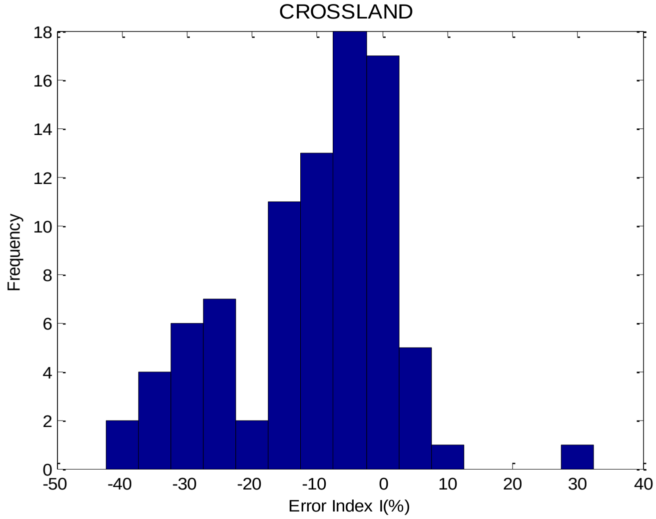

Figure 1: Histogram of fatigue index error for Crossland criterion.

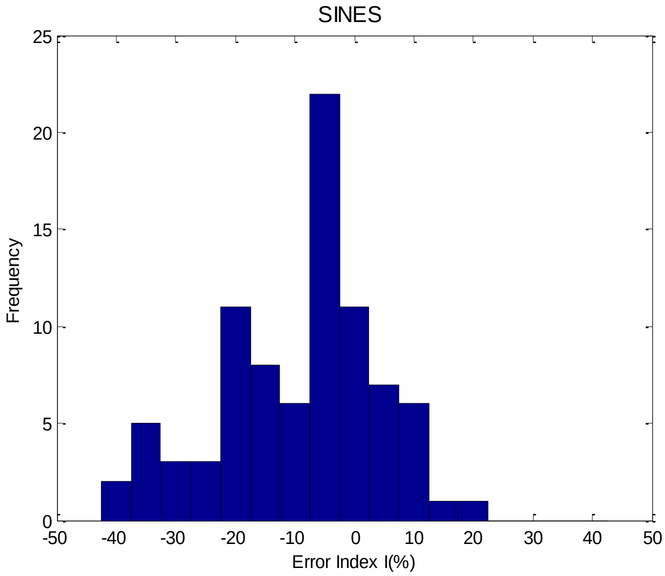

Figure 2: Histogram of fatigue index error for Sines criterion.

The analysis of results obtained on the comparison between the experimental fatigue limit and the calculated ones obtained from Crossland and Sines fatigue criteria as presented in Figure 1 and Figure 2 showed clearly their poor predictions capacity for stress states presenting mobile principal stress directions; with majority of the error index out of an error interval of 10%. Thus from theoretical considerations it appears that these criteria do not account for the influence of changes of the principal stress directions under a multiaxial loading on material fatigue. Furthermore the results are non-conservative (unsafe); thus making their usage by design and manufacturing engineers to be dangerous.

3.2 Equivalent Stress Approach

Since Crossland and Sines criteria are unsafe when used for complex stress states; Tsapi and Soh (2013) proposed a modification of the stress state for better predictions of experimental data when the multiaxial stress states presents mobility of principal stress directions.

Our approach consist of transforming complex stress states to simple equivalent stress states in such a way that the majority of the absolute error index should be within an error interval of 5%. The equivalent stress state is defined with zero out-of-phase angles for the stress components with a phase difference. The equivalent stress is normalized through parameter β, in such a way that Crossland and Sines predictions are identical in case of in phase loadings. Therefore, they associate to the shear stress τ_{xy}, equation (6), defined at the generic material point M an equivalent shear stress τ_{xyeq} equation (7).

$$\tau_{xy} = \tau_{xym} + \tau_{xya} \sin(\omega t - \varphi) \tag{6}$$

$$\tau_{xyeq} = \tau_{xym} + \tau_{xya} \left(|\cos\beta + \sin\beta| \right)^{n} \sin(\omega t) \tag{7}$$

$$\beta = \left(\delta_{0\varphi} - 1 + \varphi \right) \tag{8}$$

where n is a real; $\delta_{0\varphi}$ is the Kronecker delta symbol defined for an arbitrary phase shift angle φ as

$$\delta_{0\varphi} = \begin{cases} 1 \text{ if } \varphi = 0 \\ 0 \text{ if } \varphi \neq 0 \end{cases} \tag{9}$$

The equivalent stress they proposed is used in the implementation of the non-conservative Crossland and Sines criteria. Crossland* and Sines* criteria now represent respectively Crossland and Sines criteria when applied using the equivalent stress state approach. The study of the influence of the equivalent stress state parameter *n* in predicting fatigue strength error index trough Crossland* and Sines* criteria is done for the following values of parameter n: 1/2, 1/4, 1/8, 1/16, 1/32.

The results obtained from 87 experimental ítems for several materials under in-phase and out-of-phase alternated bending and torsion loading are presented in Figures 3-4 and Tables 10-11. These predictions are compared with predictions from other well known fatigue criteria, Robert, Dan Van 2, Fogue, and Papadopoulos 2. The results of the comparison are presented in Figure 5 and Table 12.

$\lvert\Delta I\rvert$	Crossland predictions	Crossland* predictions				
		Values of parameter n				
		1/2	1/4	1/8	1/16	1/32
5%	37.9%	44.8%	52.9%	57.5%	59.8%	59.8%
7%	44.8%	59.8%	67.8.58%	72.4%	72.4%	72.4%
10%	56.3%	77.0%	79.3%	85.1%	85.1%	85.1%
14%	69.0%	88.5%	87.4%	92.0%	92.0%	92.0%

Table 10: Recapitulative of predictions with Crossland criterion.

|ΔI|	Sines predictions	Sines * predictions				
		Values of parameter n				
		1/2	1/4	1/8	1/16	1/32
5%	29.9%	39.1%	43.7%	29.9%	46.0%	43.7%
7%	44.8%	56.3%	63.2%	44.8%	65.5%	63.4%
10%	57.5%	73.6%	75.9%	57.5%	83.9%	80.5%
14%	65.5%	82.8%	87.4%	65.5%	93.1%	89.7%

Table 11: Recapitulative of predictions with Sines criterion.

|ΔI|	Comparison of various predictions							
	Robert	Dang Van2	Fogue	Papadopoulos 2	Sines	Crossland	Sines* (n=1/16)	Crossland* (n=1/32)
5%	39.1%	41.4%	66.7%	55.2%	29.9%	37.9%	46.0%	59.8%
7%	50.6%	49.4%	80.5%	71.3%	44.8%	44.8%	65.5%	72.4%
10%	56.3%	58.6%	89.7%	82.8%	57.5%	56.3%	83.9%	85.1%
14%	71.3%	70.1%	95.4%	93.1%	65.5%	69.0%	93.1%	92.0%

Table 12: Comparison of predictions of various criteria.

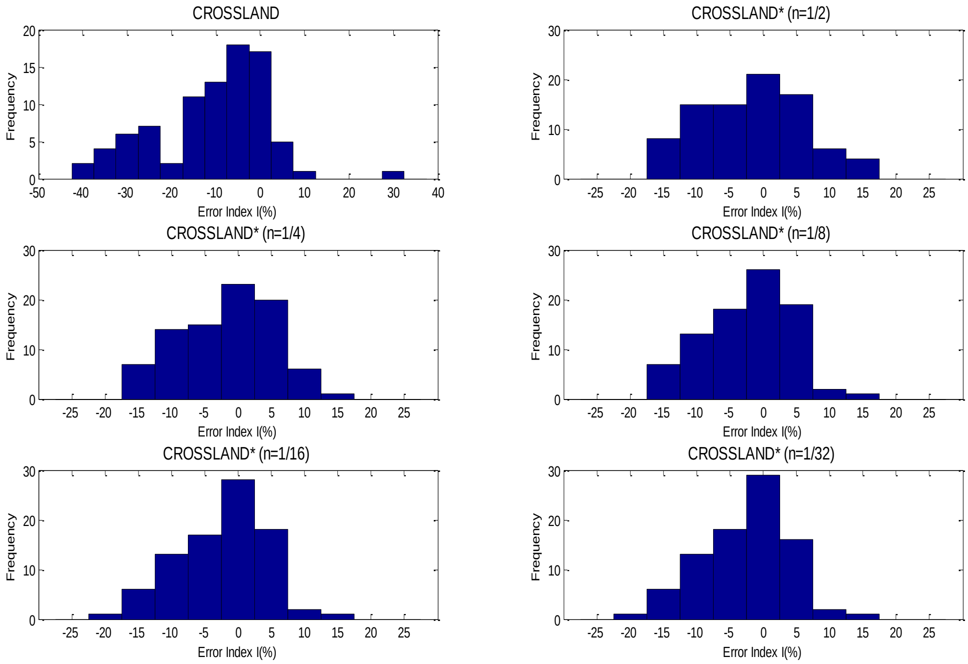

Figure 3: Histograms of fatigue index error for Crossland and modified Crossland criteria.

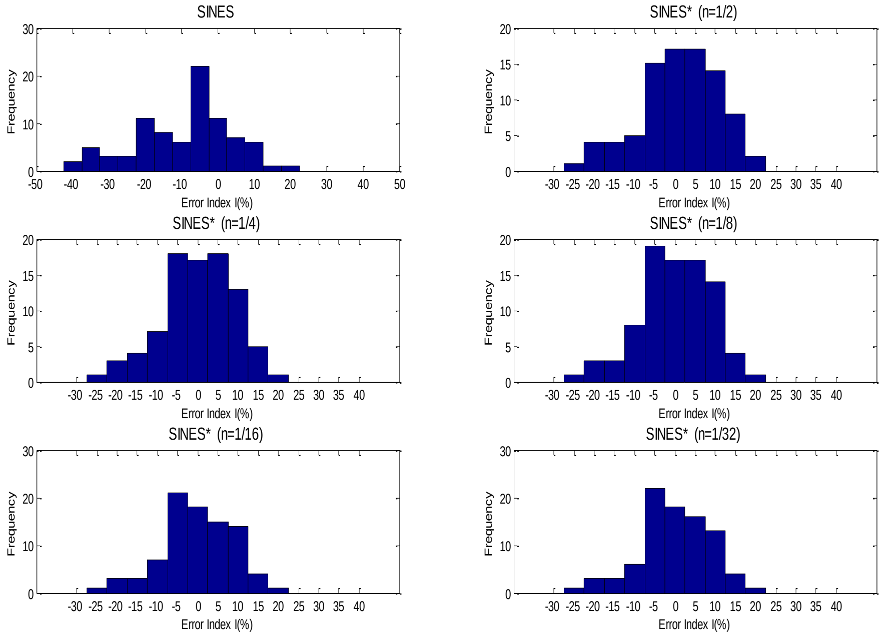

Figure 4: Histograms of fatigue index error for Sines and modified Sines criteria.

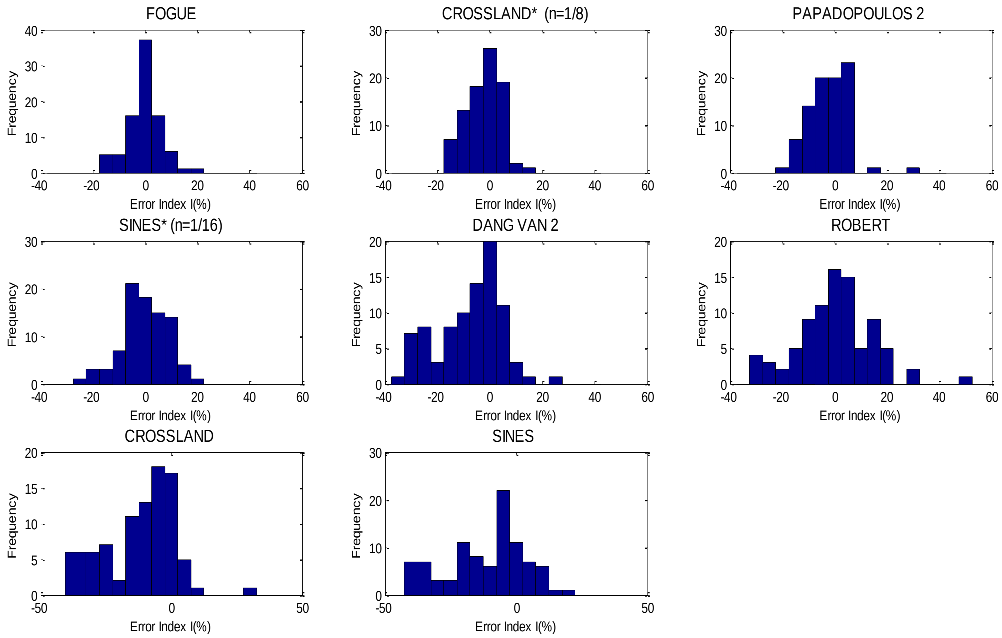

Figure 5: Histograms of fatigue index error for selected published criteria.

4 RESULTS

Figures 3–5 compares the overall values of fatigue index error obtained for individual criteria. The improved output of the Crossland and Sines solutions as shown in Figure 3 and Figure 4 when compared to other criteria improves the credibility of the equivalent stress method. The results of the Sines* Criterion is shifted to the conservative side of the fatigue index error should be noted. The Robert and Dan Van criteria provide the worst results. These methods should therefore be abandoned in cases of this kind.

The quality of the predictions deduced by the criteria examined can be evaluated through the absolute value of fatigue strength error index. The absolute value of the error index ($|\Delta I|$) is defined and the number of experimental tests whose error index falls in each interval of 5%, 7%, 10%, and 14% range are considered with respect to the values of parameter n and the total number of tests.

From a recapitulation of the results on 87 experimental items, 70 items with mobile principal stress directions, and presented in Tables 10-12, wherein we observed the following facts.

- With the proposed equivalent stress, for 10%, we passed from poor predictions with Crossland criterion, $|\Delta I|$=56.3%, to superior predictive capabilities with the proposed equivalent stress in the same error index interval, $|\Delta I|$=85.1% when n=1/32.
- With Sines criterion, for 10%, we passed from $|\Delta I|$=57.5%, to superior predictive capabilities with the equivalent stress method; $|\Delta I|$=83.9% when n=1/16.
- The influence of parameter n is seen to be meaningful for loadings with mobile principal stress directions. Especially when the causes of mobility of principal stress directions are both non-zero out-of-phase angles and mean stress effect.
- The closest value to zero of fatigue strength error index for both mobile and fixed principal stress directions are obtained respectively when n=1/32 and n=1/16 for Crossand* and Sines* criteria. And for all the values of parameter n, the Crossland* and Sines* criteria always provide better predictions estimates than the original formulations.
- From Table 13, the comparison of predictions at any material point, assumed to be the critical one for the component integrity, using Crossland, Sines, Robert, Dan Van 2, Fogue, Papadopoulos 2 criteria and the Crossland* and Sines* criteria; Fogue criterion is the best followed by the modified Crossland and Sines criteria for an error interval of 10% .

5 CONCLUSION

This paper reported on a new method for estimating more accurately the fatigue limit under complex multiaxial loading. Crossland and Sines criteria have been carefully examined in the present work. We observed that the Crossland and Sines criteria yielded good predictions for proportional loads. However, their applicability to out-of-phase loading or the loading with combined mean stress and out-of-phase load rendered very poor results. Our proposal accounting for the of out-of-phase induced mobility of principal stress direction was achieved through an equivalent stress.

Analysis of results obtained, shows that it was possible with the equivalent stress method to have superior predictive capabilities with Crossland and Sines criteria. Our approach can directly be applied by simply using explicit analytical formulations, which allow fatigue damage due to in-phase and out-of-phase biaxial loading to be calculated in a very simple way, by strongly improving the computational efficiency of the fatigue assessment process. The implementation of the Crossland and Sines criteria is very simple and efficient when compared to Fogue and Papadopoulos models.

The obtained results are promising, especially in light of the fact that, due to its nature, the equivalent stress method might also be successfully used to estimate fatigue damage in the presence of uniaxial/multiaxial variable amplitude fatigue loading. Unfortunately, more effort has still to be made to better investigate our method's accuracy in the presence of such complex loading histories.

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