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Comparison of Developed Numerical Macro and Micro Masonry Models for Static and Dynamic Analysis of Masonry-infilled Steel Frames

Abstract

First, the basic characteristics of macro and micro masonry models in numerical analyses of masonry structures are discussed. Afterwards, developed numerical macro and micro masonry models, intended for the nonlinear static and dynamic analysis of unreinforced and confined masonry walls, as well as of masonry-infilled frames, are briefly presented. The models are tested on previously performed experimental tests of masonry-infilled steel frame under horizontal static force and masonry-infilled steel frame on a shake-table. The precision of both models is compared. It is concluded that both numerical models can provide reliable results. However, the macro model has more advantages for wide practical application.

Keywords

masonry macro model, masonry micro model, comparison, masonry-infilled frames.

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1 INTRODUCTION

In the finite element analysis of masonry structures, whether they are composed of the masonry only or in combination with concrete or steel, two masonry models are mostly used: the so called macro and micro masonry models.

In the masonry macro model, this complex building material (made of brick elements, mortar and plaster) is simulated as isotropic or anisotropic homogenous material with the equivalent mechanical properties. The main problem of this model is to define representative mechanical properties of the masonry. However, it is well known that brick elements, mortar in joints and plaster have different stress-strain properties under normal and shear stresses. Besides, the behavior of the masonry is significantly influenced by the adhesive properties between these materials, which directly depends on the quality of the masonry manufacture. Therefore, in defining the corresponding representative normal and shear strength, as well as flexural and shear stiffness of the masonry, the adopted properties of the masonry modeled by the macro model should be studied in detail. The masonry macro model is advantageous since it permits large finite elements for simulation of the masonry, which reduces the number of unknowns in the system and shortens the analysis time. The macro model significantly simplifies the analysis; thus, its application is convenient in the analyses of complex structures. Deficiency of the model is in its inability to precisely describe the local effects in each material of the masonry and at their contact. However, if the corresponding representative mechanical properties of the masonry are adopted, the model can give reliable results regarding global behavior, damages and load-bearing capacity of masonry structures.

In the masonry micro model, the spatial discretization of the masonry is reduced to the simulation of brick elements and mortar in joints. More sophisticated micro models use contact elements to model the connection between the brick elements and mortar in joints. This approach provides possibility of real simulation of the masonry, with local effects in each material and at their contact. However, this approach demands a great number of relatively small finite and contact elements, i.e. a greater number of unknowns in the system and longer time of the analysis. Discretization with a great number of small finite elements, in combination with the order of numerical integration, can result in computational stiffness of the structure that quite deviates from its real stiffness. This problem can be particularly expressed in dynamic analyses of structures, due to known problems of numerical algorithms for time integration. Basically, this model has advantages in the analysis of simple structures where reliable experimental data regarding mechanical properties of each material and their contact are available. Since quality of the masonry is significantly influenced by its manufacture, this model also is not immune to this effect.

Therefore, the basic difference between the macro and micro model of the masonry is that the macro model has free spatial discretization of the masonry (independent on the size of a brick element) and this complex material is simulated as isotropic or anisotropic material with equivalent mechanical properties, while in micro model the spatial discretization is performed at the level of the brick element, mortar and their joint by using the corresponding isotropic or anisotropic constitutive models for each material and their connection.

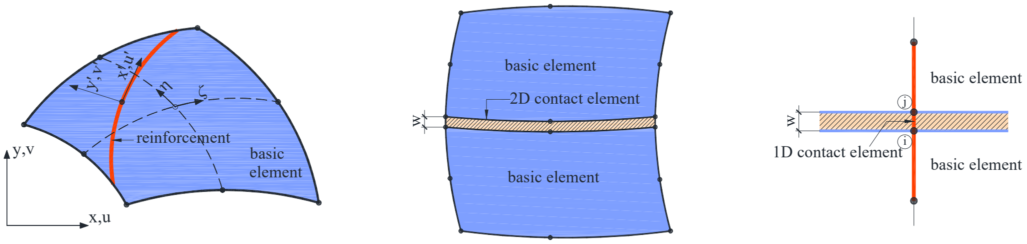
So far, a great number of macro and micro models of masonry for the analysis of simple and complex masonry structures have been developed, differing in their complexity and precision. Some of them are briefly described hereinafter. One of the first models to represent the nonlinear behavior of masonry was proposed by Page (1978, 1981, 1983). It considers the masonry as a two-phase material composed of bricks and mortar. The mortar joints are represented by linkage elements which can only deform in normal and shear directions. Lourenço et al. (1997, 1998) have proposed macro and micro models to describe the nonlinear behavior of masonry. The micro model is defined through joints which concentrate the inelastic behavior of the masonry and are able to reproduce three different types of failure mechanisms. Sayed and Shrive (1996) developed a nonlinear elasto-plastic finite-element model for face-shell-bedded hollow masonry using isoparametric shell elements. The nonlinear behavior of the masonry in compression due to progressive cracking as well as geometric and material nonlinearities were considered in the model. Zhuge et al. (1998) developed a comprehensive nonlinear finite element program to study the response of un-reinforced masonry to in-plane dynamic loads, including earthquake loads. Liu and Dawe (2003) developed and encoded for computer application an analytical technique to study the behavior of concrete masonry load-bearing walls under various loading conditions. Both geometrical and material nonlinearities to account for the moment magnification effect and the degradation of the material stiffness were included. There are many other numerical models intended for nonlinear analyses of masonry structures, such as: the micro-mechanical model for homogenization of Zucchini and Lourenço (2002), the macro model of Chen et al. (2008), the macro model of Crisafulli (1997) for confined masonry walls, the constitutive model for anisotropic quasi-brittle materials of Gesualdo and Monaco (2015), etc.

Firstly, this paper briefly presents the adopted numerical models of Radnic et al. (2011, 2013) and Baloevic et al. (2013) for static and dynamic analysis of planar unreinforced and confined masonry walls, as well as of masonry-infilled concrete and steel frames. The macro and micro models of masonry are used. Both models can simulate main nonlinear effects of the masonry. It is also possible to simulate main nonlinear effects of other materials in the structure (concrete, structural steel, soil), structure-soil interaction, formation of a structure in different time phases as well as geometric nonlinearity of the structure. Then, the adopted macro and micro masonry models are verified using the data from the two experimental tests of Baloevic (2015). The first test considered a masonry-infilled steel frame under horizontal static force. The second test considered a plastered masonry-infilled steel frame with an opening exposed to ground motion using a shake-table. Numerical results obtained by the masonry macro model and the micro model were compared to those from the experimental tests. Finally, main conclusions of this research are given.

2 PRESENTATION OF THE NUMERICAL MODEL

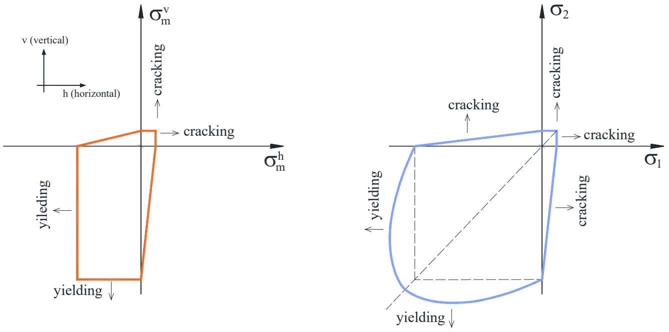
2.1 General

The adopted numerical model is described in detail in Radnic et al. (2011, 2013) and Baloevic et al. (2013), with brief presentation below. The model can simulate the main nonlinear effects of planar (2D) structures under static and dynamic loads, as stated previously. The finite element method is used for the spatial discretization of the analyzed problem. Finite difference method is adopted for the time discretization of the dynamic equation of motion. The implicit or the implicit-explicit Newmark's time algorithms can be used, developed in iterative form by Hughes. Basic 8-node serendipity finite elements are used (see Figure 1). 6-node planar and 2-node bar elements are adopted for contact elements. To include the effects of large displacements, updated Lagrange formulation is used. Convergence criterion of incremental-iterative procedure is given as a function of current displacements increment in relation to total displacements. Adopted constitutive models for considered structural materials are schematically presented in Figure 2, with a short description below.

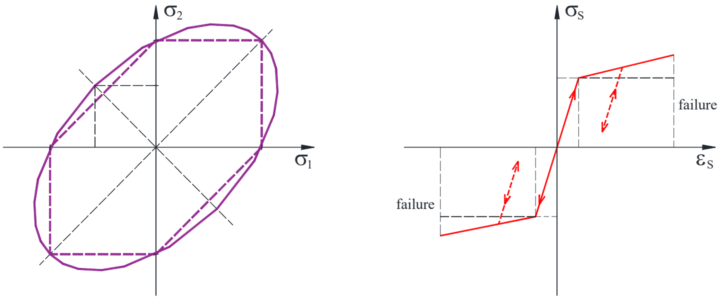


a) Basic 2D element b) 2D contact element c) 1D contact element

Figure 1: The adopted finite elements for spatial discretization of a 2D structure.



a) Masonry, Plaster, Soil b) Concrete, Soil



c) Structural steel d) Steel reinforcement

Figure 2: Adopted constitutive models for considered structural materials.

2.2 Material Models

2.2.1 Masonry Model

Macro and micro model of masonry can be used (see Figure 3). In the macro model (Figure 3b), spatial discretization is independent of brick elements and complex masonry (Figure 3a) is approximated by a representative material whose physical-mechanical properties adequately describe the actual complex masonry properties. In the micro model (Figure 3c), the spatial discretization can be performed at the level of brick elements, joints (mortar) and the contact surface between them. It is possible to use various micro models of masonry (for example, see Figure 3c1 and 3c2).

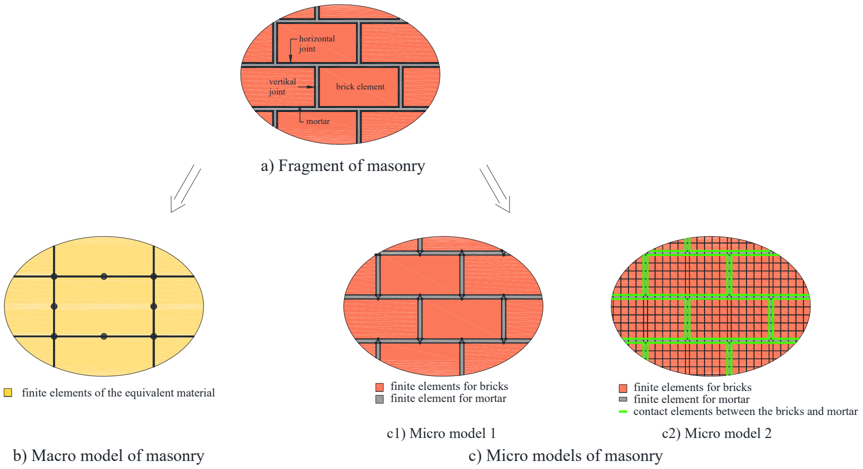


Figure 3: Macro and some possible micro models of masonry.

In the macro model, attention should be given to defining the adequate physical-mechanical parameters of representative idealized material. This material should describe the complex structures of brick elements, mortar in the joints and the connection characteristics between them. The developed constitutive model can simulate anisotropic properties of masonry, with different elasticity modulus, strengths (compressive, tensile, shear) and limit strains (compressive, tensile) in horizontal and vertical directions. The corresponding parameters of representative material should be determined based on the analysis of relevant data for brick elements, mortar and their connections.

Graphic presentation of the adopted orthotropic constitutive masonry model in compression and tension is given in Figure 2a. The separate masonry parameters in horizontal and vertical directions (compressive strengths, tensile strengths, elasticity modules and crushing compressive strains) should be defined. The effect of biaxial stresses to limit compressive strength of masonry is disregarded. An elastic-plastic model in compression and elastic-brittle model in tension are adopted. After the compressive strength of the masonry has been reached, an ideally plastic behavior of the masonry is adopted. The crushing criterion of the masonry is defined as a function of compressive strains. After the crushing criterion is reached, it is assumed that the masonry has no stiffness. The tensile stiffness of the cracked masonry is modelled in function of the tensile strain perpendicular to the crack plain. Opening and closing of cracks in the masonry is also modelled. Here, the model of fixed orthogonal smeared cracks is adopted. The transmission of compression stresses over the closed crack is modelled as in homogeneous masonry. After re-opening of the previously closed crack, the stiffness of the masonry is not taken into account. After the crushing in compression, it is assumed that the masonry has no stiffness.

Apart from tension (cracking) and compression (crushing), the collapse of the masonry due to shear is also modelled. The shear stiffness of cracked masonry is simulated by reducing the initial shear modulus of the masonry as a function of the tensile strain perpendicular to the crack plane.

The adopted micro model of masonry is analogous to the macro model of masonry, where spatial discretization of the masonry is performed at the level of brick elements, mortar in joints and their contact. Herein, brick elements and mortar are simulated with corresponding mechanical parameters of material from which they are made of, and their connection with contact elements that permit separation, penetration and sliding at the contact. The anisotropic material model can be used for brick elements that often have vertical holes. It is possible to use various micro models of masonry (some of them are presented in Figure 3), with various precision and duration of the analysis.

In the micro model 1 in Figure 3, masonry units are discretized by 8-node basic elements, and vertical and horizontal joints with 8-node basic or 6-node contact elements. In the micro model 2 in Figure 3, the masonry units and mortar are discretized by 8-node basic elements, and the contact of mortar and masonry units by thin 6-node contact elements. Other micro models i.e. different discretization of masonry can also be used.

2.2.2 Concrete Model

The adopted concrete model is graphically illustrated in Figure 2b. It is analogues to the masonry model, where it is assumed that the concrete is homogeneous and isotropic. Reinforcement (classical, cables, FRP) is simulated by bar element within basic concrete element (Figure 1a). Polygonal stress-strain relationship for bar material can be used (Figure 2d). It is assumed that there is no slipping between the concrete and reinforcement.

2.2.3 Steel Model

Biaxial failure of planar steel structures is modeled by the influence of normal stresses only. A classical elasto-plastic model for steel is used, with linear behavior in unloading (Figure 2c). Same behavior of steel in tension and compression is adopted. Von Mises yield criterion is used for steel yielding. The failure criterion of steel is defined as a function of principal strains, in an analogous way as a steel yielding.

2.2.4 Soil Model

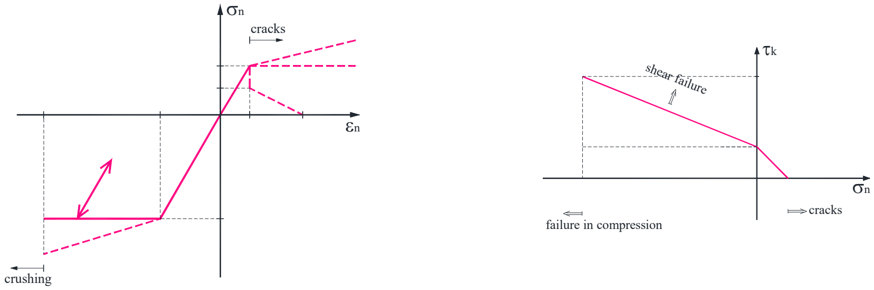
A special constitutive soil model has not been developed. A concrete model or masonry model can only be used. The material parameters should be properly defined based on the available soil parameters.

2.2.5 Plaster Model

The plaster is simulated by a simple approach where the finite elements of the masonry and plaster are overlapped. Here, the displacement compatibility exists at common node points, i.e. no sliding is permitted between the masonry and plaster. The same material models for concrete and masonry can be used to simulate the plaster. Modelling of the discrete mesh reinforcement in the plaster is the same as for the reinforcement in concrete.

2.2.6 Model for Contact Elements

Schematic graphical presentation of constitutive models for contact elements are presented in Figure 4. Planar (2D) contact elements transmit normal stress through the contact surface according to selected stress-strain relation. In this way, penetration and separation on the contact surface between different materials can be simulated. This contact element can also transmit shear stress along the contact surface, in a function of normal stress perpendicular to this surface; thus, simulating the sliding along it. 1D contact element simulates a bar that passes through the 2D contact element, and can model the transfer of normal and shear stresses according to the selected normal stress - normal strain and shear stress - normal stress diagrams.



a) Transfer of normal stresses b) Transfer of shear stresses

Figure 4: Schematic graphical presentation of constitutive models for contact elements.

3 VERIFICATION OF THE MASONRY MODELS USING EXPERIMENTAL RESULTS

3.1 Example 1

Detailed presentation of the adopted experimental test can be found in Baloevic (2015), with short description below. The basic data for one-bay, one-storey masonry-infilled steel frame that was tested is shown in Figure 5. The tested specimen represents a 1/3 scale model of a real structure, which was fabricated using real materials and construction methods. Bricks made of cellular lightweight concrete were used. The isotropic bricks were 208×83×80 mm in size. The brickwork was performed using prefabricated thin joint mortar. The steel frame was composed of rectangular hollow sections of 80×40×2.5 mm tube. All frame joints were welded and the frames were all constrained at the base. The columns and the lower beam were welded to the rigid base at the bottom in order to prevent rotation of the frame at the base.

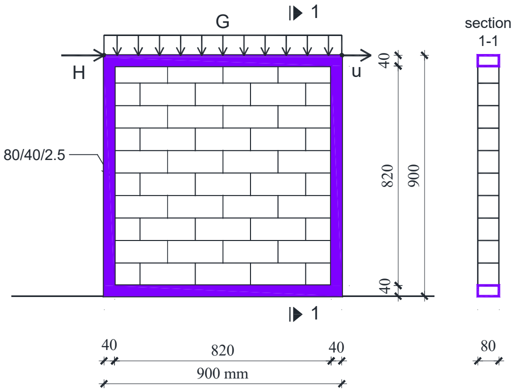


Figure 5: Basic geometry data of tested masonry-infilled steel frame in Example 1.

A dead load of G = 5 kN, made of a concrete block that simulated the load of the supported floor, was placed at the top of the frame. The horizontal force (H) was applied at the top of the frame and successively increased up to structural collapse. The frames were laterally constrained along the top steel tube to prevent lateral buckling of the columns. However, the intention was to exclude the effect of lateral buckling of the steel columns and masonry infill, and that the experiment could be adequately simulated with a 2D numerical model.

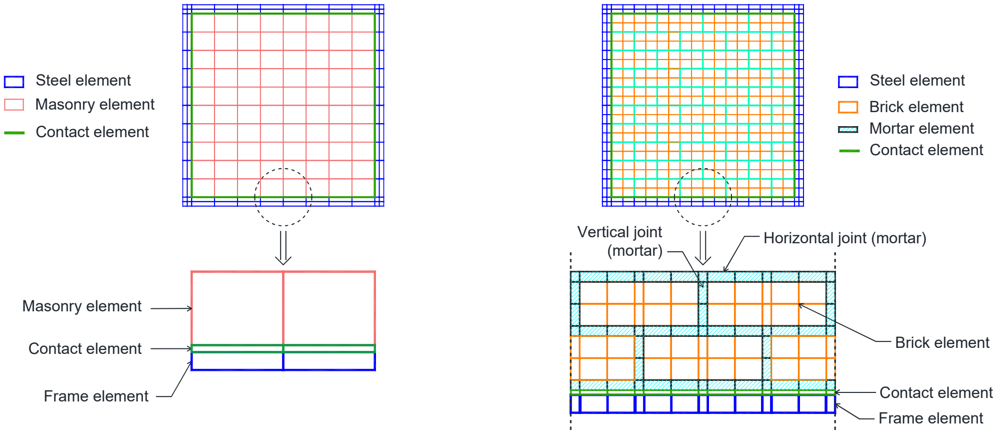
The steel had strength of 360 MPa and initial modulus of elasticity of 200 GPa. The brick elements had compressive strength of 3 MPa. The thin prefabricated mortar of class M10 had compressive strength of 11.4 MPa, and tensile strength of 3.1 MPa. Measured uniaxial compressive strength, tensile strength and the modulus of elasticity of the masonry were 2.18 MPa, 0.2 MPa and 1.02 GPa, respectively. The adopted basic material parameters for analysis with the macro and micro models are shown in Table 1.

Variable	Macromodel			Micromodel			
	Steel	Masonry	Contact	Steel	Brick	Mortar	Contact
Modulus of elasticity (GPa)	200.00	1.02	1.02	200.00	0.9	1.4	1.02
Poisson coefficient	0.30	0.15	-	0.30	0.15	0.18	-
Compressive strength (MPa)	360.00	2.18	2.18	360.00	2.30	11.4	2.18
Tensile strength (MPa)	360.00	0.2	0.02	360.00	0.21	3.1	0.02
Limit compressive strain	0.050	0.004	0.004	0.050	0.004	0.005	0.004

Table 1: Adopted main material parameters for numerical analysis.

Spatial discretization of the structure for numerical analyses is shown in Figure 6. In the macro model (Figure 6a), the frame and the masonry are modelled with basic eight-node elements. For more realistic simulation of interaction between the masonry and steel frame, six-node contact elements are used at their contact. In the micro model (Figure 6b), the frame, bricks and the joints (mortar) are modelled with basic eight-node finite elements. Six-node contact elements are placed between the frame and mortar, analogous to the macro model. The specimen with the macro model has a total of 224 elements and 733 nodes, while the specimen with the micro model has 1369 elements and 4256 nodes.

Only some results of the analysis are given and compared to the experimental results afterwards. The relationship between the horizontal force (H) and horizontal displacement of the top of the frame (u) is shown in Figure 7. It can be noticed that numerical results obtained by both models are in good agreement with the experimental results. Herein, the results obtained by the micro model are somewhat more precise than those obtained by the macro model. Up to about H = 20 kN, i.e. up to approximately 60% of the limit force H, both models show almost same results that are almost identical with the experimental results. By increasing nonlinearity in the system, a difference occurs between the experimentally determined and numerically obtained results due to the accuracy of the models. Greater accuracy of the results obtained by the micro model is a consequence of sophisticated spatial discretization of the frame and infill, as well as of applied separate constitutive models for brick elements and mortar (instead of the equivalent constitutive masonry model). Duration of the analysis with the micro model was about 4 times longer compared to the analysis with the macro model.



a) The macro model of masonry b) The micro model of masonry

Figure 6: The spatial discretization of the structure in Example 1.

Final deformation and crack state of the specimen at the end of the experiment is shown in Figure 8. Numerically obtained final crack states in the masonry for both models are shown in Figure 9. It can be noticed that numerical results for cracking and crushing zones in the masonry obtained by both models are also in good agreement with the experimental results. The experimental results indicate that damages in the masonry are dominantly located in the brick elements, which have significantly lower strengths than the mortar in joints. This was also confirmed by the numerical analyses.

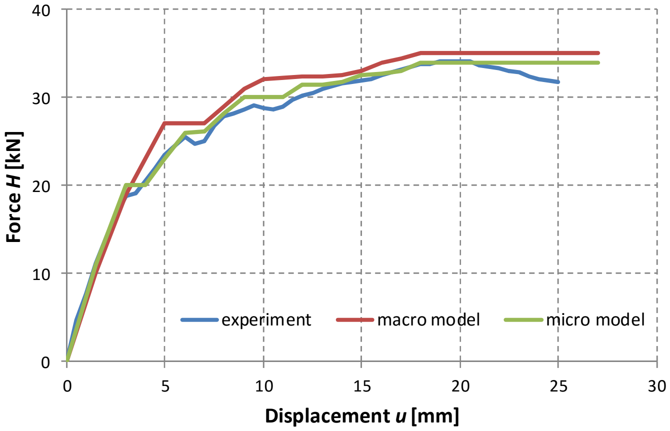


Figure 7: The comparison of experimentally measured and numerically obtained force-displacement relationship.

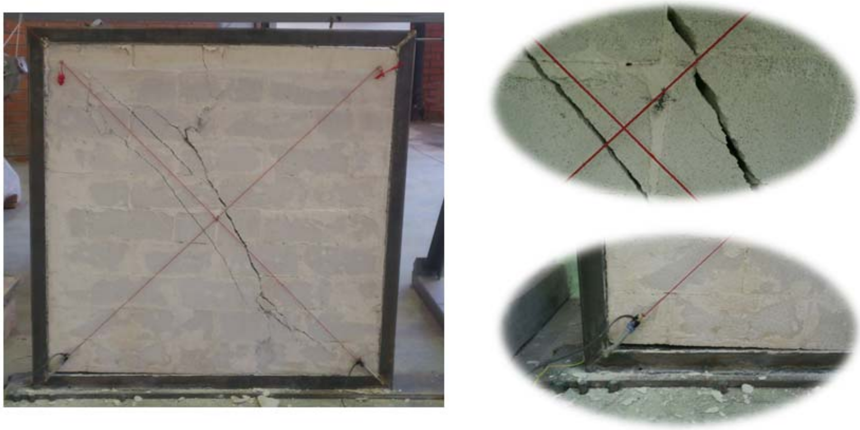
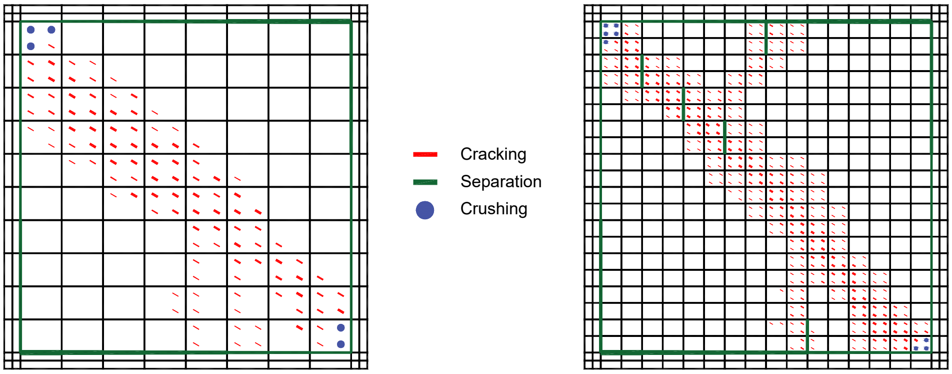


Figure 8: Final deformation and crack states at the end of experiment in Example 1.



a) The macro model

b) The micro model

Figure 9: Numerically obtained final deformation and crack states in Example 1.

3.2 Example 2

In this example, the adopted masonry macro and micro models were verified on the experimental results of a plastered masonry-infilled steel frame with an opening on a shake-table (see Figure 10). Detailed presentation of the adopted experimental test can be found in Baloevic (2015). An identical specimen as in Example 1 was used. Here, the specimen had central door opening and was strengthened with special mineral fiber-reinforced plaster in thickness of 10 mm on both sides. Additional mass of M = 3 tons made of a concrete block 1 × 1 × 1.2 m, which simulated the mass of the supported floor, was placed at the top of the frame. The block was fixed to the top of the frame. The frame was also laterally constrained at the top, same as the frame in Example 1.

Material properties of the steel, brick elements and mortar have already been described in Example 1. Measured uniaxial compressive strength, tensile strength and the modulus of elasticity of the plaster were 3.0 MPa, 1.0 MPa and 3.5 GPa, respectively. Spatial discretization of the specimen is shown in Figure 11. The same approach as in Example 1 was used for spatial discretization of the specimen using both macro and micro models. The plaster was simulated by overlapping the plaster elements with the masonry elements (macro model), i.e. with brick and mortar elements (micro model). The concrete block at the top of the frame was simulated as rigid. The specimen with the macro model has a total of 312 elements and 1048 nodes, while the specimen with the micro model has 823 elements and 2715 nodes.

The specimens were exposed to a set of horizontal base accelerations (a_g) of the shake table. The set of artificial accelerograms was created by scaling the original accelerogram in Figure 12. A total of eight artificial accelerograms were generated, where peak ground acceleration was PGA = n × 0.1 g; n = 1 - 8.

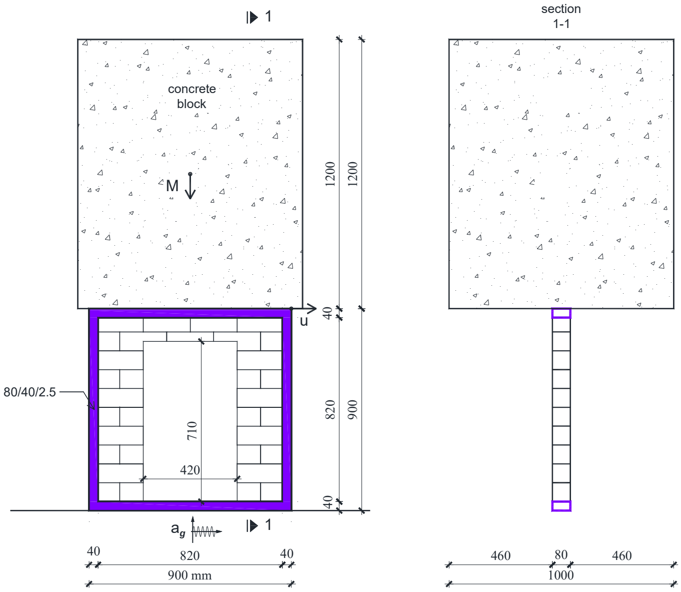
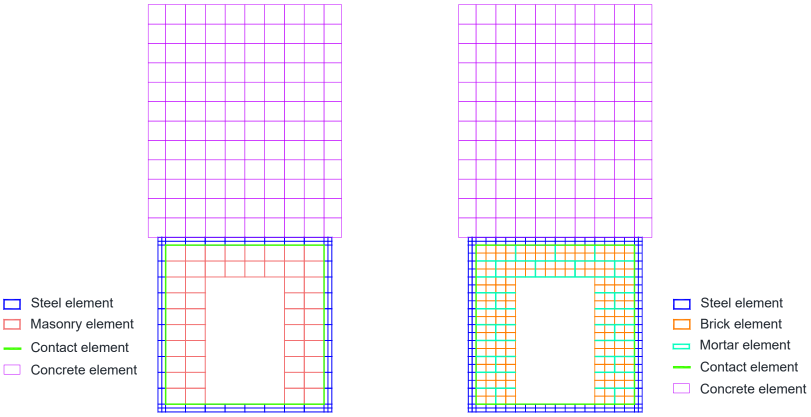
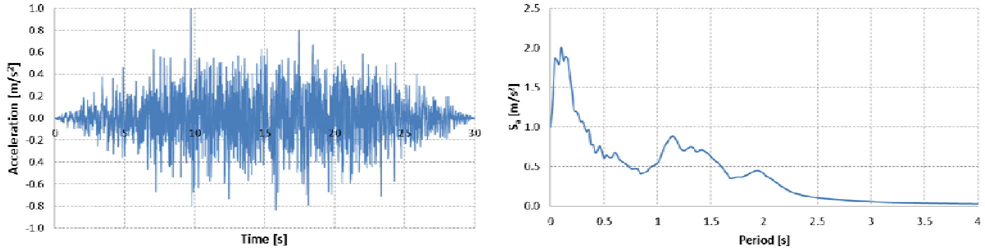


Figure 10: Basic geometry data of tested masonry-infilled steel frame in Example 2.



a) The macro model b) The micro model

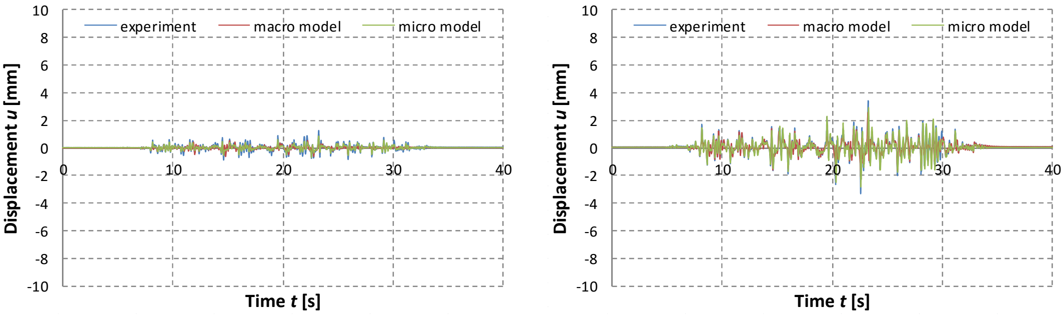
Figure 11: Spatial discretization of the tested specimen in Example 2.



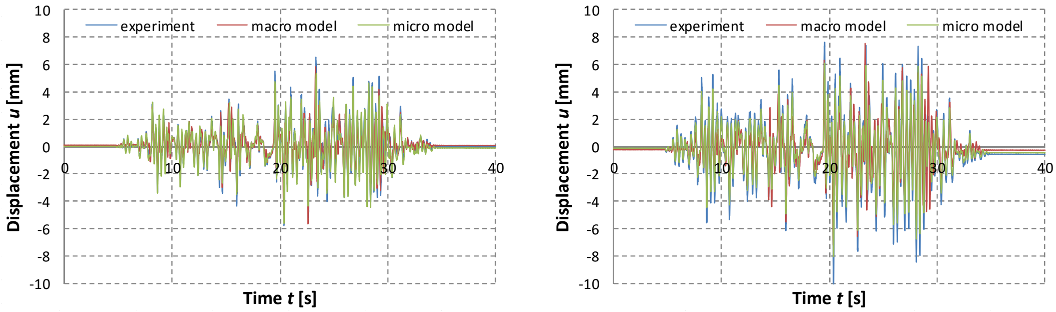
a) Acceleration time history b) Elastic response spectra

Figure 12: Artificial ground motion with PGA = 0.1 g.

Numerically obtained results are compared to the measured experimental results. The comparison of horizontal displacements at the top of the frame (u) in time is shown in Figure 13. As shown, numerical results obtained by both models are also in good agreement with the experimental results, with somewhat precise results of the micro model. For PGA = 0.8 g, maximal displacement at the top of the frame was experimentally measured as 10.0 mm, and numerically obtained values using macro and micro model were 7.5 mm and 8.0 mm, respectively. Greater accuracy of the results obtained by the micro model is probably a consequence of sophisticated spatial discretization of the frame and infill, as well as of applied separate constitutive models for brick elements and mortar (instead of the equivalent constitutive masonry model). Herein, duration of the analysis with the micro model was about 3 times longer compared to the analysis with the macro model.



a) PGA = 0.2 g b) PGA = 0.4 g

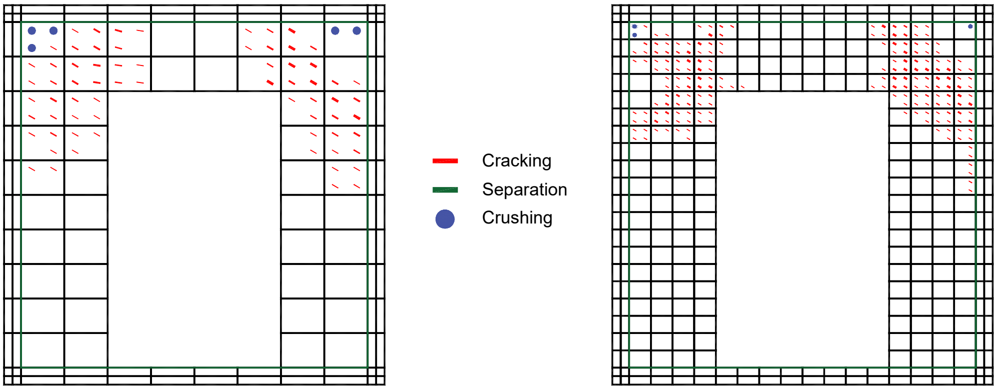


c) PGA = 0.6 g d) PGA = 0.8 g

Figure 13: The comparison of experimentally measured and numerically obtained horizontal displacements at the top of the frame in time.

Final crack state of the specimen at the end of experiment is shown in Figure 14. Numerically obtained final crack states using macro and micro models are shown in Figure 15. The numerical results for cracking and crushing zones obtained by both models are also in good agreement with the experimental results. The difference regarding the precision of the macro model and the micro model is small. Here, the failure of the masonry is also caused by overreaching the strength of the brick elements, which is significantly lower than the strength of mortar and adhesion between the mortar and the brick element.

Figure 14: Final deformation and crack states at the end of experiment in Example 2.



a) The macro model

b) The micro model

Figure 15: Numerically obtained final deformation and crack states in Example 2.

4 CONCLUSIONS

The presented numerical models for nonlinear static and dynamic analysis of planar unreinforced and confined masonry walls, as well as of masonry-infilled frames, can simulate their main nonlinear effects, such as: material nonlinearity, geometric nonlinearity (large displacements), contact problems and the formation of the structure over different phases. The model consists of separate constitutive models for concrete, steel, masonry, plaster and soil. The model is based on a small number of material parameters and can be useful in practical application. Macro and micro model can be used for masonry. In the macro model, a complex masonry is simulated by a representative homogenous isotropic or anisotropic material with equivalent mechanical properties, disregarding the local effects in each constitutive element (brick element, mortar, plaster). In the micro model of masonry, the spatial discretization can be performed at the level of brick elements, mortar in joints and their contact surface (with separate constitutive models for corresponding building materials), which provides possibility for better simulation of local effects. Verification of the considered macro and micro models of masonry was conducted on previously performed experimental tests of masonry-infilled steel frame under horizontal static force and masonry-infilled steel frame on a shake-table. Fairly good agreement is observed between experimentally determined and numerical results obtained by the macro and micro models. Herein, the masonry micro model can give somewhat more precise results. Herein, the micro model is more complex, it requires sophisticated spatial discretization and therefore greater number of nodes (unknowns) in the system, with significantly increased duration of the analysis. The micro model can be recommended for the analysis of simple structures, when mechanical characteristics of each masonry component are well known. The macro model has more advantages for wide practical application due to its simplicity and short time of the analysis. This research confirms the potential reliability of the developed numerical models for the static and dynamic analysis of planar masonry structures (unreinforced and confined masonry, infilled frames). However, further verifications of the presented model and corresponding computational software are the most welcome.

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