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Generalized Magneto-thermo-microstretch Response of a Half-space with Temperature-dependent Properties During Thermal Shock

Abstract

The generalized magneto-thermoelastic problem of an infinite homogeneous isotropic microstretch half-space with temperature-dependent material properties placed in a transverse magnetic field is investigated in the context of different generalized thermoelastic theories. The upper surface of the half-space is subjected to a zonal time-dependent heat shock. By solving finite element governing equations, the solution to the problem is obtained, from which the transient magneto-thermoelastic responses, including temperature, stresses, displacements, microstretch, microrotation, induced magnetic field and induced electric field are presented graphically. Comparisons are made in the results obtained under different generalized thermoelastic theories to show some unique features of generalized thermoelasticity, and comparisons are made in the results obtained under three forms of temperature dependent material properties (absolute temperature dependent, reference temperature dependent and temperature-independent) to show the effects of absolute temperature and reference temperature. Weibull or Log-normal.

Keywords

generalized magneto-thermoelasticity; thermal shock; microstretch, finite element method; temperature-dependent

Qi-lin Xiong [a, b]

Xiao-geng Tian [c]

[a] Department of Mechanics, Huazhong University of Science & Technology,1037 Luoyu Road, Wuhan 430074，China.

[b] Hubei Key Laboratory of Engineering Structural Analysis and Safety Assessment, 1037 Luoyu Road, 430074, Wuhan, China

E-mail: xiongql@hust.edu.cn

[c] State Key Laboratory for Mechanical Structure Strength and Vibration, Xi'an Jiaotong University, Xi'an 710049, China.

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1 INTRODUCTION

In the scopes of heat energy deposition, nuclear engineering, nuclear scrap disposition, heat design in pipeline of supply, underground tunnel fireproofing and so on, the thermal shock occurs frequently as a result of the large temperature gradient and almost all the materials involved in these engineerings possess microstructure. materials with inner microstructures, such as concrete and various composites. The large temperature gradient makes different parts of the materials expand by different amounts and then cracks may be formed once the thermal stress exceeds the strength of the material (Lu and Fleck, 1998). Considering that the engineering materials are composed of inner microstructures, the investigation on thermoelastic problems of materials with inner microstructures under thermal shock has a significant meaning for the application of these materials.

Due to failure of classical elasticity in describing the mechanical behaviour of materials with inner microstructures (i.e., oriented particles, micro-cracks, foams etc.), the linear theory of micropolar and microstretch elasticity was introduced by Eringen (1965, 1966a, 1966b, 1971), which is used to describe deformation of elastic media with inner microstructures. Subsequently, Eringen introduced thermal effects to the microstretch theory, the thermomicrostretch theory was developed (Eringen, 1990).

For non-isothermal problems of elastic body above mentioned, Biot (1956) first developed the coupled theory of thermoelasticity. The equations of elasticity and of heat conduction are coupled which deals with the defect of the uncoupled theory. However, this theory shares the defect of infinite speed of propagation for heat conduction. To eliminate the inherent defect of Biot's theory, Lord and Shulman (1967) and Green and Lindsay (1972) developed generalized thermoelastic theories with one thermal relaxation time and two relaxation times, respectively. Recently A unified treatment of both Lord–Shulman and Green–Lindsay theories was presented by Ignaczak and Ostoja-Starzewski (2009). In the last three decades, many problems that have been solved were in the context of the theory of L–S or G-L. The researches can be divided into two categories: one is to study the transient responses in time domain, such as, Sherief et al. (1981, 1986, 2004), Youssef et al. (2004, 2005, 2009), Tian et al. (2005, 2006, 2011a, 2011b), Othman et al. (2007, 2014a, 2014b) and the other is to investigate the response of steady state in frequency domain, for example, Kumar et al. (2014a, 2014b) and Sihgh (2010, 2011). Here the main attention is paied to the transient responses in time domain for thermal shock problem.

Generally speaking, high temperature does occur in the thermal shock problem. Additionally, the interaction between electromagnetism and thermoelasticity frequently happens in the modern industry (Kolumban et al. 2006), especially in nuclear industry.

At high temperatures the material characteristics such as the modulus of elasticity, Poisson's ratio, the coefficient of thermal expansion, and the thermal conductivity are no longer constants (Lomakin, 1976). Youssef (2005a, 2005b) studied the generalized thermoelasticity of an infinite body with a cylindrical cavity and variable material properties and used the state–space approach for conducting magneto-thermoelastic medium with variable electrical and thermal conductivity. Recently, Allam et al. (2010) studied the electromagneto–thermoelastic interactions in an infinite perfectly conducting body with a spherical cavity and variable material properties. However, in the work above mentioned the absolute temperature (T) was instead of the reference temperature (T_0) in the form of temperature-dependent material properties to remove the nonlinearity of the problem for achieving the solution to the problem and the results may be not pratical. And thus the thermoelastic problem with temperature-dependent material properties remains to be investigated.

In addition, in spite of many studies about magneto-thermoelastic problems, hardly any attempt is made to investigate the transient magneto-thermo-microstretch problems of a half-space with temperature-dependent material properties under thermal shock. Most importantly, the finite speed of heat propagation remains to be depicted. In the above work, the solutions of problems are obtained by the inverse integrated transformation， the finite speed of heat propagation haven't been depicted. Tian et al. (2005, 2006) solved the generalized thermoelastic problems directly in the time domain by using finite element method (FEM), in the distribution of temperature there is a distinct temperature step on thermal wave front, implies the finite speed of propagation of thermal wave is depicted perfectly by using FEM.

Thus, in the present work the magneto-thermoelastic problem of an infinite homogeneous isotropic microstretch half-space with temperature-dependent material properties whose surface is subjected to a zonal time-dependent heat shock placed in a transverse magnetic field is investigated on the basis of different generalized thermoelastic theories by using FEM. The results, including temperature, stresses, displacements, microstretch, microrotation, induced magnetic field and induced electric field are obtained under different theories of generalized thermoelasticity, Lord and Shulman (L–S) theory and Green and Lindsay (G–L) theory. Comparisons are made in the results obtained under different theories and the differences in the results predicted by different theories have been depicted graphically. The material properties of elasticity are taken to be different functions of temperature. Finally, by taking an appropriate material, the results are plotted graphically to illustrate the problem and compared the results in the case of temperature-independent mechanical properties.

2 FORMULATION OF THE PROBLEM

Let us consider an infinite isotropic thermo-microstretch half-space with temperature-dependent mechanical and thermal properties whose upper surface(z=0) is irradiated by a zonal laser pulses placed in a transverse magnetic field $\vec{H} = (0, H_0, 0)$ as shown in Figure.1 and magnetic field with constant intensity H0 acts tangent to the bounding plane. A coordinate system (x, y, z) is used for this plane problem. The constant primary magnetic field Ho is acting in the direction of the y-axis.

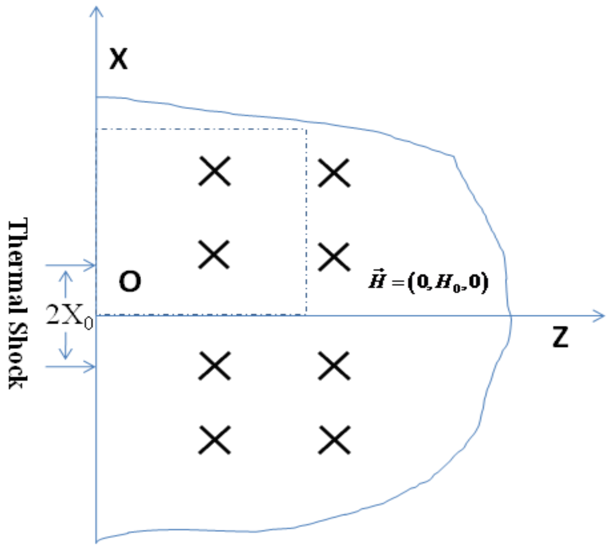

Figure 1: A half-space placed in a transverse magnetic field whose surface suffers thermal shock.

The Maxwell linear equations of electrodynamics for a homogenous isotropic conducting thermo-microstretch medium expanded by heat are the following:

$$\nabla \times \vec{h} = \vec{J} + \varepsilon_0 \dot{\vec{E}} \tag{1}$$

$$\nabla \times \vec{E} = -\mu_0 \dot{\vec{h}} \tag{2}$$

$$\vec{E} = -\mu_0 \left(\dot{\vec{u}} \times \vec{H} \right) \tag{3}$$

$$\nabla \cdot \vec{h} = 0 \tag{4}$$

Where $\vec{h}$ is the induced magnetic field vector, $\vec{J}$ is the current density vector, $\vec{E}$ is the induced electric field vector, $\vec{u}$ and $\vec{H}$ respectively are the displacement vector and the external magnetic field intensity vector, ε_0 and μ_0 respectively are the electric permeability and the magnetic permeability.

For a linear, homogenous, and isotropic thermo-microstretch medium, the generalized field equations can be presented in a unified form as (Eringen, 1990; Lord and Shulman, 1967; Green and Lindsay,1972):

Heat conduction equation

$$q_i + \tau_0 \dot{q}_i = -kT_{,i} \tag{5}$$

Energy conservation equation (without internal heat source)

$$\rho T_0 \dot{\eta} = -q_{i,i} \tag{6}$$

The equations of motion (without body force and with the Lorentz force, without body couples and without stretch force)

$$\sigma_{ji,j} + \mu_0 (\vec{J} \times \vec{H})_i = \rho \ddot{u}_i \tag{7}$$

$$\mu_{ji,j} + e_{ijk}\sigma_{jk} = \rho j \ddot{\varphi}_i \tag{8}$$

$$\lambda_{i,i} - \frac{\lambda_1}{3}\phi^* - \frac{\lambda_0}{3}u_{k,k} + \frac{\gamma_1}{3}\left(T + \tau_1 \dot{T}\right) = \frac{3\rho j}{2}\frac{\partial^2 \phi^*}{\partial t^2} \tag{9}$$

Constitutive equations

$$\sigma_{ij} = \lambda u_{k,k}\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) + 2\alpha\left(u_{j,i} - e_{ijk}\varphi_k\right) - \gamma\left[\left(T - T_0\right) + \tau_1 \dot{T}\right]\delta_{ij} + \lambda_0 \phi^* \delta_{ij} \tag{10}$$

$$\mu_{ij} = \varepsilon \varphi_{k,k}\delta_{ij} + \beta \varphi_{i,j} + \gamma' \varphi_{j,i} \tag{11}$$

$$\lambda_i = \alpha_0 \phi^*_{,i} \tag{12}$$

$$\rho\eta = \gamma u_{k,k} + \left(\frac{k}{\kappa T_0}\right)\left[\left(T - T_0\right) + \tau_0 \dot{T}\right] + \gamma_1 \phi^* \tag{13}$$

Where T、T_0 respectively the absolute temperature and reference temperature, σ_{ij}、μ_{ij}、λ_i respectively components of stress tensors, couple stress tensors and microstretch tensors. u_i、φ_i、ϕ^* respectively components of displacement vector, microrotation vector and the scalar microstretch. ρ density, $\gamma=(3\lambda+2\mu+2\alpha)\alpha_{t1}$、$\gamma_1=(3\lambda+2\mu+2\alpha)\alpha_{t2}$, λ、μ Lame's elastic constants, α_{t1}、α_{t2} the coefficients of linear thermal expansion, k thermal conductivity, c_E specific heat at constant strain. κ the thermal diffusivity($\kappa = k / \rho c_E$). τ、τ_0、τ_1 thermal relaxation times. α、ε、β、γ' micropolar material constants. α_0、λ_0、λ_1 microstretch material constants. δ_{ij}、e_{ijk} respectively Kronecker delta function and permutation symbol, j the micro-inertia coefficient, i(j、k) takes 1、2、3.

Setting $\tau_0=\tau_1=\tau_2=0$, we get field equations for the classical coupled theory of thermoelasticty, whereas when $\tau_0>0$ and $\tau_1=\tau_2=0$ the equations reduce to the Lord and Shulman (L-S) model and when $\tau_0=0$ but when τ_1 and τ_2 are nonvanishing the equations reduce to the Green and Lindsay (G-L) model. The classical coupled thermoelastic theory will not be discussed in details.

Now we suppose the form of temperature-dependent mechanical, thermal properties in the plate we will consider as follows:

$$\lambda\left(\mu,\gamma,\alpha,\lambda_0,\lambda_1\right)=\lambda_0\left(\mu_0,\gamma_0,\alpha_0,\lambda_{00},\lambda_{10}\right)f\left(T\right),\ k=k_0 f\left(T\right)$$

where k is the thermal conductivity, k_0, λ_0, μ_0, γ_0, λ_{00} and λ_{10} are considered to be constants; f(T) is given in a non-dimensional function of temperature. In the case of temperature-independent properties, f(T)=1. We will consider other two forms of temperature-dependent properties, i.e. the form of reference temperature-dependent properties (Youssef , 2005a, 2005b; Allam et al. 2010) ($f\left(T\right)=1-\alpha^* T_0$) and absolute temperature-dependent properties in this work ($f\left(T\right)=1-\alpha^* T$), where α^* is called the empirical material constant.

Substituting Eqs. (10-11) and temperature-dependent properties into the equation Eqs. (7-8), we can get

$$\begin{aligned}&f\left(T\right)_{,j}\begin{Bmatrix}\lambda_0 u_{k,k}\delta_{ij}+\mu_0\left(u_{i,j}+u_{j,i}\right)+2\alpha_0\left(u_{j,i}-e_{ijk}\varphi_k\right)\\-\gamma_0\left[\left(T-T_0\right)+\tau_1\dot{T}\right]\delta_{ij}+\lambda_{00}\phi^*\delta_{ij}\end{Bmatrix}+\\&f\left(T\right)\begin{Bmatrix}\lambda_0 u_{k,k}\delta_{ij}+\mu\left(u_{i,j}+u_{j,i}\right)+2\alpha_0\left(u_{j,i}-e_{ijk}\varphi_k\right)\\-\gamma_0\left[\left(T-T_0\right)+\tau_1\dot{T}\right]\delta_{ij}+\lambda_{00}\phi^*\delta_{ij}\end{Bmatrix}_{,j}+\mu_0(\vec{J}\times\vec{H})_i=\rho\ddot{u}_i\end{aligned}\tag{14}$$

$$f\left(T\right)_{,j}\begin{Bmatrix}\varepsilon_0\varphi_{k,k}\delta_{ij}+\beta_0\varphi_{i,j}\\+\gamma_0'\varphi_{j,i}\end{Bmatrix}+f\left(T\right)\begin{Bmatrix}\varepsilon_0\varphi_{k,k}\delta_{ij}+\beta_0\varphi_{i,j}\\+\gamma_0'\varphi_{j,i}\end{Bmatrix}_{,j}=\rho j\ddot{\varphi}_i\tag{15}$$

From Eqs. (5), (6), (13) and temperature-dependent properties, we obtain the equation of heat conduction

$$\left[k_0 f\left(T\right)T_{,i}\right]_{,i}=\left(1+\tau_0\frac{\partial}{\partial t}\right)\left[\frac{k_0 f\left(T\right)}{\kappa}\left(\dot{T}+\tau_2\ddot{T}\right)+\gamma_0 T_0 f\left(T\right)u_{k,k}+\gamma_{10}T_0\phi^*\right]\tag{16}$$

For the two dimensional problem we will consider, the components of the displacement, magnetic field are defined by

$$u_x=u\left(x,z,t\right),\ u_y=0,\ u_z=u\left(x,z,t\right),\ \varphi_x=\varphi_z=0,\ \ \varphi_y=\varphi\left(x,z,t\right),\ \ \phi^*=\phi^*\left(x,z,t\right),\ \vec{H}=\left(0,H_0,0\right)$$

The induced field components in the thermoelastic solid are obtained from Eqs. (1)–(4) in the forms

$$\vec{E} = \mu_0 H_0 \left(\dot{u}_z, 0, -\dot{u}_x\right)$$

$$\vec{h} = H_0 \left(0, e_{kk}, 0\right)$$

$$\vec{J} = \left(\left(H_0 e_{,z} - \varepsilon_0 \mu_0 H_0 \ddot{u}_z\right), 0, \left(-H_0 e_{,x} + \varepsilon_0 \mu_0 H_0 \ddot{u}_x\right)\right),$$

$$\vec{F} = \mu_0 \left(\vec{J} \times \vec{H}\right) = \mu_0 \left(H_0^2 \left(-e_{,x} + \varepsilon_0 \mu_0 \ddot{u}_x\right), 0, H_0^2 \left(e_{,z} - \varepsilon_0 \mu_0 \ddot{u}_z\right)\right)$$

The constitutive equations for a two dimensional plane problem are expressed as follows

$$\sigma_{xx} = \left[\lambda_0 u_{k,k} + 2\left(\mu_0 + \alpha_0\right)\frac{\partial u_x}{\partial x} - \gamma_0 T_0 \left(\theta + \tau_1 \dot{\theta}\right) + \lambda_{00}\phi^*\right] f\left(T\right)$$

$$\sigma_{zz} = \left[\lambda_0 u_{k,k} + 2\left(\mu_0 + \alpha_0\right)\frac{\partial u_z}{\partial z} - \gamma_0 T_0 \left(\theta + \tau_1 \dot{\theta}\right) + \lambda_{00}\phi^*\right] f\left(T\right)$$

$$\sigma_{xz} = \left[\mu_0 \left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right) + 2\alpha_0 \left(\frac{\partial u_z}{\partial x} + \varphi\right)\right] f\left(T\right)$$

$$\sigma_{zx} = \left[\mu_0 \left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right) + 2\alpha_0 \left(\frac{\partial u_x}{\partial z} - \varphi\right)\right] f\left(T\right)$$

$$\mu_{xy} = \left(\gamma_0' \frac{\partial \varphi}{\partial x}\right) f\left(T\right), \quad \mu_{yx} = \left(\beta_0 \frac{\partial \varphi}{\partial x}\right) f\left(T\right)$$

$$\mu_{yz} = \left(\beta_0 \frac{\partial \varphi}{\partial z}\right) f\left(T\right), \quad \mu_{zy} = \left(\gamma_0' \frac{\partial \varphi}{\partial z}\right) f\left(T\right)$$

For numerical convenience, the following dimensionless quantities are introduced:

$$x_i^* = c'\zeta x_i; \quad u_i^* = c'\zeta u_i; \quad t^*(\tau^*, \tau_0^*, \tau_1^*) = c'^2 \zeta t(\tau, \tau_0, \tau_1); \quad \theta^* = \frac{T - T_0}{T_0}; \quad \sigma^* = \frac{\sigma}{\mu_0}$$

Where $c'^2 = \dfrac{\lambda_0 + 2\mu_0}{\rho}$, $\zeta = \dfrac{\rho c_E}{k_0}$. Without causing ambiguity, the asterisk symbol of the non-dimensional variables is dropped off in the following for the sake of brevity. In the above equations, a comma followed by a suffix denotes material derivative and a superposed dot denotes the deriva-

tive with respect to time.
From the preceding description and the problem we will consider, the initial and boundary conditions may be expressed as:
Initial conditions (t = 0):

$$u_x\left(x,z,t\right) = u_z\left(x,z,t\right) = \dot{u}_x\left(x,z,t\right) = \dot{u}_z\left(x,z,t\right) = 0$$

$$\varphi_y\left(x,z,t\right) = \dot{\varphi}_y\left(x,z,t\right) = 0, \ \phi^*\left(x,z,t\right) = \dot{\phi}^*\left(x,z,t\right) = 0$$

$$\theta\left(x,z,t\right) = \dot{\theta}\left(x,z,t\right) = 0$$

Boundary conditions:

$$\text{At } z = 0, \ \theta\left(x,z,t\right) = \theta_0 H\left(0.4 - \left|x\right|\right), \ \sigma_{zz} = \sigma_{xz} = 0, \ \lambda_z = 0$$

$$\text{At } x = 0, \ u_x\left(x,z,t\right) = 0, \ \frac{\partial\theta}{\partial x} = 0$$

$$\text{At } z \to \infty \ , \ x \to \infty \ , \ u_x = u_z = \varphi_y = \phi^* = \theta = 0$$

Where θ_0 is a given temperature, H (*) is the Heaviside function. L is the thickness of the plate.

3 SOLUTIONS OF FINITE ELEMENT EQUATIONS

According to the equations of balance of motion, constitutive equations and boundary conditions, using generalized variational principle, we obtain at any time

$$\begin{aligned}
&\int_v \left[\delta\left\{\varepsilon\right\}^T\left\{\sigma\right\} + \delta\left\{\omega\right\}^T\left\{\mu\right\} + \delta\left\{\theta'\right\}^T(q+\tau\dot{q}) + \delta\left\{\varpi\right\}^T\left\{\lambda\right\} - \delta\theta\rho T_0\left(\dot{\eta}+\tau\ddot{\eta}\right)\right]dV \\
&= \int_v \delta\left\{u\right\}^T(-\rho\left\{\ddot{u}\right\} + \mu_0(\vec{J}\times\vec{H}))dV + \int_v \delta\left\{\varphi\right\}^T(-\rho j\left\{\ddot{\varphi}\right\})dV \\
&+ \int_v \delta\left\{\phi^*\right\}^T(-3\rho j\ddot{\phi}^*/2)dV + \int_{A_\sigma}\delta\left\{u\right\}^T\left\{T'\right\}dA + \int_{A_\omega}\delta\left\{\varphi\right\}^T\left\{T''\right\}dA + \int_{A_\theta}\delta\theta\overline{q}dA + \int_{A_\varphi}\delta\phi^*\overline{\lambda}dA
\end{aligned} \tag{17}$$

At the right side of above equation, and T' 、 T'' 、 $\overline{q}$ 、 $\overline{\lambda}$ respectively, external force vector、external rotation vector、 external heat flux and external moment vector. And then the finite element governing equations can be established and solved in time domain directly with the aids of the corresponding initial conditions and boundary conditions. The convergence of the approximate solution to the exact one can in principle be achieved by either increasing the number of nodes per element or by refining the mesh, i.e. increasing the number of the element (Xiong and Tian, 2011).

4 NUMERICAL RESULTS AND DISCUSSION

Now we present some numerical results. The Magnesium crystal-like material medium was chosen as the material for the purposes of numerical computation, the physical data for which are the following:

$$\lambda_0 = 9.4\times10^{10}\,Nm^{-2},\ \mu_0 = 4.0\times10^{10}\,Nm^{-2},\ \alpha_0 = 0.5\times10^{10}\,Nm^{-2},\gamma_0' = 0.779\times10^{-9}\,N\ ,j = 0.2\times10^{-19}\,m^2$$

$$,\alpha_{00} = 0.779\times10^{-9}\,Nm^{-2}\ ,\ \lambda_{00} = 0.5\times10^{10}\,Nm^{-2}\ ,\ \lambda_1 = 0.5\times10^{10}\,Nm^{-2}\ \ \alpha_{t1} = \alpha_{t2} = 2.05\times10^{-5}\,K^{-1},$$

$$\rho = 1740kgm^{-3},\ k_0 = 170Wm^{-1}K^{-1},\ \ c_E = 1040Jkg^{-1}K^{-1},\ \varepsilon_0 = 10^{-9}/\left(36\pi\right)Fm^{-1},\ \mu_0 = 4\pi\times10^{-7}\,Hm^{-1}$$

$$,\ H_0 = 10^7/\left(4\pi\right)Am^{-1},\ \alpha^* = 0.001K^{-1}$$

According to the reference (Vedavarz et al. 1994), the value of thermal relaxation time $\tau(\tau_0, \tau_1)$ for various materials is of the order of seconds (porous materials) to picoseconds (metals). For metals, the value of thermal relaxation time $\tau(\tau_0, \tau_1)$ range from 10^{-14} to 10^{-11}s at room temperature and smaller than 10—14s at high temperatures. The thermal relaxation time $\tau\left(\tau_0,\tau_1\right) = 4.695\times10^{-14}\left(s\right)$ and the non-dimensional thickness of plate takes 3.0.

Firstly, it is essential to verify the accuracy and credibility of the FEM adopted in the present study. Thus, the example in reference (Sherief and Youssef, 2004) is repeated by using FEM and the comparison is presented in Figure.2. From Figure.2, the present results are in good agreement with the analytical solutions given in reference (Sherief and Youssef, 2004) except for the location of thermal wave front. This shows that FEM proposed here can predict the dynamic response owing to the thermal shock accurately. Additionally, according to the non-dimensional speed of thermal wave ($V_{thermal} = \sqrt{\frac{1}{\tau}} = 7.071$), the location of the thermal wave front should be at x = 0.7071 when t = 0.1. The result given in Figure.2 is in good agreement with the prediction. This further shows the results obtained by using FEM are accurate and reliable.

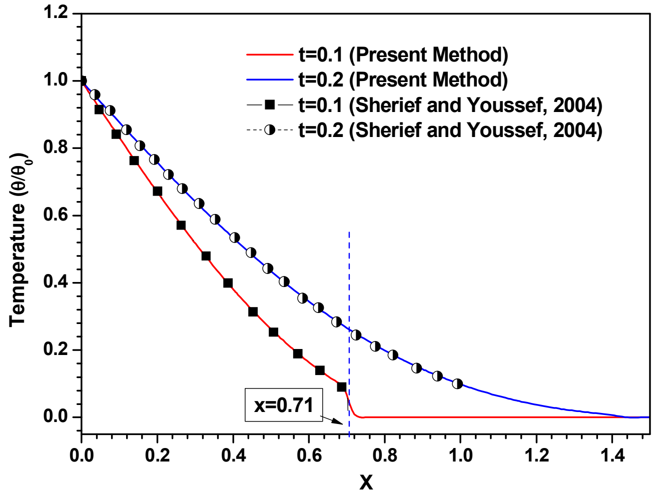

Figure 2: Comparison between temperature distributions predicted by numerical transformation and present method (FEM).

Distributions of temperature, displacement, stress, microstretch, microrotation, the induced magnetic field and induced electric field with distance z and x for L–S and G–L theories with temperature-dependent properties have been shown by different symbol lines, and G-L with $f(T)=1-\alpha^{*}T$ represents G–L theory and the absolute temperature-dependent properties are applied, G-L with $f(T)=1-\alpha^{*}T_{0}$ represents G–L theory and the reference temperature-dependent properties are applied, for the L-S theory, it is the same as the G-L.

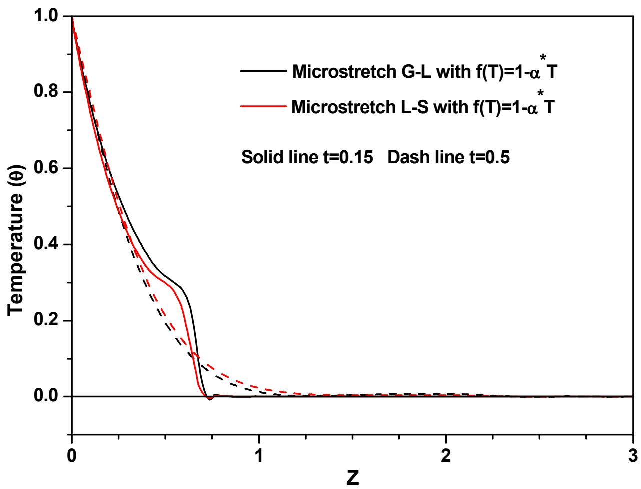

Figure 3: Temperature distribution at x=0 versus z under different generalized thermoelastic theories.

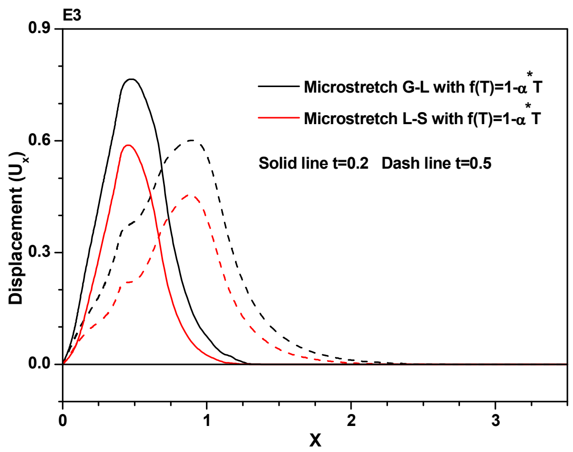

Figure 4: Displacement U_x distribution at z=0 versus x under different generalized thermoelastic theories.

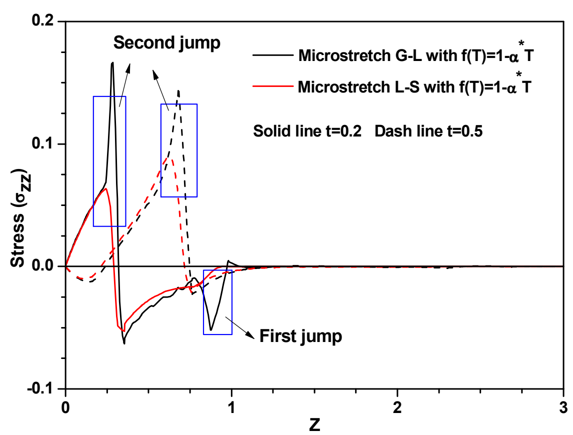

Figure 5: Stress (σ_{zz}) distribution at x=0 versus z under different generalized thermoelastic theories.

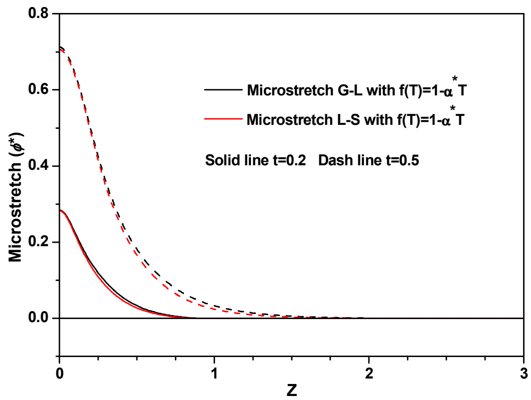

Figure 6: Microstretch (ϕ^*) distribution at x=0 versus z under different generalized thermoelastic theories.

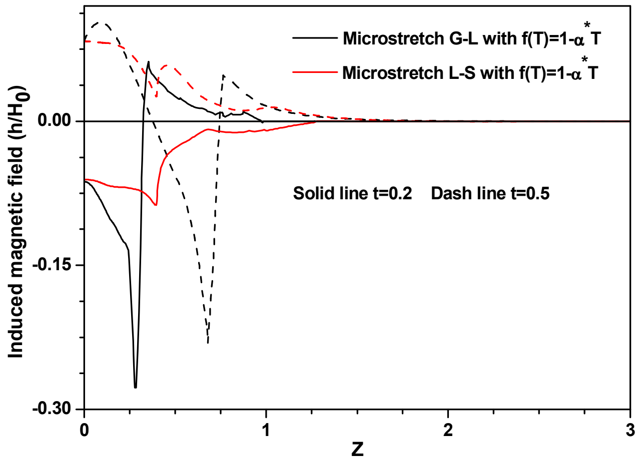

Figure 7: Induced magnetic field (h/H_0) distribution at x=0 versus z under different generalized thermoelastic theories.

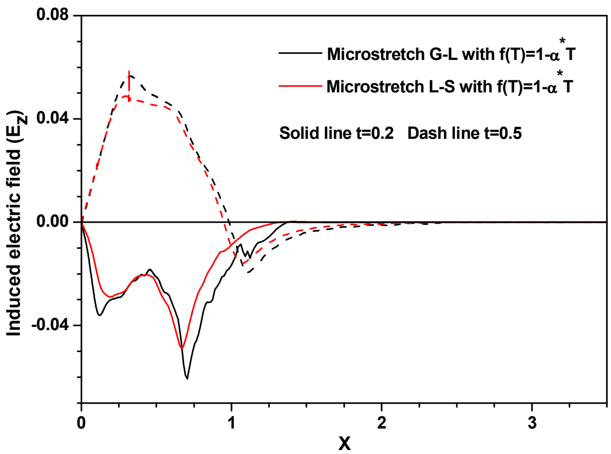

Figure 8: Induced electric field (E_z) distribution at z=0 versus x under different generalized thermoelastic theories.

Figures.3–8 present the dimensionless values of displacement, temperature, stresses, microstretch, induced magnetic field and induced electric field distributions for different values of coordinate distance and for different time points, namely for t = 0.15, 0.2 and 0.5.

In Figure.3, it can be seen that there exists a distinct temperature step on thermal wave front in the distribution of temperature at t=0.15 for generalized theory (G-L or L-S), while there isn't this phenomenon and the finite speed of propagation of thermal wave isn't depicted in the previous studies (Sherief et al. 1981, 1986, 2004; Youssef et al. 2004, 2005, 2009; Othman et al. 2007, 2014a, 2014b). The temperature step on thermal wave front in the distribution of temperature predicted by generalized theory (G-L or L-S) becomes indistinct along with the passage of time (such as shown at t=0.5 in Figure.1). And we can also find that the temperature predicted by G-L theory is higher than that predicted by L-S theory unlike the conditions without temperature dependent properties (Xiong and Tian, 2011a), which illustrates the two theories in dealing with problems of heat conduction of generalized thermoelasticity are not equivalent when the temperature dependent properties are considered.

Figure.4 gives the distributions of displacement U_x at z=0 versus x under two theories at different time points. Due to the symmetry of thermal load, the displacement Ux is zero at x=0 and increase to the maximum value near the edge of thermal shock zone and then starts to decrease with the increase of distance x and then decreases finally to diminish as shown in Figure.4. After thermal shock applied at the x-y surface of z=0, the media in the front of the stress shock wave induced by thermal shock is compressed and the region behind the stress shock wave is expended due to the increase of temperature.

Figure.5 represents the distributions of stress σ_{zz} at x=0 versus z at different times. It can be seen that stress σ_{zz} predicted by G-L theory is larger than that predicted by L-S theory in Figure.5. At t=0.15 the first jump (from right to left) in stress σ_{zz} distribution is caused by heat wave front in the Figure.2 for the generalized theory(G-L or L-S), it can be seen that the location of the first jump is very close to the location of temperature step in Figure.2. The second jump in stress σ_{zz} distribution is elastic wave front. According to the speed of propagation of elastic wave, the location of the elastic wave front should be z=0.2 at t=0.2, we see that results given by Figure.6 are in good agreement with the predictions, while these phenomenon didn't appear in the previous studies (Sherief et al. 1981, 1986, 2004; Youssef et al. 2004, 2005, 2009; Othman et al. 2007, 2014a, 2014b). But at time point t=0.5, the location of the elastic wave front cannot be predicted exactly, this is because that the temperature dependent properties make the speed of the elastic wave increase significantly at time point t=0.5.

Figure.6 shows the distribution of the microstretch at x=0 versus z under two theories at different times. From the Figure.6, it can be seen that the microstretch obtained by L-S theory and G-L theory are close, but there exits an evident difference in microrotation predicted by G-L theory and L-S theory.

Figures.7-8 show the distribution of the induced magnetic field at x=0 versus z and induced electric field at z=0 versus x under two theories at different times respectively. From Figures. 7-8, it can be seen that the induced electric field obtained by L-S theory and G-L theory are close, but there exits an evident difference in induced magnetic field predicted by G-L theory and L-S theory. And it should be noted that the absolute value of all the physical responses predicted by the G-L theory is larger shown in Figures.3-8.

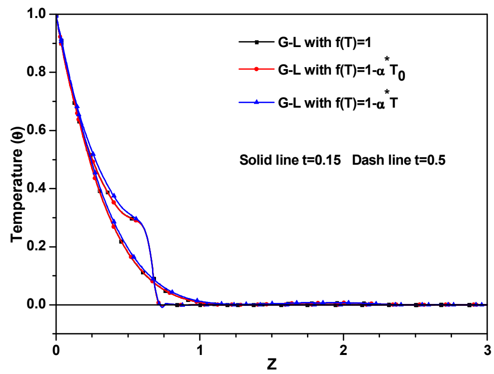

Figure 9: Temperature (θ) distribution at x=0 versus z under different forms of temperature dependent material properties.

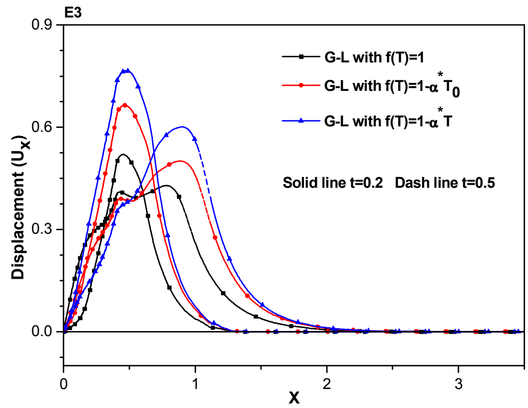

Figure 10: Displacement (U_x) distribution at z=0 versus x under different forms of temperature dependent material properties.

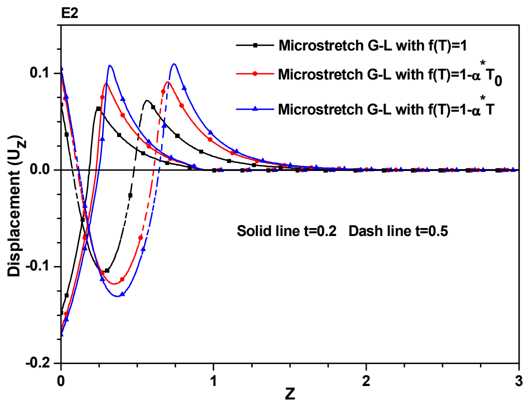

Figure 11: Displacement (U_z) distribution at x=0 versus z under different forms of temperature dependent material properties.

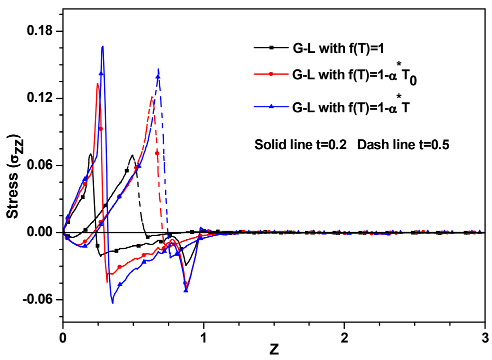

Figure 12: Stress (σ_{zz}) distribution at x=0 versus z under different forms of temperature dependent material properties.

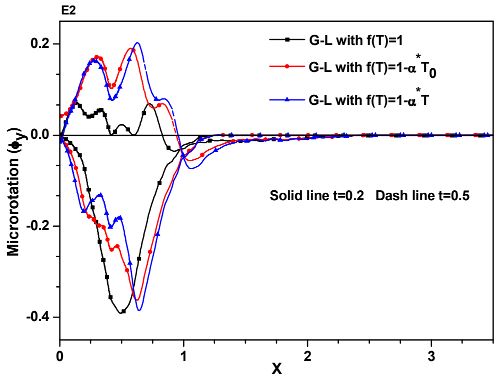

Figure 13: Microrotation (ϕ_y) distribution at z=0 versus x under different forms of temperature dependent material properties.

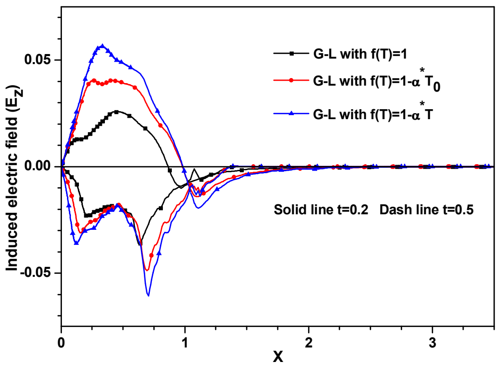

Figure 14: Induced electric field (E_z) distribution at z=0 versus x under different forms of temperature dependent material properties.

In order to more clearly express meanings of graphics – the effect of different forms of temperature dependent properties, the Figures.9-14 only show the numerical results obtained under G-L theory.

Figures.9–14 present the results obtained under three forms of temperature-dependent material properties (absolute temperature-dependent, reference temperature-dependent, temperature-independent), regarding displacement, temperature, stresses, microrotation, induced magnetic field and induced electric field at different time points, namely for t = 0.15, 0.2 and 0.5.

From the Figure.9, it can be found that in comparison with the case of temperature-independent properties (f(T)=1) there don't exit a difference in temperature distribution when the material properties is a linear function of reference temperature ($f(T)=1-\alpha^{*}T_0$), while there exists a difference when the material properties is a non-linear function of absolute temperature($f(T)=1-\alpha^{*}T$) and temperature is higher in this case to show the effect of absolute temperature has a great influence on temperature. And so we know that the absolute temperature (T) shouldn't be instead of the reference temperature (T_0) in the form of temperature-dependent material properties.

In Figures.10-14, the results obtained, including displacement, stresses, microrotation, induced magnetic field and induced electric field under the condition the material properties is a linear function of reference temperature ($f(T)=1-\alpha^{*}T_0$) are larger than that in the case of temperature-independent properties (f(T)=1), and under the condition that the material properties is a non-linear function of absolute temperature ($f(T)=1-\alpha^{*}T$) the results are the largest. The results obtained under the two conditions (reference temperature-dependent $f(T)=1-\alpha^{*}T_0$, absolute temperature-dependent $f(T)=1-\alpha^{*}T$) are different, and this further shows the absolute temperature (T) isn't instead of the reference temperature (T_0) in the form of temperature-dependent material properties simply when we need to consider the material properties is temperature-dependent.

5 CONCLUSIONS

Finite element governing equations of magneto-microstretch thermoelasticity are established. The thermoelastic responses of an infinite homogeneous isotropic microstretch half-space with temperature-dependent material properties subjected to a zonal time-dependent heat shock placed in a transverse magnetic field is studied by solving the finite element equations directly in time domain. The results, including temperature, stresses, displacements, microstretch, microrotation, induced magnetic field and induced electric field are presented graphically. Comparisons are made in the results obtained under two theories of generalized thermoelasticity. The results show that the two theories (G-L and L-S) are not equivalent when dealing with problems of generalized thermoelasticity when the temperature dependent properties, the response (temperature, displacement, stress and so on) predicted by the two theories is different, and that predicted by the G-L theory is larger. And we also know that the reference temperature-dependent material properties has no influence on temperature but an influence on the other responses, while the absolute temperature-dependent material properties has a great influence on all the thermoelastic responses and the influence become greater along with the passage of time. And the two forms of temperature-dependent material properties are not equivalent. So when we study the thermoelastic response of all materials with temperature-dependent materials properties caused by thermal shock, the absolute temperature (T) should be applied and shouldn't be instead of the reference temperature (T_0) in the form of temperature-dependent material properties.

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