Latin American Journal of Solids and Structures
www.lajss.org

Simple Formulae for Control of Industrial Time Delay Systems

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Abstract
Control of time delay integrating systems is a challenging and ongoing research. In this paper a new structure for control of stable and integrating time delay systems is presented. The control design process is as simple as selection of some constant gains, for which simple formulae are introduced. The design methods are derived analytically, while no fractional approximation for the time delay term of the plant transfer function is used. Simulation, as well as, experimental studies reveal the exceptional effectiveness of the proposed methods in achieving a robust and well-performing tracking, even when the plant pure time delay is very large.

Keywords
Time-delay systems; Integrating processes; Tuning formulae; Uncertainty; Robustness; Input cost; Servo/regulator tradeoff.

http://dx.doi.org/10.1590/1679-78253032

Received 29.04.2016
In revised form 09.08.2016
Accepted 10.08.2016
Available online 29.08.2016

1 INTRODUCTION

Time delay is very often encountered in various industrial systems, such as pneumatic and hydraulic networks, chemical processes, long transmission lines, robotics, etc. A large group of industrial processes are stable, with a possible integrating and time delay nature, e.g., in a fluid level or distillation column level control problems (Alfaro and Vilanova, 2012). Control of time-delay systems has always been difficult, and if the system has integrating characteristics, this difficulty would be doubled, for the balanced relationship between the input and output may be easily destroyed by an external disturbance (Liu and Gao, 2011).

Smith predictor is one the oldest and most popular methods of control for time delay systems. Although the original method is only applicable to stable systems (Smith,1959), more recent development on the Smith structure can be applied to unstable time delay systems as well. Some of such methods are limited to integrating first order pure time delay systems (IFOPTD) (Kaya, 2003; Majhi and Atherton, 2000; Normey-Rico and Camacho, 2009; Shamsuzzoha and Moonyong , 2008; Uma and Rao , 2014) and some others involve complex algorithms (Garca andAlbertos, 2008; Hang, Wang and Yang, 2003; Kwak, Sung and Lee, 2001; Matausek andMicic, 1996; Matausek andRibi, 2012). Due to such complexities, compared to the original Smith Predictor, discrete-time version of time-delayed plants are used in many practical applications (Garca andAlbertos, 2013; Normey-Rico and Camacho, 2009; Torrico andNormey-Rico, 2005).

Application of PID controllers for time delay systems are proposed by many other researchers, although they are either applicable to stable plants such as in (Cvejn, 2013) or do not provide acceptable tracking and disturbance rejection properties (Ali andMajhi, 2010; Shamsuzzoha andLee, 2007; Wang, Hang and Yang, 2001). Since most of the PID-based methods, are based on the Pade' approximation of the time delay term, they provide poor performance when long time delays are involved (Tan, Marquez and Chen, 2003; Vanavil, Chaitanya and Seshagiri Rao, 2015). Similarly, many methods which are based on the internal model principle, are also based on the Pade' approximation and, therefore, provide acceptable disturbance rejection and reference tracking properties only for rather small time delays (Jin and Liu, 2014; Liu and Gao, 2011; Tan et al., 2003; Vanavil et al., 2015; Zhang, Rieber and Gu, 2008). Considering the well-known drawbacks of the existing methods, the objective of this paper is to provide a simple control structure with straightforward tuning guidelines, in which the closed loop performance and stability are guaranteed. The process of tuning the control parameters are very simple and only include substitution in some pre-specified formulas. The proposed method is tailored for application to the case of frequently seen industrial plants, as described in Section 2. The results of simulations are compared with some of other existing methods.

This paper is organized as follows: Problem statement and the proposed control structure are given in Section 2. In Section 3, the tuning rules are given for prescribed standard plant models. Closed loop performance of the proposed method is studied in Section 4. In Section 5, the results of simulations are compared with some methods reported in the recent literatures, and their strengths and weaknesses are investigated. An experimental case study is described in Section 6 where the speed control of an AC servo motor with deliberately induced long time delay is considered. A comparison between simulation and experimental studies is also given in Section 6. Concluding remarks are given in Section 7.

2 PROBLEM STATEMENT

Many industrial time delay stable and integrating systems can be approximated by one of the following simplified forms (Skogestad, 2003;Shamsuzzohaa and Skogestad, 2010):

1. Pure Time Delay System (PTD):

$$G_1(s) = ke^{-\theta s} \tag{1}$$

2. First Order Pure Time Delay System (FOPTD):

$$G_2(s) = k\frac{e^{-\theta s}}{\tau s + 1} \tag{2}$$

3. Integrating Pure Time Delay System (IPTD):

$$G_3(s) = k\frac{e^{-\theta s}}{s} \tag{3}$$

4. Integrating First Order Pure Time Delay System (IFOPTD):

$$G_4(s) = k\frac{e^{-\theta s}}{s(\tau s + 1)} \tag{4}$$

5. Double Integrating Pure Time Delay System (DIPTD):

$$G_5(s) = k\frac{e^{-\theta s}}{s^2} \tag{5}$$

where k is the system gain, τ is the time constant and θ is the dead time parameter.

The main purpose of this article is to provide a series of analytical tuning rules for such systems, which can guarantee the closed-loop stability and an acceptable level of performance and robustness.

3 PROPOSED METHOD

The proposed control structure is shown in Figure 1. In this figure, R is the reference input, v_1 is the plant input disturbance, v_2 is the plant output disturbance, y is the system output, $T(s)$ is the inner loop stabilizing controller, $C(s)$ is the main forward controller, and $\tilde{G}(s)$ is a feed-forward controller. The closed-loop response of the system in Figure 1, is given by

$$y = \frac{C(s)\,G(s)}{\Delta(s)}r + \frac{G(s)\left(1 + C(s)\tilde{G}(s)\right)}{\Delta(s)}v_1 + \frac{1 + C(s)\tilde{G}(s)}{\Delta(s)}v_2 \tag{6}$$

where,

$$\Delta(s) = 1 + C(s)\,G(s) + C(s)\tilde{G}(s) + T(s)\left(1 + C(s)\tilde{G}(s)\right)G(s) \tag{7}$$

The inner loop controller $T(s)$ is designed to guarantee the internal stability. Simple formulae for the controllers $C(s)$ and $\tilde{G}(s)$ are introduced, such that the closed-loop stability and performance of the systems 1-5 are guaranteed.

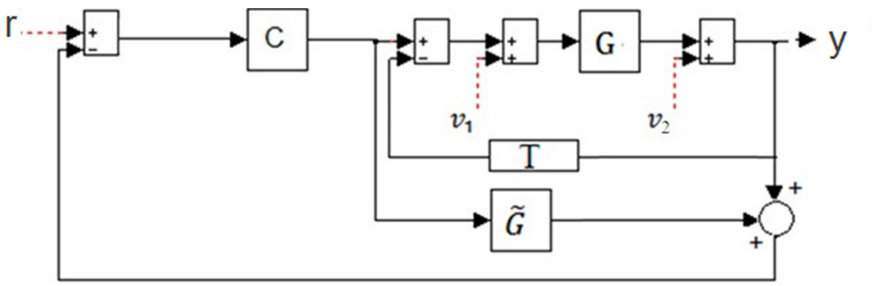


Figure 1: Proposed control structure.

For each of the systems (1)-(5), suitable controllers and tuning rules are proposed in the sequel.

3.1 PTD and FOPTD Plants

Since a PTD and FOPTD plants are stable, the inner loop controller in Figure 1 can be selected as $T(s)=0$. Since PTD plants are special cases of FOPTD plants, with $\tau=1$, similar control design methodologies can be used for the remaining controllers, $c(s)$ and $\tilde{G}(s)$, i.e.,

$$C(s)=k_d\frac{1}{\beta s+1}\frac{\tau s+1}{s(\lambda s+1)} \tag{8}$$

and

$$\tilde{G}(s)=k_p\frac{ks}{\tau s+1}e^{-\theta s} \tag{9}$$

The closed-loop characteristic equation is then given by

$$\Delta(s)=1+k_d k\frac{1}{\beta s+1}\frac{k_p s+1}{s(\lambda s+1)}e^{-\theta s} \tag{10}$$

Here, β is a to-be-tuned parameter, which must be selected according to the desired trade-off between the performance, robust stability and input cost. By selecting $\beta<1$, the following approximation holds (Skogestad, 2003)

$$1/(\beta s+1)\approx e^{(-\beta s)}$$

Then, (10) can be approximately written as

$$\Delta(s)\approx 1+k_d k\frac{k_p s+1}{s(\lambda s+1)}e^{-(\theta+\beta)s} \tag{11}$$

By using the results in (Matausek and Micic, 1996) the following lemma can be deduced:

Lemma 1: Consider the closed loop characteristic (11). Let

$$k_p=\alpha(\theta+\beta),\quad 0\le\alpha<1$$

Then, for $\lambda=k_p/10$, the closed loop stability is guarantied if the controller gain k_d is chosen as

$$k_d=\frac{\frac{\pi}{2}-\phi_m}{k(\theta+\beta)\sqrt{(1-\alpha)^2+(\frac{\pi}{2}-\varphi_m)^2\alpha^2}} \tag{12}$$

where, φ_m is the desired phase margin, $0\le\alpha<1$, and β is a to-be-tuned parameter.

It can be verified that, the closed-loop stability and robustness can be satisfied by selecting typical values $\alpha = 0.4$ and $\varphi_{\mathrm{m}} = 64^{\circ}$ (Matausek and Micic, 1996), the resulting control gain would then be as

$$k_d = \frac{0.724}{k(\theta+\beta)} \tag{13}$$

It also turns out that

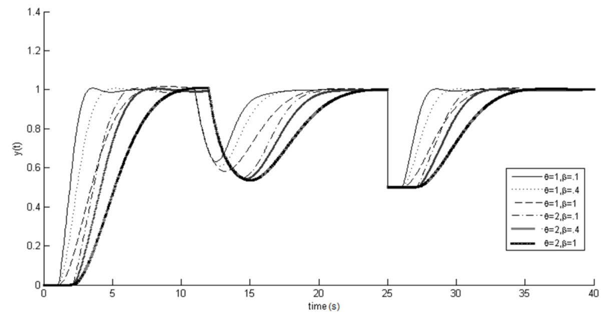
$$\lim_{t\to\infty} y_r(t) = \lim_{s\to 0} \frac{C(s)G(s)}{\Delta(s)} = 1$$

and

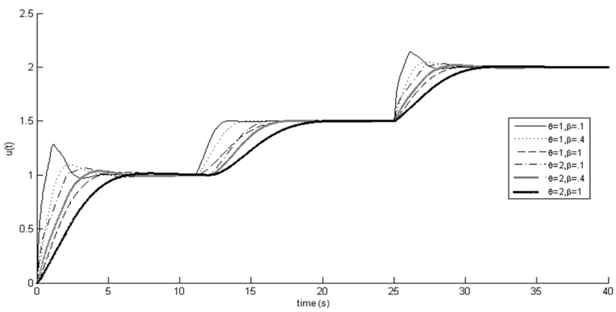
$$\lim_{t\to\infty} y_{v_1}(t) = \lim_{t\to\infty} y_{v_2}(t) = 0$$

This provides step disturbance rejection and step tracking properties.

Figures 2(a) and 2(b), respectively, depict the closed-loop step response and control input for different values of β and θ, and for $\tau = 1$ and $k = 1$. The time response due to two consecutive step disturbances at times 10sec and 25sec are also shown. It can be seen that the closed-loop settling time is increased for larger values of β. The associated resulting control input signals are also shown in Figure 2(b).



(a)



(b)

Figure 2: Effects of β on the time response (a) and control input (b) for $\tau = 1$, $k = 1$ and $\theta = 1,2$.

3.2 IPTD Plants

In order to preserve the stability of the inner loop, a constant gain controller $T(s)=k_i$ is selected, i.e.,

$$1+T(s)G(s)=1+k_i\frac{k}{s}e^{-\theta s} \tag{14}$$

In order to achieve a phase margin of 60° and a gain margin of 3, the following gain is chosen:

$$k_i=\frac{0.5236}{k\theta} \tag{15}$$

The controllers $\mathrm{C}(s)$ and $\widetilde{G}(s)$ are then obtained as

$$C(s)=k_d\frac{1}{\beta s+1}\frac{\left(1+\frac{0.5236}{\theta s}e^{-\theta s}\right)}{\lambda s+1} \tag{16}$$

and

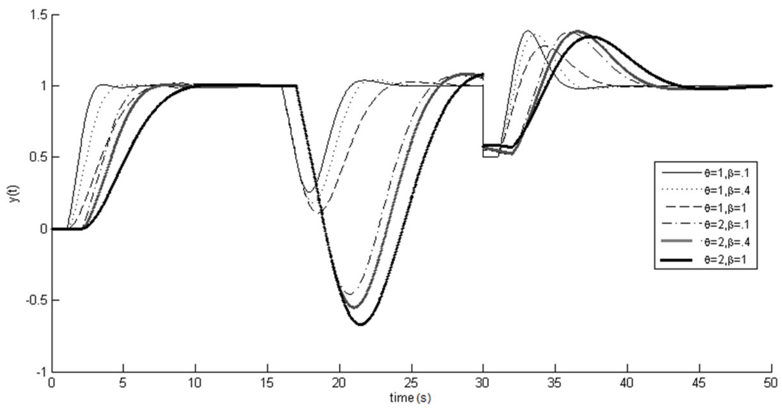
$$\widetilde{G}(s)=k_pk\frac{e^{-\theta s}}{\left(1+\frac{0.5236}{\theta s}e^{-\theta s}\right)} \tag{17}$$

The resulting closed-loop denominator is

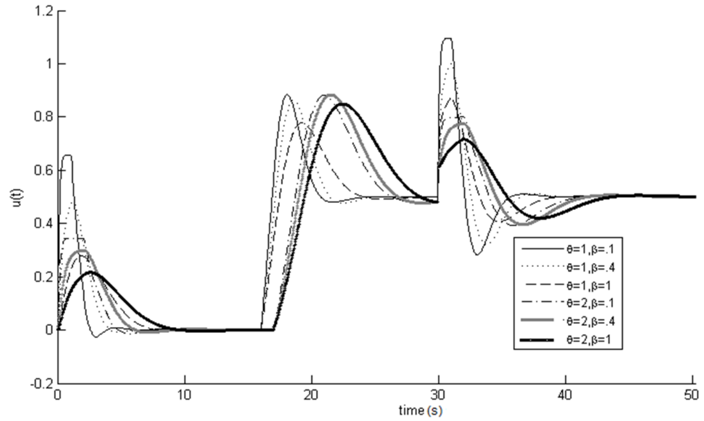
$$\Delta(s)=\left(1+k_dk\frac{1}{\beta s+1}\frac{k_ps+1}{s(\lambda s+1)}e^{-\theta s}\right)\left(1+\frac{0.5236}{\theta s}e^{-\theta s}\right) \tag{18}$$

Again, the value of parameter k_d is determined using (12). In particular, for $\alpha=0.4$ and $\varphi_\mathrm{m}=64^\circ$, k_d can be obtained from (13). It can be simply verified that, $\lim_{t\to\infty}\mathrm{y_r}(\mathrm{t})=1$ and $\lim_{t\to\infty}\mathrm{y}_v(\mathrm{t})=0$, as is desired.

In Figure 3(a), for several values of the parameters β and θ, andwith $k=1$, the closed-loop step response due to two consecutive step disturbances are depicted. It can be seen that, the closed-loop settling time increases for larger values of β. The effects on the control signal is shown in Figure 3(b). It can be seen that, the closed-loop performance for input tracking and disturbance rejection is worsened with an increase in β, although it leads to a smoother control signal and reduced overshoot.



(a)



(b)

Figure 3: Effects of β on the time response (a) and control input (b) for $k=1$.

3.3 IFOPTD Plants

To preserve the stability of the inner-loop, $T(s)=k_i\dfrac{\tau s+1}{\gamma s+1}$ is selected, then

$$1+T(s)G(s)=1+k_i k\frac{e^{-\theta s}}{s(\gamma s+1)} \tag{19}$$

By selecting $\gamma=\tau/10$, the effect of the low-pass filter in the above equation can be neglected for computation of the phase and gain margins; therefore, parameter k_i can be obtained from (15).

As before, the controllers $C(s)$ and $\widetilde{G}(s)$ are selected such that the closed-loop stability and performance are satisfied, i.e.,

$$C(s)=k_d\frac{1}{\beta s+1}.\frac{\tau s+1}{\lambda s+1}\left(1+\frac{0.5236}{\theta s(\gamma s+1)}e^{-\theta s}\right) \tag{20}$$

$$\tilde{G}(s) = k_p k \frac{e^{-\theta s}}{\left(1 + \frac{0.5236}{\theta s(\gamma s+1)} e^{-\theta s}\right)(\tau s+1)} \tag{21}$$

The closed-loop denominator is obtained as

$$\Delta(s) = \left(1 + k_d k \frac{1}{\beta s+1} \frac{k_p s+1}{s(\lambda s+1)} e^{-\theta s}\right)\left(1 + \frac{0.5236}{\theta s(\gamma s+1)} e^{-\theta s}\right) \tag{22}$$

By using (12), the value of the parameter k_d can be obtained. Through simulation studies,it can be further concluded that an increase in β leads to a smoother control signal and a slower time response.

3.4 DIPTD Plants

To preserve the stability of the inner-loop, $T(s) = k_i \frac{T_d s+1}{(T_d/Ns+1)s^2}$ is selected. Then,

$$1 + T(s)G(s) = 1 + k_i k \frac{T_d s+1}{(T_d/Ns+1)s^2} e^{-\theta s} \tag{23}$$

N is a largenumber and chosensuchthat$T_d/N \ll 1$. Also selecting $\tau = 8\theta$ makesthispossible to use the PD structure proposed in (Skogestad, 2003). Therefore, the control parameters are found as

$$k_i = \frac{0.0625}{k} \frac{1}{\theta^2} \quad and \quad T_d = 8\theta \tag{24}$$

For retaining the closed-loop stability, disturbances rejection and reference tracking properties, the controllers $C(s)$ and $\tilde{G}(s)$ can be simply obtained as

$$C(s) = k_d \frac{1}{\beta s+1} \frac{s}{\lambda s+1}\left(1 + \frac{0.0625}{\theta^2} \frac{T_d s+1}{(sT_d/N+1)s^2} e^{-\theta s}\right) \tag{25}$$

and

$$\tilde{G}(s) = k_p \frac{k}{s} \frac{e^{-\theta s}}{\left(1 + \frac{0.0625}{\theta^2} \frac{T_d s+1}{(T_d/Ns+1)s^2} e^{-\theta s}\right)} \tag{26}$$

Again, the value of parameter k_d is determined using (12). In particular, for $\alpha = 0.4$ and $\varphi_m = 64^\circ$, k_d is obtained. It can be simply verified that, $\lim_{t\to\infty} y_r(t) = 1$ and $\lim_{t\to\infty} y_v(t) = 0$, as is desired.

In Figure 4, for several values of the parameters β, $\theta = 1$ and with $k = 1$, the closed-loop step response due to two consecutive step disturbances are depicted. It can be seen that, the closed-loop settling time increases for larger values of β. The effects on the control signal are shown in Figure 5(a) and Figure 5(b). It can be seen that, the closed-loop performance for input tracking and disturbance rejection is worsened with an increase in β, although it leads to a smoother control signal and reduced overshoot.

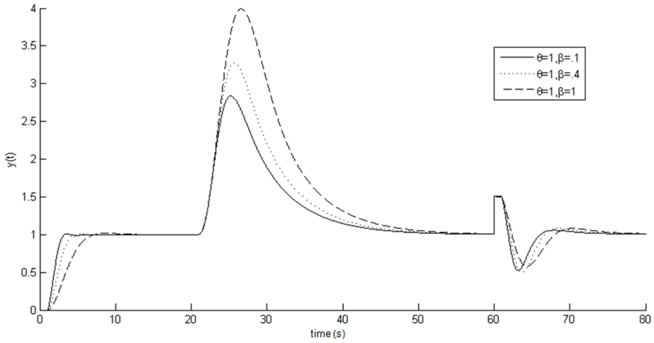
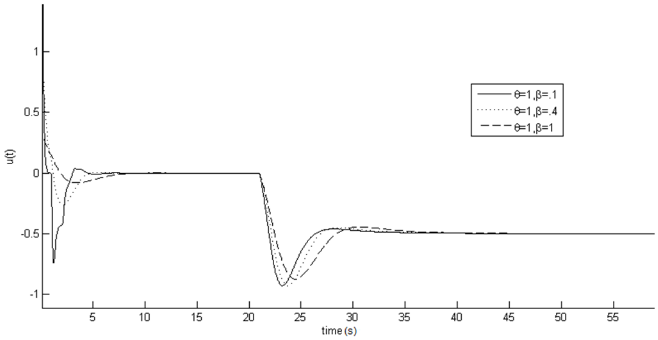
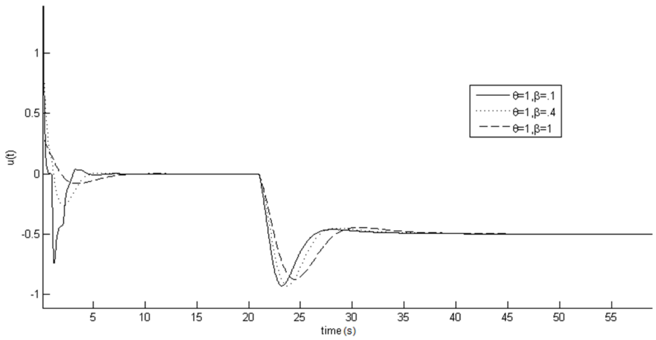


Figure 4: Effects of β on the time responses, with $k = 1$.



(a)



(b)

Figure 5: Effects of β on the control signal for rejection of disturbances v_1 (a) and v_2 (b), for $k = 1$.

3.5 Performance and Robustness

Time domain performance and robustness of the proposed method are studied in this section.

3.5.1 Time Response Index

The integral absolute error (IAE), defined for the error signal $y-y_s$, is an important index for assessment of the closed-loop system performance, which is defined as

$$IAE=\int_0^{\infty}|e(t)|\,dt \tag{27}$$

Numerical solutions (by using the Matlab regression toolbox) are employed for calculation of this index for the controlled system, considering various kind of plants as described in (1)-(5). The reference and disturbance inputs are considered as unit steps. Parametric study on the effects of θ is also carried out and using the regression method, simple correlations with respect to θ are reported in table 1. It can be seen that, the IAE varies from 1.8θ for systems given by (1)-(4), and up to of 2.8θ for the system given by (5) whereas based on the results obtained from the so-called SIMC method (Skogestad, (2003)), the IAE varies from 2.17θ to 7.92θ. The IAE(y) value for load disturbance v_1, varies from $1.8\,k\,\theta$ to $3.8\,k\theta$. Based on results obtained from SIMC method, the IAE value due to the disturbance v_1 varies from 2.17θ to $128\,\theta^3$, which indicates the high sensitivity of the IAE value to an increase in the system time delay.

3.5.2 Control Input

In order to evaluate the smoothness of the required control input, the index TV is defined as

$$TV(u)=\int_0^{\infty}\left|\frac{du}{dt}\right|dt\approx\sum_{i=0}^{\infty}\left|u_i-u_{i-1}\right| \tag{28}$$

This index characterizes the overall variations of $u(t)$, which should be reasonably small. This ensures that the un-modeled higher order dynamics of the plant is not excited by the control input.

The index TV values due to a unit step command R, and a unit step disturbance v_1, are listed in the table 1. Based on the results, TV (u) value ranges from 1 (for PTD plants) up to 2.9 (for IFOPTD plants). Parametric study on the effects of θ is also carried out and using the regression method, simple correlations with respect to θ are reported in table 1.The TV(u) value for a unit step command ranges from 1 (for PTD plants), to $\dfrac{.81\theta^2+32\theta-21}{k(\theta+0.1)^2}$ (for DIPTD plants). In deriving these results, $\beta=0.1$ was assumed. By making changes to β, a desired trade-off between the time response and the smoothness of the control input can be achieved.

Based on the obtained results, the values of TV(u) for the set-point and the disturbance v_1 are in the satisfactory level, and the control usage in the beginning is of a small value order, which is desirable from practical point of view. Next, the control signal changes are studied through some examples and compared with required control usage of other methods.

We would see through simulation studies that some of the methods reported in the recent literatures require an unbounded control signal for rejection of plant output disturbances (Jin and Liu, 2014), (Alcantara et al., 2013).

3.5.3 Robustness

Sensitivity and complementary sensitivity functions, respectively denoted by $S(s)$ and $CS(s)$, are two conventional criteria for evaluation of closed-loop system robustness. For the general structure of the proposed controller of Figure 1, those functions are obtained as

$$S(s)=\frac{1+C(s)\widetilde{G}(s)}{\Delta(s)}, CS(s)=1-S(s) \tag{29}$$

The maximum sensitivity function is defined as $M_S=\|S(j\omega)\|_{\infty}$, the M_S value is equal to the inverse of the shortest distance from point -1 in the open loop Nyquist diagram. Typical values of M_S should be in the range of 1.4-2 (Astrom and Hagglund, 1995). Furthermore, $M_{CS}=\|CS(j\omega)\|_{\infty}$ is inversely related to the step response overshoot, and also to the PM and GM through the following relations:

$$GM \geq 1+\frac{1}{M_{CS}} \quad , \quad PM \geq 2sin^{-1}(\frac{1}{2M_{CS}})$$

According to table 1, for each of the systems (1)-(5), $M_{CS}=1.05$, i.e., P$M \geq 56.8$ and $GM \geq 1.95$. The values of M_S for DIPTD, IPTD, IFOPTD systems exceed the upper bound value of 2, yet, lead to large reductions on the IAE(y). Next, the effect of this parameter on the rejection of the input disturbance and robustness against uncertainty will be shown through some examples and compared with other methods. The controllers are designed for a nominal value of θ, but the actual value of this parameter may change during the system's operation. Thus, a robust controller should be effective in a wide range of uncertainty in θ, therefore, the term $\Delta\theta/\theta$ can be considered as alimiton system stability. As shown in Table 1, the value of this term is 0.5 for DIPTD models, and could vary up to 1.85 for IPTD and IFOPTD models. In other words, the proposed method is robust against the time delay uncertainty of about 50% to 185%.The results obtained throughout this section are summarized in Table 1.

4 SIMULATION STUDIES

In this section, the effectiveness of the proposed method is shown via detailed comparisons with other methods proposed in the recent literature. In Example 1, a FOPTD plant with large time delay is considered. Example 2 considers the comparison to a classical approach applied to a PTD plant. An IPTD plant with large time delay is studied in Examples 3, and finally a comparison study is carried out on a DIPTD plant.

Plant	k_d	λ	k_i	γ	T_d	M_S	M_{CS}	$\Delta\theta/\theta$	IAE_s[a]	TV_s[b]	IAE_l[c]	TV_l[d]
FOPTD[e]	$\frac{0.724}{k(\theta+\beta)}$	$0.04(\theta+\beta)$	-	-	-	1.756	1.05	1.85	1.8 θ	$\frac{6.5}{k}$	1.8 k. θ	1
PTD	$\frac{0.724}{k(\theta+\beta)}$	$0.04(\theta+\beta)$	–	-	-	1.76	1.05	1.85	1.8 θ	$\frac{1}{k}$	1.9 k. θ	1
IPTD	$\frac{0.724}{k(\theta+\beta)}$	$0.04(\theta+\beta)$	$\frac{0.5236}{k.\theta}$	-	-	2.7	1.05	0.56	1.8 θ	$\frac{1.52}{k(\theta+.1)}$	3 k. θ	2.9
IFOPTD[f]	$\frac{0.724}{k(\theta+\beta)}$	$0.04(\theta+\beta)$	$\frac{0.5236}{k.\theta}$	$\tau/10$	-	2.71	1.05	0.53	1.8 θ	$\frac{.4}{k(\theta+0.1)^2}$	2.9 k. θ	2.9
DIPTD	$\frac{0.724}{k(\theta+\beta)}$	$0.04(\theta+\beta)$	$\frac{0.5236}{k.\theta}$	-	8. θ	2.73	1.05	0.5	2.8 θ	$\frac{0.81\theta^2+32\theta-21}{k(\theta+.1)^2}$	3.8 k. θ	2.8

[a] The IAE values for unit step command with $\beta = 0.1$

[b] The TV values for unit step command with $\beta = 0.1$

[c] The IAE values for unit step load disturbance, v_1 with $\beta = 0.1$

[d] The TV values for unit step load disturbance, v_1 with $\beta = 0.1$

[e,f] Here, for calculation of M_S, M_{CS}, $\cdots$, IAE_l^b, the assumption $\tau = 4\theta$ was made. The proposed values for k_d, λ and k_i were obtained independent of the parameters τ and β.

Table 1: Proposed method: Settings and performance indices for various time delay plants.

Example 1 (FOPTD plant with large dead time)

Consider

$$G(s) = \frac{0.5}{1.5s+1}e^{-9s}$$

which is in the form of (2). The proposed controllers are in the form of (8) and (9), for which the required parameters are very simple to find from Table 1. In particular, for $\beta = 0.1$, the values of $k_d = 0.159$ and $\lambda = 0.364$ are obtained.

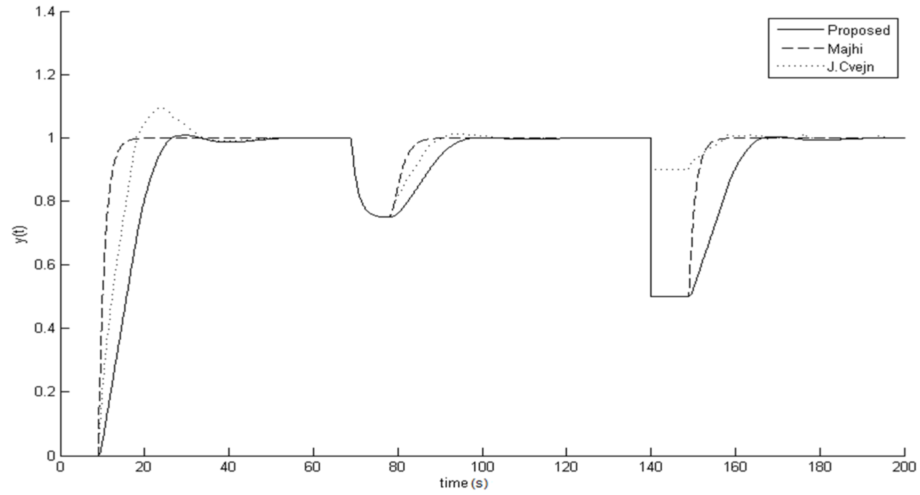
For the purpose of comparison, the methods of Maghi (Majhi and Atherton, 2000) and Cvejn (Cvejn, 2013) are also considered, where the former approach provides controllers

$$G_m(s) = \frac{1}{3s+2}, \quad G_c(s) = 2\frac{1.5s+1}{1.5s}$$

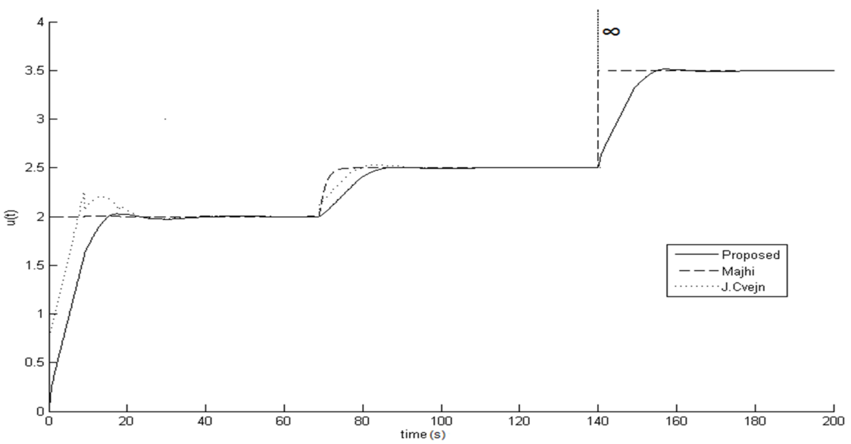
and the latter method gives rise to the PID controller

$$C(s) = \frac{1}{6}\frac{4.5s^2+4.5s+1}{s}$$

The responses of the system to a unit step command and disturbances are shown in Figure 6(a). Results show that the proposed method has a better time response compared to the method of Cvejn. Method of Maghi has a good performance in terms of reference input tracking and disturbance rejection. Figure 6(b) shows the control input signal for the three studied methods, based on which, the Cvejn'smethod needs a larger control input for rejection of the output step disturbance. Maghi's method needs a non-zero control signal at the beginning, which may not be desirable from practical point of view. The required control input with the proposed control system is completely smooth and without overshoot, and for step disturbances, v_1 and v_2 (see figure 6(c)) remains in an acceptable range. In order to study the robustness of the proposed method, the system responses to a unit set-point and step disturbance are illustrated in Figure 6(d), with 30% increase in the presumed time delay. Results show that Maghi's method is not resistant to time delay uncertainty and leads to instability in the closed-loop system. Cvejn's method is more resistant to the variations of θ, albeit with a more sluggish time response.



(a)



(b)

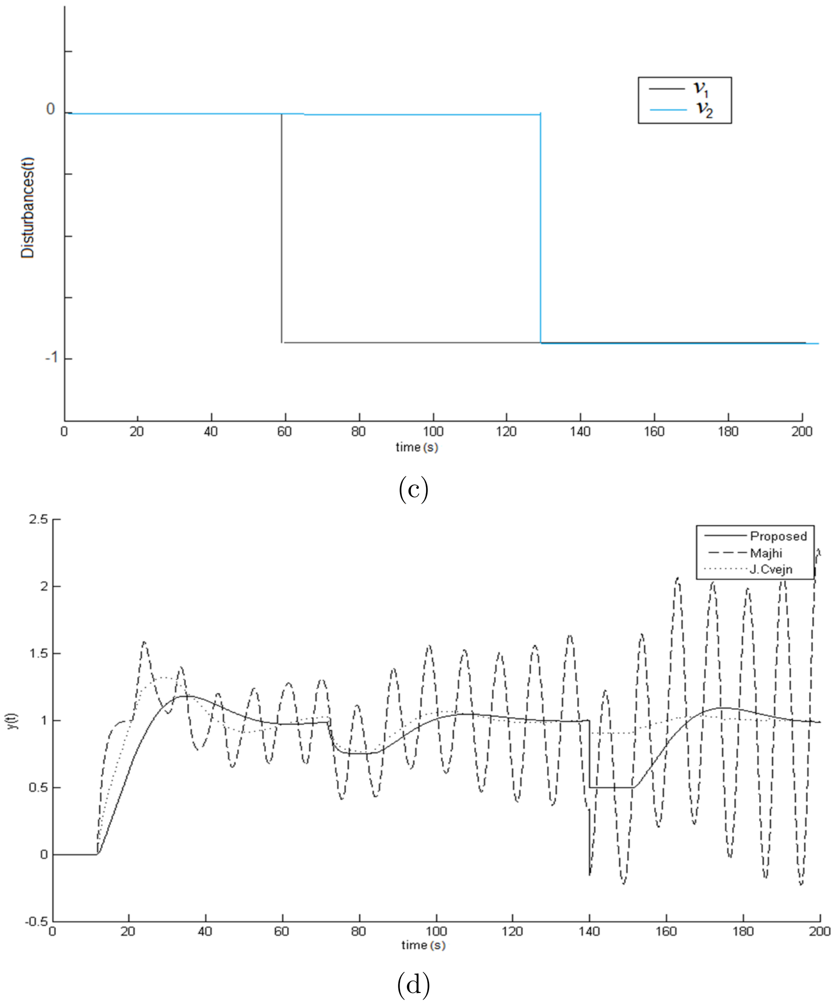


Figure 6: Effects of β on time response (a) and control input (b) on rejection of disturbances v_1 and v_2 (c), in Example 1. Also, the time response to a unit set-point and step disturbances with +30% increase in θ is shown (d), which clearly shows the effectiveness of the proposed method.

Example 2 (PTD plant)

Consider

$$G(s) = e^{-s}.$$

which is in the form of (1). The proposed controllers are in the form of (8) and (9), for which the required parameters are found from Table 1. In particular, for $\beta = 0.1$ and $\tau = 0.01$, the values of $k_d = 0.685$ and $\lambda = 0.044$ are obtained.

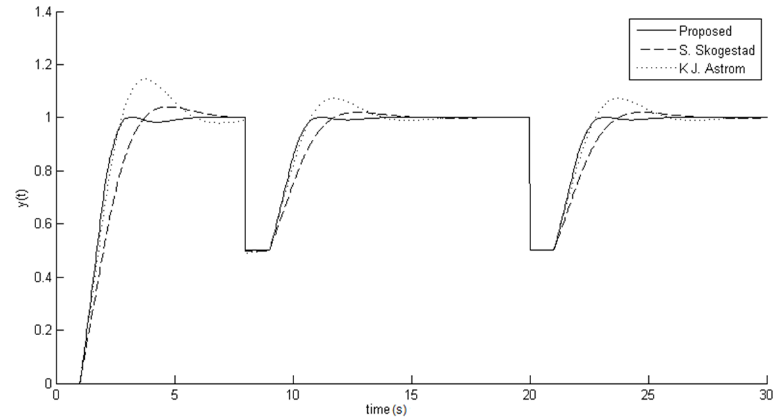
For the purpose of comparison, the methods of Astrom (Astrom et al., 1995) and Skogestad (Skogestad, 2003) are also considered, for which the required controllers are respectively found as

$$C(s) = \frac{0.5}{s}$$

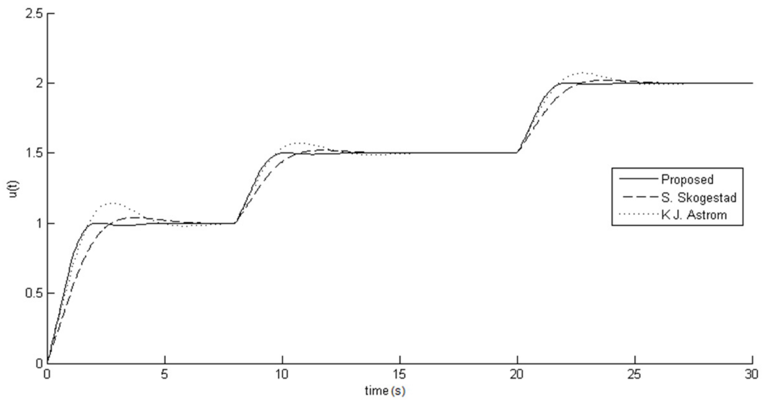
and

$$C(s) = 0.16 + \frac{0.4724}{s}.$$

The responses to unit step command and disturbances are shown in Figure 7(a). The achieved results show that the proposed method is superior in terms of performance indices for reference tracking and disturbances rejection. Figure 7(b) shows the control input signal for the three studied methods, where, the required control input with the proposed control system turns out to be desirable from practical point of view.



(a)



(b)

Figure 7: Time responses due to a unit set-point and step disturbances, v_1 and v_2 (a), and the corresponding control inputs (b), in Example 2.

Example 3 (IPTD plant with long dead time)

Consider

$$G(s)=\frac{e^{-7.4s}}{5s}.$$

which is in the form of (3). The proposed controllers are in the form of (16) and (17), for which the required parameters are found from Table 1. In particular, for $\beta=0.5$, the values of $k_d=0.458$, $k_i=0.354$ and $\lambda=0.316$ are obtained.

The proposed method is compared with the methods of Zhang (Zhang et al., 1999), Ali (Ali and Majhi,2010), Kaya (Kaya, 2003) and Jin (Jin and Liu,2014). The method of Zhang provides the PID controller

$$C(s)=0.365\frac{407s+10}{\mathrm{s}(999s+490)}$$

The method of Ali gives rise to the controller

$$C(s)=0.696(1+\frac{1}{23.46\,s}+\frac{3.626\,s}{0.3626s+1})$$

The Kaya's controllers, with the notation used in (Kaya,2003), are derived as

$$G_{c1}=1+\frac{1}{0.5\,s}$$

$$G_{c2}=4.43$$

and

$$G_d=0.642(1+4.68s)$$

for $\alpha=0.633$ and $\varphi_{\mathrm{m}}=65^{\circ}$ Finally, the PI controller obtained by the method of Jin is

$$C(s)==0.384\left(1+\frac{1}{35.79\,s}\right)$$

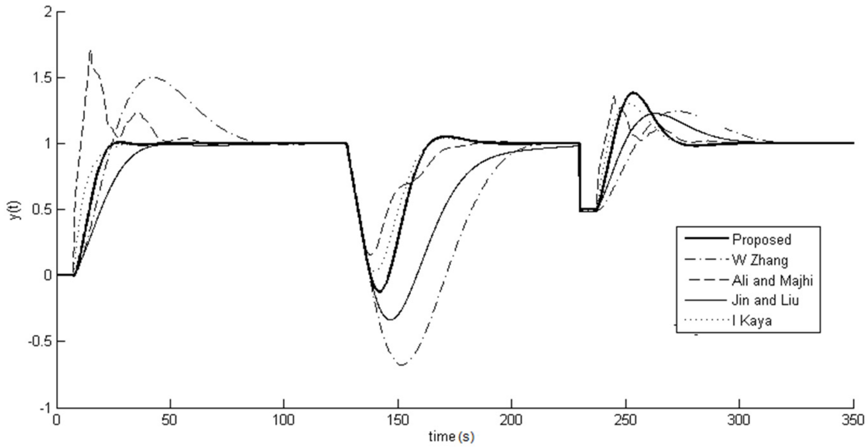
and the corresponding reference input filter turns out to be as

$$W(s)=\frac{14.32s+1}{35.79s+1}$$

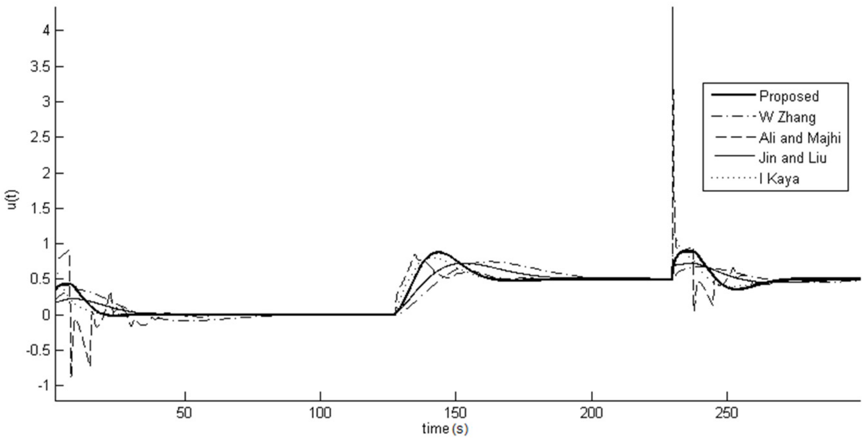
Figure 8(a) shows the time response to a unit step command and disturbance. The method proposed by Zhang has a weak performance in set point tracking and disturbance rejection, the method given by Ali also provides a poor performance in set point tracking, yet a suitable performance in disturbance rejection. The method of Kaya has a good performance in both set-point tracking and disturbance rejection. The method of Jin provides a good performance in set-point tracking, yet a poor performance in the rejection of plant-input disturbance.

The method proposed in this research provides very good performance in terms of set-point tracking and disturbance rejection. The required control input for the aforementioned controllers are shown in Figure 8(b). The required control input with the proposed method turns out to be superior compared to others. The control input signal with the proposed method can be further improved by tuning the β parameter, so that a better trade-off between the closed-loop performance and required control input can be achieved.

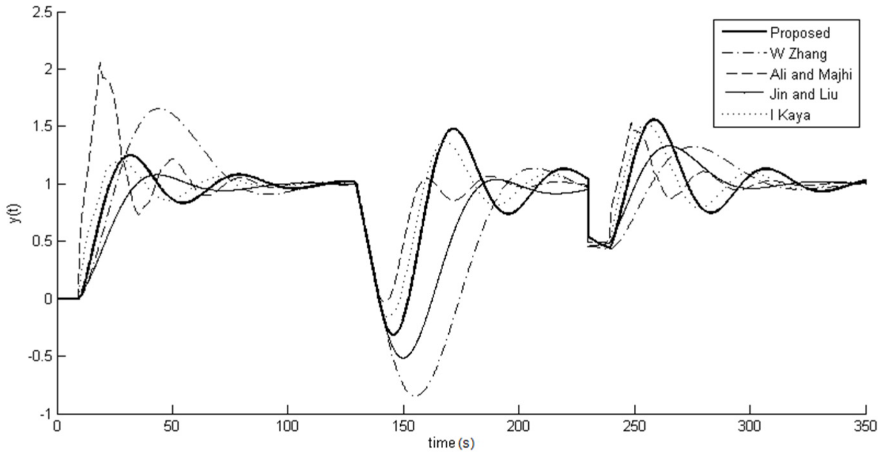
In order to assess the robustness of various studied methods, a $+25\%$ perturbation in θ is considered, and the corresponding time responses are shown in Figure 8(c). It can be concluded that the method of Kaya is not robust against perturbation in the values of θ, while, the method of Jin provides a good performance in reference tracking. On the other hand, the method of Ali provides a good performance in disturbance rejection, while the method proposed in this paper provides a superior performance compared to others.



(a)



(b)



(c)

Figure 8: Time response (a) and control input (b) for rejection of disturbances v_1 and v_2, in Example 3. Also, the time response to a unit set-point and step disturbances with +25% increase in θ is shown (c), which clearly shows the effectiveness of the proposed method.

Example 4 (DIPTD plant)

Consider

$$G(s)=\frac{e^{-s}}{s^2}$$

which is in the form of (5). The proposed controllers are in the form of (25) and (26), for which the required parameters are very simple to find from Table 1. In particular, for $\beta=0.1$, the values of $k_{\mathrm{i}}=0.0625$, $k_d=0.483$ and $\lambda=0.044$ are obtained.

The improved SP structure proposed by Uma (Uma and Rao, 2014) gives the following controllers

$$G_{cs}(s)=(1+\frac{0.25}{s}+1.51s)(\frac{1}{0.25s+1})$$

$$G_{cd}(s)=(0.29+\frac{0.045}{s}+0.77s)(\frac{1}{0.023s+1})$$

with parameters $\lambda_s=1.7$ and $\lambda_d=1.5$. Set-point weighting constant and the filter parameter are chosen 0.38 and 6 respectively.

The PID controller proposed in (Ali and Majhi, 2010) is given in a PID form, i.e.,

$$C(s)=0.125(1+\frac{1}{10\,s}+\frac{4\,s}{0.4s+1})$$

Similarly, an IMC-based controller designed by the method of (Jinand Liu, 2014) can be found as

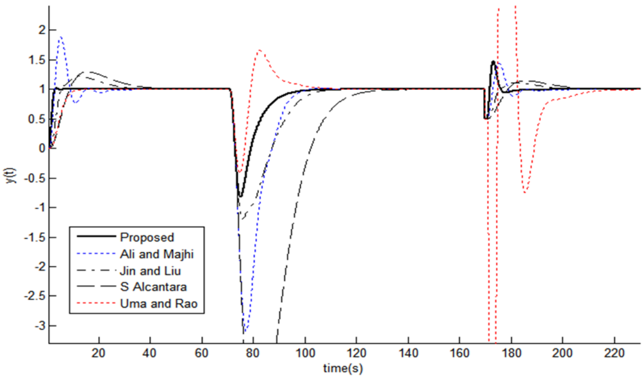
$$C(s) = 0.191\left(1 + \frac{1}{12.013\,s} + 4.281\,s\right)$$

Using the method of Alcantara (Alcantara et al., 2013) another PID controller is obtained as

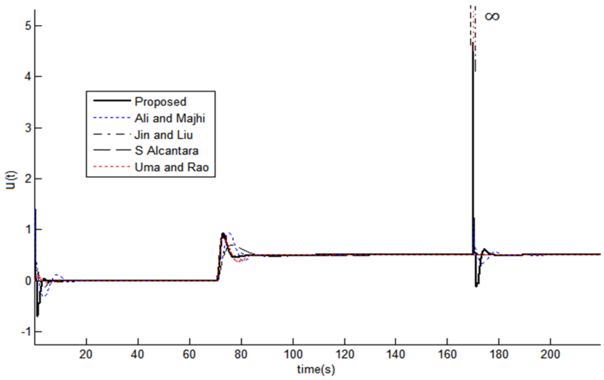
$$C(s) = 0.07\left(1 + \frac{1}{16.6\,s} + 5.9\,s\right)$$

Time response associated with each of the considered methods is shown in Figure 9(a). The superiority of the proposed method in servo tracking and disturbance rejection is obvious. The control input signals are shown in Figure 9(b), where, the methods of Jin, Uma and Alcantara require larger control inputs, compared with the method proposed in this research. The proposed method provides a good set-point tracking with moderate input usage together with a good disturbance rejection.

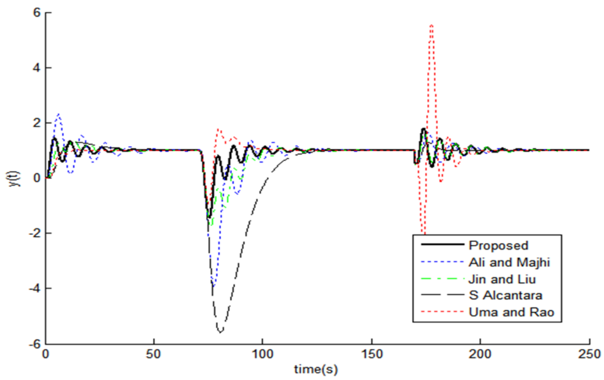
In order to assess the robustness of various studied methods, a +40% perturbation in the time delay is considered, and the corresponding time responses are shown in Figure 9(c). This figure clearly depicts the far superior performance of the proposed method.



(a)



(b)



(c)

Figure 9: Time response (a) and control input (b) for rejection of disturbances v_1 and v_2, in Example 4. Also, the time response to a unit set-point and step disturbances with +40% increase in θ is shown (c).

5 EXPERIMENTAL VERIFICATION

This section deals with theoretical analysis and experimental studies of an AC servo motor in the real time. Use has been made of the Modbus RTU protocol for communication between the controller (a PC) and the motor driver.The schematic of the experimental setup is shown in Figure 10 where τ and τ' are two variable communication time delays, in the range of 30-400 mili-seconds. In order to make the problem more challenging, a fictitious time delay (θ) and an integrator term $(s^i, i = 0,1)$were incorporated in the real-time. In section 5.1 i=0 and θ =3, and in section 5.2 i=1 and θ =3 are chosen. The Servo motor has the specification given in Table 2.

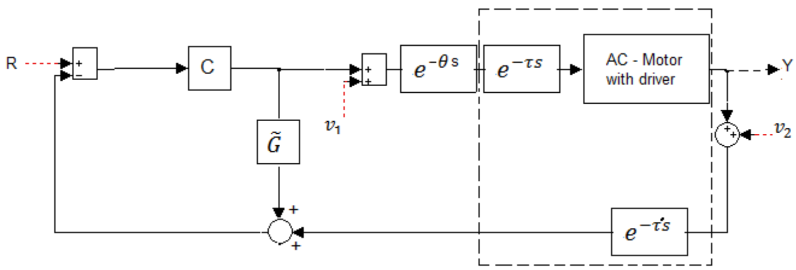


Figure 10: flow diagram in Matlab/Simulink, τ and τ' are variable communication delays and θ is an add artificial delay.

Model	BONMET - SA3LO6B
Input voltage	AC 3 phases, 50/60 HZ, 200-230 V
Output voltage	AC 3 phases, 0-230 V
Smps	6 A

Table 2: servo motor specifications.

In the first step, the transfer function of the servo motor was identified experimentally, by applying a random input voltage to the servo motor, and measuring the velocity, and analyzing the results using the MATLAB identification toolbox, with 83 % fitness index, as given below:

$$P(s) = \frac{859.8850\ e^{-\theta s}}{s^i\,(s^4 + 8.1993s^3 + 92.8620s^2 + 395.5641s + 864.7907)} \tag{30}$$

where, $\theta = 0$ and $i = 0$.

In order to evaluate the effectiveness of the proposed control method for FOPTD and IFOPTD plants, two experimental studies were considered as follows.

5.1 Plant Modeled as FOPTD

For $i = 0$ and $\theta \neq 0$ in 30 and using (Steadman andHymas, 1979), the plant given by (30) can be formed as follows

$$G(s) = \frac{e^{-(0.356+\theta)s}}{0.156s + 1}$$

which is in the form of (2). The proposed controllers should be in the form of (8) and (9), for which the required parameters are found from Table 1. In particular, for $\beta = 0$ and $\theta = 3$, the values of $k_d = 0.217$ and $\lambda = 0.133$ are obtained.

Time responses to a unity step commandand disturbance changes are obtained from simulation, as well as, experimental implementation, and the results are shown in Figure 11. Results show an exceptional similarity between the simulation and experimental results, while both have desirable closed loop performance and robustness.

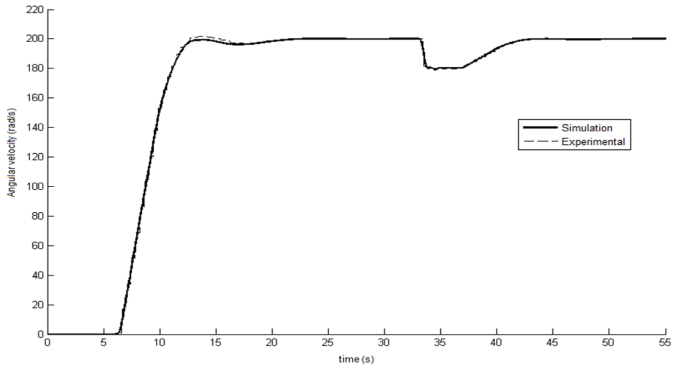


Figure 11:Comparison between simulation and experimental results, by modeling the plant as an FOPTD system.

5.2 Plant Modeled as IFOPTD

In the new experiment, $\mathrm{i} = 1$ and $\theta = 3$ are considered in (30). The servo motor transfer function is considered as an IFOPTD system, as given in (Steadmanand Hymas, 1979), i.e.,

$$G(s) = \frac{e^{-3.356s}}{s(0.156s+1)}$$

The proposed controllers are in the form of (16) and (17), for which the required parameters are very simple to find from Table 1. In particular, for $\beta = 0$, the values of $k_i = 0.157$, $k_d = 0.217$ and $\lambda = 0.133$ are obtained.

By using table 1 and for $\theta = 3$ and $\beta = 0$, values $k_i = 0.157$, $k_d = 0.217$, and $\lambda = 0.1324$ are obtained.

Time responses to a unity step command and disturbance changes are obtained from simulation, as well as, experimental implementation, and the results are shown in Figure 12. Once again, the results show an exceptional similarity between the simulation and experimental results, while both have desirable closed loop performance and robustness.

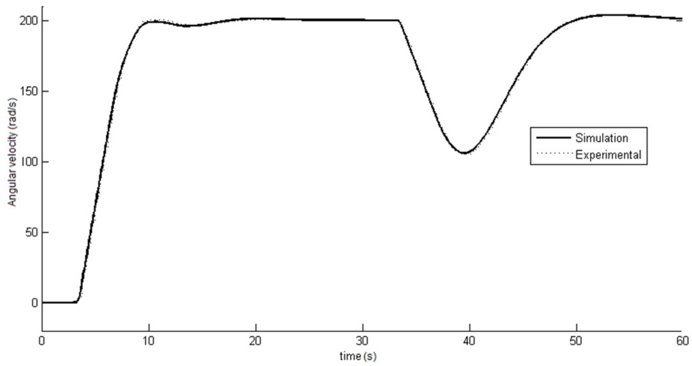


Figure 12: Comparison between simulation and experimental results, by modeling the plant as an IFOPTD system.

6 CONCLUSIONS

In this paper a new and simple method for control of stable and integrating systems with time delay was proposed. The controller design process includes designing unknown gains, for which very simple tuning formulae were proposed. The controller design process was studied in through simulation studies and comparison with some recent methods proposed in the literature. Based on the implemented studies, the proposed method was shown to have a very good performance in terms of the input tracking, disturbances rejectionand robustness against uncertainty in the time delay, and control input requirements, as compared to the five other methods proposed in the literature. The results of simulations revealed that some of the recently introduced methods need an excessive input usage to preserve the disturbance rejection property of the closed-loop, and hence, they may not be efficient methods from practical point of view.

The main advantages of the proposed control scheme were shown to be the simplicity of the design procedure and tuning of the control parameters, which ensure a robust behavior in the tracking and disturbance rejection properties of the closed-loop system. Experimental verifications also provide clear evidences on the effectiveness of the proposed method under practical limitations and uncertainties.

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