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Semi-empirical procedures for estimation of residual velocity and ballistic limit for impact on mild steel plates by projectiles

Abstract

This paper deals with the development of simplified semi-empirical relations for the prediction of residual velocities of small calibre projectiles impacting on mild steel target plates, normally or at an angle, and the ballistic limits for such plates. It has been shown, for several impact cases for which test results on perforation of mild steel plates are available, that most of the existing semi-empirical relations which are applicable only to normal projectile impact do not yield satisfactory estimations of residual velocity. Furthermore, it is difficult to quantify some of the empirical parameters present in these relations for a given problem. With an eye towards simplicity and ease of use, two new regression-based relations employing standard material parameters have been discussed here for predicting residual velocity and ballistic limit for both normal and oblique impact. The latter expressions differ in terms of usage of quasi-static or strain rate-dependent average plate material strength. Residual velocities yielded by the present semi-empirical models compare well with the experimental results. Additionally, ballistic limits from these relations show close correlation with the corresponding finite element-based predictions.

Keywords

projectile, mild steel plate, semi-empirical, residual velocity, ballistic limit, normal and oblique impact.

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1 INTRODUCTION

In addition to experimental and numerical-based studies, formulae for estimating residual velocity can be useful tools for design of metallic armour plates. It is obvious that rather than predicting the detailed geometry of failure, what often matters in design is whether perforation will take place for a given impact condition as well as the estimation of residual velocity and ballistic limit. To facilitate this latter objective of engineering design of perforation-resistant armour plates, a number of semi-empirical relationships [1–4, 7, 8] have been developed by various researchers. However, most of these relations rely on empirical material parameters which are difficult to determine. Thus, following a review of existing relations for estimation

NOMENCLATURE

$m = M = G$	projectile mass
m_p	plug mass
V_i	projectile impact velocity
V_r	projectile residual velocity
$V_{bl} = V_c = V_{oc}$	ballistic limit or critical velocity
$H = h = b = t$	target plate thickness
a	radius of punch or projectile
d	diameter of punch or projectile
p	depth of penetration
R	radius of target plate
D and q	Cowper-Symonds strain rate hardening parameters
W_{os}	critical transverse displacement for failure by shear
$\sigma_u = \sigma_f$	ultimate or failure tensile strength
$\sigma_y = Y$	yield stress or strength of material
$\sigma_m = (\sigma_y + \sigma_u)/2$	mean strength of material
$\tau_u = \tau_a$	shear stress
τ_s	effective shear strength of target plate material
$\gamma_u = \gamma_c = \gamma_i$	critical shear strain
n	work hardening index
A_s	shear area
F_u	critical transverse shear force
F_c	static collapse load
K_m	membrane stiffness
M_o	fully plastic bending moment per unit length
N_o	fully plastic membrane force per unit length
δ	width of shear band
α	angle of impact

of projectile residual velocity, new expressions are presented for estimating projectile residual velocity by using standard material parameters (such as yield and failure strengths) of a target mild steel plate. It is assumed in the latter relations which use alternatively quasi-static or strain rate-dependent material parameters that plate failure is caused predominantly by shear plugging. Residual velocities yielded by the current semi-empirical relations are found to compare well with the corresponding test data and finite element-based results. Additionally, ballistic limits obtained from these relations match well with finite element-based predictions for a target mild steel plate for which experimental ballistic limit is not available.

2 SOME AVAILABLE SEMI-EMPIRICAL MODELS FOR PROJECTILE RESIDUAL VELOCITY PREDICTION

Recht and Ipson [1, 8] had proposed an analytical relation to predict projectile residual velocity in the following form:

$$V_r = \frac{m}{m + m_p}\sqrt{V_i^2 - V_{bl}^2} \tag{1}$$

where, m is projectile mass, m_p denotes plug mass, V_i and V_r are respectively projectile impact and residual velocities, and V_{bl} is ballistic limit.

Eq. (1) represents an energy-based model for estimating projectile residual velocity; however, the difficulty in its use lies in the requirement of *a-priori* knowledge of ballistic limit which is often a parameter to be estimated. Perhaps, this relation can be more useful for ballistic limit prediction if limited tests are conducted on a target plate for determining residual velocities for given impact velocities.

Expressions which can be used directly for ballistic limit and residual velocity prediction have been given, for example, by Wen and Jones [7], Bai and Johnson [2], Chen and Li [3], and Gupta et al. [4] who considered plate and projectile parameters also in their relations.

Wen and Jones [7] proposed a semi-analytical model to study the behaviour of punch-impact-loaded metal plates. Based on the principle of virtual work, load-deflection relationships were derived and used to predict energy absorbing capacity of plates subjected to low velocity impact by blunt projectiles causing perforation and penetration. The relation for obtaining critical velocity (equivalent to ballistic limit), V_c by Wen and Jones [7] is given as follows:

$$V_c = \sqrt{\frac{2\sigma_y d^3}{G}}\left[1 + \left(\frac{2W_{os}V_c}{3\sqrt{2}DRa\ln^2\left(\frac{R}{a}\right)}\right)^{\frac{1}{q}}\right]^{\frac{1}{2}} \times \left[A\left(\frac{H}{d}\right) + B\left(\frac{H}{d}\right)^2 + C\left(\frac{H}{d}\right)^3\right]^{\frac{1}{2}} \tag{2}$$

where,

$$A = 0.138\lambda^2 \ln\left(\frac{R}{a}\right) \tag{3}$$

$$B = 0.27\lambda^2 \ln\left(\frac{R}{a}\right) + \frac{0.451\lambda\gamma_c}{1+n} \tag{4}$$

$$C = 0.132\lambda^2 \ln\left(\frac{R}{a}\right) + \frac{0.44\lambda\gamma_c}{1+n} - \frac{\pi}{12}\ln\left(\frac{R}{a}\right)\left[1 + \frac{\left(1+\frac{\sqrt{3}}{2}\right)}{\ln\left(\frac{R}{a}\right)}\right]^2 \tag{5}$$

$$\lambda = \frac{\sigma_u}{\sigma_y} \tag{6}$$

$$W_{os} = \frac{F_u - F_c}{K_m} \tag{7}$$

$$F_u = \tau_u \times A_s \tag{8}$$

$$\tau_u = \lambda\sigma_y \times \left(0.41 \times \left(\frac{H}{d}\right) + 0.42\right) \tag{9}$$

$$A_s = \pi d H \tag{10}$$

$$F_c = \frac{4}{\sqrt{3}} \times \pi M_0 \times \left(1 + \frac{\left(1 + \frac{\sqrt{3}}{2}\right)}{\ln\left(\frac{R}{a}\right)}\right) \tag{11}$$

$$M_0 = \frac{\sigma_y H^2}{4} \tag{12}$$

$$K_m = \frac{2\pi N_0}{\ln\left(\frac{R}{a}\right)} \tag{13}$$

$$N_0 = \sigma_y H \tag{14}$$

The various parameters in Eqs. (2) through (14) are defined as: H is plate thickness, a and d are radius and diameter of punch respectively, G is projectile mass, R is radius of target plate, D and q are Cowper-Symonds strain rate hardening parameters, W_{os} is critical transverse displacement for failure by shear, σ_u is ultimate tensile strength, σ_y is static yield stress, n is work hardening index, γ_c is critical shear strain, F_u is critical transverse shear force, τ_u is critical shear stress, A_s is shear area, F_c is static collapse load, K_m is membrane stiffness, M_0 is fully plastic bending moment per unit length, and N_0 is fully plastic membrane force per unit length.

As will be shown later, the semi-empirical relation given by Eq. (2) and subsequent application of Eq. (1) can yield a good prediction of residual velocity; however, without proper estimation (which may be challenging) of material parameters such as D and q as well as the critical shear strain, γ_c, the predictions may not be realistic.

Bai and Johnson [2] developed a model for plugging of metal plates based on adiabatic shear instability. The model consists of three basic elements: kinetic energy equation connecting the projectile and plug, a constitutive relation, and a relationship between displacement of plug and shear strain. This model requires parameters such as depth of penetration, and width of shear band which are not readily available. The energy E absorbed by a target plate is estimated by Bai and Johnson [2] is as follows:

$$E = \frac{MV_i^2}{2} - \frac{M + m}{2} V_r^2 \tag{15}$$

where, M represents punch mass, m denotes plug mass, and V_i and V_r are respectively projectile impact and residual velocities.

The critical velocity of projectile, V_{oc}, is obtained from Eq. (15) by setting $V_r = 0$; in this case, E can be interpreted as the critical energy, e_u of a given target plate. With these observations, the critical velocity, V_{oc} can be written as

$$V_{oc} = \sqrt{\frac{2e_u}{M}} \tag{16}$$

where, according to Bai and Johnson [2],

$$e_u = \int_0^p 2\pi ab\tau_a dp \tag{17}$$

$$\tau_a = \tau_m \left(\frac{1-n}{n\gamma_u}\frac{p}{a}\right)^n \exp\left\{\frac{n}{1+n}\left[1-\left(\frac{1-n}{n\gamma_u} \times \frac{p}{a}\right)^{n+1}\right]\right\} \tag{18}$$

where, p is depth of penetration, a is radius of punch, b is thickness of plate, τ_a is shear stress, $\gamma_u = p/\delta$ is critical shear strain, δ is width of shear band, and n is work hardening index. It should be noted that the width of the shear band is difficult to estimate.

Chen and Li [3] have presented relations for predicting ballistic limit (V_{bl}) and residual velocity (V_r) of circular metallic target plates impacted by blunt shaped projectiles by taking into account effects of transverse shear, bending, and membrane deformations on the perforation process. The relation for ballistic limit is given as (assuming pure shear velocity field):

$$V_{bl} = 2\left(\sqrt{\frac{2k\chi(1+\eta)}{\sqrt{3}}}\right)\sqrt{\frac{\sigma_y}{\rho}} \tag{19}$$

where, ρ is density of plate material, σ_y is yield stress, $\chi = H/d$ is plate thickness to projectile diameter ratio, η is ratio of the plug mass to projectile mass (i.e. $\frac{\pi\rho d^2 H}{4G}$), d is diameter of projectile, G is projectile mass, H is plate thickness, and k is an empirical parameter in the shear failure criterion.

The residual velocity definition given by Chen and Li [3] is same as that by Recht and Ipson [1, 8] in the form of Eq. (1) and requires the value of ballistic limit for computing the residual velocity. It may be pointed out that the value of the parameter k is not readily known.

A pair of relations due to Gupta, Ansari and Gupta [4] can directly predict both ballistic limit and residual velocity assuming failure by shear plugging and energy balance. These relations, which are given below in the form of Eqs. (20) and (21), also have parameters (i.e. k and n) which are empirical in nature and values for which were suggested for aluminium target plates of thickness 0.5 to 2 mm:

$$V_{bl} = \sqrt{\frac{kdY}{m}h_0^n} \tag{20}$$

$$V_r = \sqrt{v_i^2 - \frac{kdY}{m}h_0^{2n}} \tag{21}$$

where, d is diameter of projectile, m is projectile mass, h is plate thickness, k and n are constants which can be estimated using experimental data, and Y is yield strength of target plate.

It may be pointed out that although some of the foregoing relations have been derived using a substantial degree of limit-state mechanics and can yield good prediction of residual velocity and ballistic limit, they still contain empirical parameters the values of which may be difficult to obtain especially for problems for which only standard material data is known.

3 ESTIMATION OF RESIDUAL VELOCITY USING AVAILABLE SEMI-EMPIRICAL MODELS

Estimations of residual velocity using the semi-empirical models given by Wen and Jones [7], Bai and Johnson [2], Chen and Li [3], and Gupta et al. [4] have been compared for perforation of mild steel plates of different grades and thicknesses studied experimentally by Gupta and Madhu [5]. The values of empirical parameters in these models given by Eqs. (2) through (21) have been obtained from the relevant literature and are given in Tables 1 through 4 with the indicated sources.

The computed residual velocities according to Wen and Jones [7] relations using yield strength-based and ultimate strength-based Cowper-Symonds parameters (i.e. D and q) are given in columns 6 and 7 respectively of Table 5. It is seen from these columns that the predicted results correlate relatively well with the corresponding test residual velocities given in [5]. However, uncertainties may exist in the choice of material strength values (i.e nominal or true), Cowper-Symonds parameters, critical shear strain (γ_c), etc. and may significantly affect the predicted results. It may be noted that in the current study, the Cowper-Symonds parameters (i.e. D and q) have been estimated using the stress-strain behaviours of MS1, MS2 and MS3 plates given in Figs. 2 through 5 in [6]. Due to the long mathematical expressions involved and the implicit nature of Eq. (2), the critical velocity for a given case in Table 5 has been obtained through an iterative algorithm using the Mathematica package and the residual velocity was then computed from Eq. (1) by ignoring the plug mass (based on the observation that for tests conducted by Gupta and Madhu [5] with ogival-head projectiles, well-formed plugs were absent).

The values of projectile residual velocity in column 8 of Table 5 have been obtained using the Bai and Johnson [2] relations (i.e. Eqs. (16) through (18)). Moderate correlation is obtained with test data for plates of lower thicknesses (i.e. 4.7 mm and 6 mm), however, the disagreement between predicted and test residual velocities is substantial for thicker target plates. The fact that higher residual velocities are consistently obtained using the Bai and Johnson [2] approach can be at least in part due to the non-accountability of strain rate effects on stress-strain behaviour of plate material.

The predicted residual velocities using the Chen and Li [3] approach are given in column 9 of Table 5. It is seen that the predicted results according to this model differ significantly from the corresponding test data and hence cannot be relied upon for the prediction of residual velocities for the test cases involving mild steel target plates considered in [5].

Strictly speaking, the test data-based empirical relation suggested by Gupta, Ansari and Gupta [4] should be applicable to the type of aluminium target plates considered by the said authors. It is therefore not surprising that poor correlation (including residual velocity being not real) is obtained using the relations of Gupta, Ansari and Gupta [4] for the mild steel plates considered here. Due to the simplicity and easy to compute the ballistic limit and residual velocity, this model has been cited in this paper. Also, as in the cases of Bai and Johnson [2], and Chen and Li [3] relations, there is no provision for incorporating strain rate effects on target material behaviour in the relations (i.e. Eqs. (20) and (21)) proposed by Gupta, Ansari and Gupta [4].

The observations given above point to the need for simpler but reliable semi-empirical relations for generating preliminary estimates of residual velocity as well as of ballistic limit. Thus, two new relations, one with quasi-static and the other with strain rate-dependent average plate strength, are proposed in the next section for prediction of residual velocity of projectile immediately following plate perforation. The relations are applicable to both normal and oblique impact of projectile on target plate, and contain, in addition to a material parameter, basic geometric parameters of plate and projectile. It is noted that the existing semi-empirical relations discussed here are only applicable to normal impact of projectiles on target plates.

Table 1 Values of parameters for Wen and Jones [7] approach corresponding to MS1, MS2 and MS3 plates.

Parameter		MS1	MS2	MS3
Yield strength, σ_y (MPa) (from [6])		205	360	305
Failure strength, σ_u (MPa) (from [6])		380	505	465
Projectile diameter,d(mm)		7.8		
Projectile mass, G (grams)		5.2		
Radius of target plate, R (mm)		100 (plate length/2)		
Cowper-Symonds: D and q	Yield stress based	103330, 4.23	100079, 4.22	105651, 4.25
	Failure stress based	79240, 2.49	82443, 2.50	91373, 2.56
Work hardening index, n (from [7])		0.25		
Critical shear strain, γ_c (from [7])		0.8		

Table 2 Values of parameters for Bai and Johnson [2] approach corresponding to MS1, MS2 and MS3 plates.

Parameter	MS1	MS2	MS3
Yield strength, σ_y (MPa) (from [6])	205.0	360.0	305.0
Failure strength, σ_u (MPa) (from [6])	380.0	505.0	465.0
Mean strength, $\sigma_m = (\sigma_y + \sigma_u)/2$ (MPa)	292.5	432.5	385.0
Shear strength, $\tau_u = \sigma_m/2$ (MPa)	146.3	216.3	192.5
Projectile diameter,d(mm)	7.8		
Projectile mass, G (grams)	5.2		
Work hardening index,n(from [2])	0.28		
Critical shear strain,γ_i(from [2])	1.94		

Table 3 Values of parameters for Chen and Li [3] approach corresponding to MS1, MS2 and MS3 plates.

Parameter	**MS1**	**MS2**	**MS3**
True yield strength, σ_y (MPa) (from [6])	205.4	360.0	305.0
Projectile diameter, d (mm)		7.8	
Projectile mass, G (grams)		5.2	
Empirical parameter, k (from [3])		1.0	

Table 4 Values of parameters for Gupta et al. [4] approach corresponding to MS1, MS2 and MS3 plates.

Parameter	**MS1**	**MS2**	**MS3**
True yield strength, σ_y (MPa) (from [6])	205.4	360.0	305.0
Projectile diameter, d (mm)		7.8	
Projectile mass, G (grams)		5.2	
Constant, k (from [4])		7.0	
Constant, n (from [4])		0.89	

4 PROPOSED REGRESSION-BASED RELATIONS FOR RESIDUAL VELOCITY AND BALLISTIC LIMIT PREDICTION

Assuming a rigid projectile and removal of plate material primarily due to shear plugging, the energy balance equation for the plate-projectile system can be written as:

$$\frac{1}{2}mv_i^2 - \frac{1}{2}mv_r^2 = C \times \frac{\pi dt}{\cos^2 \alpha} \times \tau_s t \tag{22}$$

where, m represents projectile mass, v_i and v_r are respectively projectile impact and residual velocities, d is projectile diameter, t is thickness of target plate, τ_s is effective shear strength of target plate material, C is a regression coefficient (i.e. an empirical constant), and α is the angle of impact.

Analogous to the maximum shear stress theory, the average plate shear stress causing plugging failure in target steel plate is assumed as

$$\tau_s = \frac{\sigma_m(\dot{\varepsilon})}{2} \tag{23}$$

In Eq. (23), the strain rate-dependent mean flow stress, $\sigma_m(\dot{\varepsilon})$, of plate material is interpreted as

$$\sigma_m(\dot{\varepsilon}) = \frac{\sigma_y(\dot{\varepsilon}) + \sigma_f(\dot{\varepsilon})}{2} \tag{24}$$

In Eq. (24), $\sigma_y(\dot{\varepsilon})$ and $\sigma_f(\dot{\varepsilon})$ are respectively the plate yield and failure strengths at a given strain rate, $\dot{\varepsilon}$.

Table 5 Estimations of residual velocity using available semi-empirical models.

Plate material in [5]	**Plate thickness (mm)**	**Angle of impact (deg)**	**Impact velocity (m/s)**	**Residual velocity (m/s)**					
				Test [5]	**Wen and Jones** [7]		**Bai and Johnson** [2]	**Chen and Li** [3]	**Gupta et al.*** [4]
					Using yield stress-based D **and** q **in Eq.**(2)	**Using ultimate strength-based** D **and** q **in Eq.** (2)			
MS1	4.7	00	821.0	758.6	775.8	763.8	810.22	602.0	702.7
MS2	6.0	00	866.3	792.2	791.7	800.5	841.7	580.6	575.4
MS3	10.0	00	827.5	702.2	642.5	666.2	796.8	444.6	No real solution
		15	815.0	690.4	Not applicable for oblique impact				
		30	825.7	654.0					
		45	790.0	500.0					
	12.0	00	818.0	661.5	575.1	607.1	780.6	394.0	No real solution
		15	842.7	671.6	Not applicable for oblique impact				
		30	801.8	598.0					
	16.0	00	819.7	562.0	487.7	529.5	769.5	319.5	No real solution
		15	817.3	544.4	Not applicable for oblique impact				
		30	817.7	496.3					

*strictly speaking, the empirical parameters for this case are applicable to aluminium plates.

Substituting Eq. (23) in Eq. (22) and rearranging, the expression for residual velocity is obtained as

$$v_r^2 = v_i^2 - C\frac{\pi\sigma_m(\dot{\varepsilon})\,dt^2}{m\cos^2\alpha} \tag{25}$$

In Eq. (25), $\dot{\varepsilon}$ can be considered as an average strain rate that the target plate is subject to during the process of being perforated by a projectile and can be approximated as follows:

$$\dot{\varepsilon} = \frac{v_i + v_r}{2t} \tag{26}$$

The average flow stress in Eq. (25) can be obtained as per Eq. (24) by estimating the yield and ultimate strengths of the target plate material for a given strain rate, $\dot{\varepsilon}$. It may be noted that the value of v_r is known for a case in which a physical test or finite element analysis has been carried out. However, when only Eq. (25) is used for prediction ofv_r, the average strain rate, $\dot{\varepsilon}$ given by Eq. (26) will not be known *a-priori*. In such a situation, an initial value of v_r as per Eq. (25) can be calculated by assuming $v_r = 0$ for estimating $\dot{\varepsilon}$ as per Eq. (26). Based on this initial value of v_r, an improved estimate of $\dot{\varepsilon}$ can be obtained according to Eq. (26) and v_r can be recomputed using Eq. (25). This procedure can be repeated until v_r does not change significantly between two successive iterations.

The empirical constant, C, which is incorporated in Eq. (22) to ensure that test results of residual velocity can be reasonably predicted, is determined here by linear regression according to the least square error method. According to this approach, for n test cases, the sum of error-squares, E, can be written as follows:

$$E = \sum_{k=1}^{n}(Y_k - Z_k - CX_k)^2 \tag{27}$$

where,

$$Y_k = \lfloor V_r^2 \rfloor_k \text{ (test-based)}, \quad Z_k = \lfloor V_i^2 \rfloor_k, \text{ and } X_k = -\left[\frac{\pi dt^2\sigma_m}{m\cos^2\alpha}\right]_k.$$

The error parameter E in Eq. (27) has been minimized with respect to C by considering the twelve test cases listed in Table 6.

If the effect of strain rate is ignored and only the quasi-static values of yield and ultimate strengths of plate are considered, a value of 0.70 is obtained for C if Eq. (25) is used for predicting the test values of residual velocity given in [5] for ogival-head projectiles and mild steel plates of different thicknesses given in Table 6. If average strain rate, $\dot{\varepsilon}$ is computed using Eq. (26) and the values of $\sigma_y(\dot{\varepsilon})$ and $\sigma_f(\dot{\varepsilon})$ are estimated from Figs. 2 through 4 in [6], linear regression analysis using the test cases in Table 6 yields a value of 0.40 for C. The noticeably different value of C obtained in the latter case points out to the significant influence of high strain rates on the behaviour of target plates.

Using the regression-based values of C as mentioned above on the right side of Eq. (25), the final expressions for residual velocity can be written as:

$$v_r = \left[v_i^2 - \frac{2.20\sigma_m^o dt^2}{m\cos^2\alpha}\right]^{\frac{1}{2}}, \text{ by considering quasi-static value of } \sigma_m \text{ indicated as } \sigma_m^o; \tag{28}$$

$$v_r = \left[v_i^2 - \frac{1.26\sigma_m\left(\dot{\varepsilon}\right)dt^2}{m\cos^2\alpha}\right]^{\frac{1}{2}}, \text{ by using the strain rate-based value of } \sigma_m \text{ indicated as } \sigma_m\left(\dot{\varepsilon}\right). \tag{29}$$

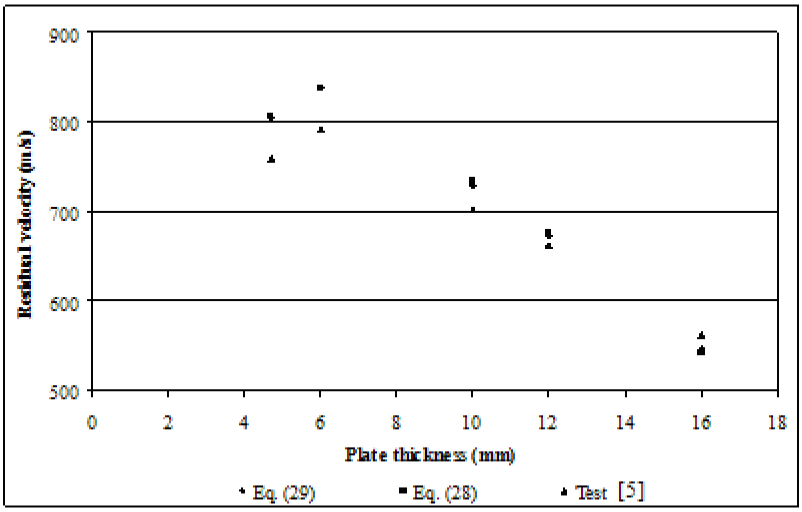

Figure 1 Predicted residual velocity comparison.

A comparison is given in Figure 1 between test-based residual velocities and those predicted by semi-empirical relations, (28) and (29). The predictions obtained from Eq. (28) which does not account for the effect of strain rate on plate material strengths are given in the seventh column of Table 6 and those from the strain-rate based model given by Eq. (29) are given in the last column of the same table. It is seen from the last three columns of Table 6 that the semi-empirical models given here yield reasonably good prediction of test residual velocities. As an example of the iterative procedure involved in the usage of the semi-empirical relation with strain rate-based average strength of target plate, the values of v_r obtained in successive steps of iteration are listed in Table 7 for the first case in Table 6 corresponding to mild steel target plate of type MS1 designated in [5]. All other values of v_r in the last column of Table 6 have been obtained following a similar approach.

A quantitative assessment of the degree of correlation of the residual velocities obtained with the present semi-empirical relations with respect to test data can be carried out with the aid of the following gross 'Correlation Index', CI:

$$CI = 1 - \left\{\frac{\sum e_i^2}{\sum V_r^2}\right\}^{\frac{1}{2}} \tag{30}$$

Table 6 Comparison of projectile residual velocities for normal and oblique impact (Projectile diameter, $d = 7.8$ mm, and mass, $m = 5.2$ grams).

Plate material in [5]	**Average quasi-static strength, σ_m^o (MPa)**	**Plate thickness, t (mm)**	**Angle of impact, α (degrees)**	**Impact velocity, v_i (m/s)**	**Estimated strain rate $\dot{\varepsilon}$ (S^{-1})**	**Residual velocity, v_r (m/s)**		
						Test [5]	**Quasi-static strength model, Eq. (28)**	**Strain rate-based model, Eq. (29)**
MS1	340.0	4.7	00	821.0	172910.92	758.6	806.3	804.4
MS2	521.0	6.0	00	866.3	142040.59	792.2	837.3	838.2
MS3	464.0	10.0	00	827.5	77781.20	702.2	732.9	728.1
			15	815.0	76066.47	690.4	711.4	706.3
			30	825.7	75789.16	654.0	696.4	690.1
			45	790.0	67746.02	500.0	573.4	565.1
		12.0	00	818.0	62075.15	661.5	675.7	671.8
			15	842.7	63861.66	671.6	694.5	690.0
			30	801.8	58209.22	598.0	599.6	595.2
		16.0	00	819.7	42679.25	562.0	542.3	546.0
			15	817.3	41725.23	544.4	512.9	518.3
			30	817.7	38650.59	496.3	405.9	419.1

Table 7 Computation of residual velocity for MS1 target plate in the last column of Table 6.

Iteration number	**Strain rate,** $SR = \frac{(V_i+V_r)}{2t}$ **(s^{-1})**	**Yield stress,** σ_y^{true} (computed using Fig. 2 of ref.[6]) **(MPa)**	**Failure stress,** σ_f^{true} (computed using Fig. 2 of ref.[6]) **(MPa)**	**Mean stress,** $\frac{(\sigma_y^{true}+\sigma_f^{true})}{2}$ **(MPa)**	**Residual velocity,** V_r **(m/s)**
0	87340.43	406.68	802.20	604.44	0
1	87340.43	406.68	802.20	604.44	805.5
2	173030.28	437.26	858.15	647.70	804.4
3	172910.92	437.23	858.09	647.66	804.4

where, V_r is the test residual velocity, e_i is the difference between computed and test residual velocities, and the summation is carried out over the number of cases for which a combined index of correlation is sought. It is apparent from Eq. (30) that as the degree of correlation increases (i.e. e_i decreases), CI (based on the $L2$ norm of error) approaches unity. The values of CI are calculated for the last two columns in Table 6 and are listed in Table 8 indicating that both the semi-empirical relations presented here are effective in predicting test residual velocities of projectiles for impact on mild steel target plates, although the model that accounts for strain rate effects on average strength of target plate performs marginally better.

Table 8 Correlation indices computed using Eq. (30).

Method of prediction of residual velocity	**Correlation Index (CI)**
Semi-empirical based prediction using strain rate-based model, Eq. (29)	0.95
Semi-empirical based prediction using quasi-static model, Eq. (28)	0.93

Ballistic limit for a given mild steel target plate can be obtained from Eq. (25) by setting $V_r = 0$, so that $V_i = V_{bl}$. Thus,

$$V_{bl} = \left[C \frac{\pi \sigma_m (\dot{\varepsilon}) dt^2}{m \cos^2 \alpha} \right]^{\frac{1}{2}} \tag{31}$$

Ballistic limits computed using Eq. (31) for MS1 plates studied in [5] for normal impact are listed in Table 9. It may be noted that no information on experimental ballistic limits of mild steel target plates is given in [5]. Thus, for two different cases of projectile diameter and mass, the semi-empirically computed ballistic limits are compared with the corresponding finite element-based values given in Fig. 14 in [6]. It is seen in Table 9 that though the two sets of semi-empirically predicted ballistic limits compare well with the simulation-based values, the ones obtained via the strain rate-based model are more conservative and hence, can be preferred for design.

Table 9 Comparison of predicted ballistic limits for 4.7 mm thick MS1 target plate.

Plate thickness (mm)	Projectile diameter (mm)	Projectile mass (grams)	Ballistic limit (m/s)		
			Numerically predicted [6]	Eq. (31) and, Eq. (28) or (29) and V_i= 821 m/s	
				Quasi-static strength-based, Eq. (28)	Dynamic strain rate-based, Eq. (29)
4.7	10	11.1	118.0	122.0	112.6
	15	37.6	79.0	81.1	73.6

5 CONCLUSIONS

In the present paper, semi-empirical relations for projectile residual velocity prediction, applicable to normal impact on a target plate, presented by earlier investigators have been reviewed. Two new regression-based relations employing an average strength of target plate and other readily-available parameters (i.e. mass and diameter of projectile, impact velocity and angle of impact, and plate thickness) have been presented for prediction of projectile residual velocity. The material parameter can be easily estimated from yield and failure strengths which are given by the supplier of the plate material. The present semi-empirical expressions differ in terms of usage of a quasi-static or strain rate-dependent average strength of target plate. The empirical constant appearing in either relation is determined by regression analysis of test data for mild steel target plates. For targets of other materials, the empirical constants may need to be re-derived. However, given the complexity of the present category of problems and reliance on test data for modelling material behaviour, the approach outlined here appears to be practical and useful for armour plate design. It has been shown that, in addition to residual velocities, the current semi-empirical relations are also capable of predicting ballistic limits.

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