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Stress State at the Vertex of a Composite Wedge, One Side of Which Slides Without Friction Along a Rigid Surface

Abstract

For studying the stress-strain state at singular points and their neighborhoods new concept is proposed. A singular point is identified with an elementary volume that has a characteristic size of the real body representative volume. This makes it possible to set and study the restrictions at that point. It is shown that problems with singular points turn out to be ambiguous, their formulation depends on the combination of the material and geometric parameters of the investigated body. Number of constraints in a singular point is redundant compared to the usual point of the boundary (it makes singular point unique, exclusive). This circumstance determines the non-classical problem formulation for bodies containing singular points. The formulation of a non-classical problem is given, the uniqueness of its solution is proved (under the condition of existence), the algorithm of the iterative-analytical decision method is described. Restrictions on the state parameters at the composite wedge vertex, one generatrix of which is in non-friction contact with a rigid surface are studied under temperature and strength loading.

The proposed approach allows to identify critical combinations of material and geometric parameters that define the singularity of stress and strain fields close to singular representative volumes. The constraints on load components needed to solution existence are established. An example of a numerical analysis of the state parameters at the wedge vertex and its neighborhood is considered. Solutions built on the basis of a new concept, directly in a singular point, and its small neighborhood differ significantly from the solutions made with asymptotic methods. Beyond a small neighborhood of a singular point the solutions obtained on the basis of different concepts coincide.

Keywords

Composite structures; non-classical tasks; singular points; material point, representative volume.

V. Pestrenin [a, *]
I. Pestrenina [a]
L. Landik [a]

[a] Perm State National Research University, Perm, Russia,
pestreninvm@mail.ru
ipestrenina@gmail.com
lidialandik@gmail.com

* Corresponding author

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1 INTRODUCTION

Singular points of elastic bodies are vertexes of cracks, wedges, cones, pyramids, lines of surface generatrix crossing (ribs), line (surface) of edge points of the composite structural elements connections, etc. Singular points are potential stress concentrators, contribute to premature failure of the structure.

The research of the state parameters (stresses, strains) of deformable bodies in the singularity vicinity attracts a great interest from the authors, whose studies are divided into two approaches.

1. The classical (asymptotic) approach. This approach includes methods of the operational calculus are used in Bogy(1971) and Sinclear (2004), complex-variable functions in Parton and Perlin (1981), Airy functions in Chobanyan(1987), integral equations in Uflyand (1967) and Andreev (2013), separation of variables in Aksentyan (1967), series expansion by different functions in Kovalenko (2011), He and Kotousov (2016), etc. Authors of numerical approaches realize the asymptotic idea through unlimited grid model refinement of the area close to the singular point. Also there are studies by finite element method in Koguchia and Muramoto (2000), Xu and Tong(2016)), method of boundary elements in Mittelstedt (2005), Koguchi (2010), method of boundary conditions in Ryazantseva (2015). Mathematical problems, concerning the justification of asymptotic methods for studying of mechanics problems of a deformable solid body with singular points, were considered and successfully resolved in studies Kondratiev (1967), Mazya(1976).

However, the classical approach does not guarantee the reliability of the research results in a small neighborhood of the singular point. Indeed, in the classical approach, the corresponding values for the state parameters at a singular point are taken as limit ones. It means that a singular point in the classical approach is considered as a mathematical point (with zero volume), because in the limit transition to a singular point distance to this point tends to zero.

Models of real bodies are studied in solid mechanics (SM). The model is a continuum whose physic-mechanical properties are determined by the properties of the representative volume of the real body. A representative volume has a linear scale (characteristic size). This scale of the representative volume is also a characteristic dimension of the elementary volume of the continuum, to which the stresses and strains obtained in the solution should be referred. This means that at a mathematical point (a point with zero volume) the notion of stresses and strains has no mechanical content. No constraints (for example, boundary conditions) can be imposed on the state parameters at such a point. The absence of a mechanical content in the solution for a singular point does not allow us to use the asymptotic approach with respect to real bodies in a small neighborhood of this point. Our studies show that the characteristic size of such a neighborhood turns out to be equal to 5-7 characteristic dimensions of the representative volume of body material.

2. The non-classical approach to the study of the state parameters directly at special points and their vicinity is being developed by the authors of this article (2015). The approach is based on the concept of a singular point as an elementary volume (material point) with a characteristic size of the representative volume of a real body. In the new approach it is possible to determine state parameters at the singular point and to formulate the constraints at them.

Such restrictions are a system of algebraic equations. The study of these equations shows that the formulation of the solid mechanics problem for a body with a singular point is ambiguous. It is determined by a combination of material characteristics and geometric parameters of the object under consideration. Originality (uniqueness) of the singular point is manifested in the redundancy (in comparison with the boundary point of the classical problem) of the constraints given in it. This feature makes the solid mechanics problem for a body with a singular point a non-classical.

The non-classical approach was used by the authors of (2013–2017) to study the stress-strain state in homogeneous planar and composite wedges, composite spatial ribs, and internal singular points of plane structural elements. In this paper we give a general formulation of the non-classical (in the sense indicated above) problem for deformable solid body with singular points, and prove the uniqueness of its solution (under the condition of its existence). The features of stress distribution near and directly at the tip of a composite wedge, one of the sides of which slides without friction along a rigid surface under the thermal or force load are studied.

2 PROBLEM STATEMENT

A part of the considered structure element represents a wedge composed of two isotropic linearly elastic elements 1, 2. Side of the wedge element 1 is oriented by the unit vector $\bar{n}$. The unit vector $\bar{n}'$ is orthogonal $\bar{n}$ and directed along the side, which can be loaded with the surface forces with the density $\bar{p}_n = p_n\bar{n} + \tau_n\bar{n}'$. Side of the wedge element 2 is oriented by the unit vector $\bar{m}$. The unit vector $\bar{m}'$ is orthogonal to the unit vector $\bar{m}$ and directed along the side, which slides without friction along the rigid surface (Figure 1).

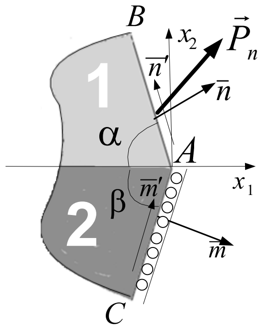


Figure 1: Composite wedge

Angles α, β of elements, constituting a wedge, are subject to the conditions

$$0<\alpha<2\pi \quad 0<\beta<2\pi \quad \alpha+\beta\leq 2\pi \tag{1}$$

In accordance with the accepted concept, representative volumes of attached bodies 1,2, which are located at A vertex of the wedge, are singular points. Accepted designations: $\sigma_{ij}^{(k)}, \varepsilon_{ij}^{(k)}$ - components of stresses and strains, correspondingly; $E_k, G_k, \nu_k, \omega_k$ – Young's modulus, shear modulus, Poisson's ratio and linear coefficient of thermal expansion in k-th ($k = 1,2$) wedge constituent element; σ_n, $\tau_{n'}$,

σ_m, $\tau_{m'}$ – normal and shear stresses on wedge sides oriented by unit vectors $\bar{n}$, $\bar{m}$; ΔT– temperature increment. It is also accepted that the structural element under consideration is in a generalized plane stressed state. At the point A (wedge vertex) the orthonormal Cartesian coordinate system x_1, x_2 is introduced. Axis x_1 directed tangential to the connection line of elements constituting wedge. At the wedge vertex state parameters (stresses, strains, displacements) of representative volumes (singular points) are subject to the following restrictions:

a) at the area element with the normal $\bar{n}$ are set normal and tangential stresses

$$\sigma_n = p_n \quad \tau_{n'} = \tau_n \tag{2}$$

b) at the area element with the normal $\bar{m}$ the tangential stress vanishes

$$\tau_{m'} = 0 \tag{3}$$

and the projection value of the displacement vector $\bar{u}$ to the direction $\bar{m}$

$$\bar{u} \cdot \bar{m} = 0 \tag{4}$$

c) the following is performed on the line of elements 1,2 connection

1) stress continuity condition

$$\sigma_{12}^{(1)} = \sigma_{12}^{(2)} = \sigma_{12} \qquad \sigma_{22}^{(1)} = \sigma_{22}^{(2)} = \sigma_{22} \tag{5}$$

2) strain continuity condition (the equality of relative extensions of linear elements directed along the connection line)

$$\varepsilon_{11}^{(1)} = \varepsilon_{11}^{(2)} \tag{6}$$

3) displacement continuity condition.

Conditions (2), (3), (5), (6) are recorded by the system of linear inhomogeneous equations relatively to the parameters $\sigma_{11}^{(1)}, \sigma_{11}^{(2)}, \sigma_{12}, \sigma_{22}$

$$\begin{gathered}
\sigma_{11}^{(1)} \sin^2\alpha + 2\sigma_{12}\sin\alpha\cos\alpha + \sigma_{22}\cos^2\alpha = p_n \\
-\sigma_{11}^{(1)} \sin\alpha\cos\alpha - \sigma_{12}(\cos^2\alpha - \sin^2\alpha) + \sigma_{22}\sin\alpha\cos\alpha = \tau_n \\
\sigma_{11}^{(2)} \sin\beta\cos\beta - \sigma_{12}\cos 2\beta - \sigma_{22}\sin\beta\cos\beta = 0 \\
\frac{1}{E_1}\sigma_{11}^{(1)} - \frac{1}{E_2}\sigma_{11}^{(2)} - \left(\frac{\nu_1}{E_1} - \frac{\nu_2}{E_2}\right)\sigma_{22} = (\omega_2 - \omega_1)\Delta T
\end{gathered} \tag{7}$$

The matrix determinant of the system of equations (7) can be written by the equation

$$\Delta = -\left[\frac{1}{E_1}\cos 2\alpha \sin\beta\cos\beta + \frac{1}{E_2}\cos 2\beta \sin\alpha\cos\alpha + \right. \\ \left. + \left(\frac{1-\nu_1}{E_1} - \frac{1-\nu_2}{E_2}\right)\sin^2\alpha\sin\beta\cos\beta\right] \tag{8}$$

The task is to find possible variants of equation system (7) solutions, depending on the load, as well as geometry and material parameters of the connected elements. Such solutions are specified constraints on parameters of the state in representative volumes adjoining the wedge vertex. In each variant to determine the type (classical, non-classical) of the respective SM task.

3 THERMAL LOADING OF THE STRUCTURAL MEMBER

In this item, it is accepted that parameters p_n, τ_n vanish in the column of free members of the equation system (7).

3.1 Research of Solutions of Equations (7), Depending on the Rank of the Augmented Matrix

Determinants Δ_i (i=1–4), obtained through sequential substitution of matrix columns of the equations system (7) by the free members column, have the values

$$\begin{aligned} \Delta_1 &= -(\omega_2 - \omega_1)\Delta T\cos^2\alpha\sin\beta\cos\beta \\ \Delta_2 &= (\omega_2 - \omega_1)\Delta T\sin\alpha(\cos 2\beta\cos\alpha - \sin\beta\cos\beta\sin\alpha) \\ \Delta_3 &= 0.25(\omega_2 - \omega_1)\Delta T\sin 2\alpha\sin 2\beta \\ \Delta_4 &= -(\omega_2 - \omega_1)\Delta T\sin\beta\cos\beta\sin^2\alpha \end{aligned} \tag{9}$$

The analysis of equations (9) shows that there are five variants of wedge elements connections, in which all the determinants (9) vanish:

$$\begin{gathered} 1)\ \alpha = \pi/2 \quad \beta = \pi/2 \quad 2)\ \alpha = \pi/2 \quad \beta = \pi \quad 3)\ \alpha = \pi/2 \quad \beta = 3\pi/2 \\ 4)\ \alpha = \pi \quad \beta = \pi \qquad 5)\ \alpha = 3\pi/2 \quad \beta = \pi/2 \end{gathered} \tag{10}$$

At these values α, β the determinant Δ (8) vanishes too. Let us note that determinants Δ_i (i = 1-4) (9) simultaneously vanish only at points (10). At the same time, the determinant (8) can vanish not only at these points. In this connection, such variants of the behavior of solutions of equations (7) are possible.

1. Parameters α, β do not fall within the group (10), and the determinant (8) vanishes. The matrix rank of the equations system (7) does not coincide with the rank of the augmented matrix, equations are incompatible. In the present case, vanishing of the determinant (8) is a critical condition in the

following sense. When the combination of material and geometrical parameters included in the formula (8) is aimed at vanishing of the determinant, stresses in singular points increase with no limit.

2. Parameters α, β do not fall within the group (10), the determinant (8) does not vanish, the system of equations (7) has the single solution:

$$\sigma_{11}^{(1)} = \Delta_1 / \Delta \qquad \sigma_{11}^{(2)} = \Delta_2 / \Delta \qquad \sigma_{12} = \Delta_3 / \Delta \qquad \sigma_{22} = \Delta_4 / \Delta$$

In this case, three components of the stress tensor (typically, two are set) and condition (4) turn to be set in the singular points for each element constituting the wedge. The number of defined conditions is redundant, the problem is non-classical.

Parameters α, β fall within the group (10), determinants (8) and Δ_i (i=1–4) vanish. The following restrictions on state parameters in singular points correspond to each of variants (10):

1) $\alpha = \pi / 2, \beta = \pi / 2$. Ranks of the matrix and the augmented matrix are equal to three. From the equations (7) follow dependences

$$\sigma_{11}^{(1)} = 0 \qquad \sigma_{12} = 0 \qquad -\frac{1}{E_2}\sigma_{11}^{(2)} - \left(\frac{\nu_1}{E_1} - \frac{\nu_2}{E_2}\right)\sigma_{22} = (\omega_2 - \omega_1)\Delta T \tag{11}$$

Due to the fact that the equation (5) is true, conditions (11) represent five restrictions in singular, one more restriction (4). Set number of restrictions is redundant. Therefore, the problem under consideration is non-classical. In the particular case, when between material parameters there is a dependence

$$E_2\nu_1 = E_1\nu_2 \tag{12}$$

the number of set restrictions in the singular point is reduced to five, nevertheless, the problem remains to be non-classical.

2) $\alpha = \pi / 2 \quad \beta = \pi$ The matrix rank of the equations system (7) coincides with the rank of the augmented matrix and is equal three. Dependencies between the stresses coincide with (11). Therefore, results of the preceding item are true.

3) $\alpha = \pi / 2 \quad \beta = 3\pi / 2$ The matrix ranks of the equations system (7) and its augmented matrix coincide and are equal three. Restrictions on stress components at a singular point are represented by equations (11), therefore the results, given in item 1, are true for this case too.

4) $\alpha = \pi \quad \beta = \pi$ The matrix rank of the equations system (7) is equal to three and is equal to the augmented matrix rank. Restrictions on the stress components derived from the equations (7) are written by equations

$$\sigma_{22} = 0 \qquad \sigma_{12} = 0 \qquad \frac{1}{E_1}\sigma_{11}^{(1)} - \frac{1}{E_2}\sigma_{11}^{(2)} = (\omega_2 - \omega_1)\Delta T \tag{13}$$

Taking into account equations (4), (5), the number of set independent restrictions on stress components in singular points is redundant, the task is non-classical.

5) $\alpha = 3\pi / 2 \quad \beta = \pi / 2$ The matrix rank of the equations system (7) and its augmented matrix coincide and are equal to three. Restrictions on the state parameters in singular points are set by equations (11), therefore analysis results of the solution of equations (7) are the same as in the item 1.

3.2 Particular Cases of Bonding Elements of the Wedge

It is preferable to carry out the analysis of conditions of vanishing determinant (8) numerically, due to a significant number of parameters determining it. At the same time, the analytical method effective in the often met in the practice cases of bonding elements 1, 2 of the wedge (Figure 1).

Realized below approach to the analysis of solution of equations (7) can be used to detect the combination of critical parameters of the wedge elements connection and in other cases.

1) **$\alpha = \beta$.**

Determinant (8) becomes

$$\Delta = -\left[\left(\frac{1}{E_1} + \frac{1}{E_2}\right)\cos 2\alpha + \left(\frac{1-\nu_1}{E_1} - \frac{1-\nu_2}{E_2}\right)\sin^2\alpha\right]\sin\alpha\cos\alpha \tag{14}$$

Two equations follow from the equation $\Delta = 0$

$$\sin\alpha\cos\alpha = 0 \tag{15}$$

$$\left(\frac{1}{E_1} + \frac{1}{E_2}\right)\cos 2\alpha + \left(\frac{1-\nu_1}{E_1} - \frac{1-\nu_2}{E_2}\right)\sin^2\alpha = 0 \tag{16}$$

Within the area of admissible values $\alpha\,(0 < \alpha \le \pi)$ the equation (15) has roots

$$\alpha = \pi / 2 \quad \alpha = \pi \tag{17}$$

The solution of equations (7) at such angles α is considered in the item 2.1.

Equation (16) within the area of admissible values α has two roots determined by equations

$$\begin{aligned} \sin\alpha_1 = \sin\alpha_2 &= \left\{\frac{E_1 + E_2}{3E_1 + E_2 + (\nu_1 E_2 - \nu_2 E_1)}\right\}^{\frac{1}{2}} \\ \cos\alpha_1 = -\cos\alpha_2 &= \left\{\frac{2E_1 + (\nu_1 E_2 - \nu_2 E_1)}{3E_1 + E_2 + (\nu_1 E_2 - \nu_2 E_1)}\right\}^{\frac{1}{2}} \end{aligned} \tag{18}$$

If $(\omega_1 - \omega_2)\Delta T \neq 0$, angles (18) of the wedge bonding are to be considered critical, since in this case the matrix rank of the equations system (7) turns out to be less than the rank of the the augmented matrix. Therefore, when a jointing members material parameters combination approach to the equations performing (18), the solution of equations (7) increases with no limit. But if $(\omega_1 = \omega_2)$

or $\Delta T = 0$, the system of equations (7) turns to be homogeneous, its rank at angles (18) is equal to three and, hence, three components of stresses can be expressed through the fourth, for example, so

$$\sigma_{11}^{(1)} = ctg^2\alpha_i\sigma_{22} \quad \sigma_{11}^{(2)} = -(ctg^2\alpha_i - 2)\sigma_{22} \quad \sigma_{12} = -ctg\alpha_i\sigma_{22} \quad (i{=}1{,}2) \tag{19}$$

Conditions (19), (4) and (5) at the wedge vertex represent six independent restrictions on the state parameters. This task is classical. If in the case under consideration ($\alpha = \beta$), the angle α is not determined by equations (17), (18), the matrix rank of the equation system (7) is equal to four. Therefore, (in the case $(\omega_1 = \omega_2)$ or $\Delta T = 0$) this system has zero solution only.

$$\sigma_{11}^{(1)} = 0 \quad \sigma_{11}^{(2)} = 0 \quad \sigma_{12} = 0 \quad \sigma_{22} = 0 \tag{20}$$

Equations (20) jointly with (4) and (5) impose seven restrictions on the state parameters at the singular point. The number of restrictions exceeds the number of restrictions set in the classical case, which is equal to six.

2) $\alpha + \beta = \pi / 2$.

Determinant (8) becomes

$$\Delta = \left[\left(\frac{1}{E_2} - \frac{1}{E_1}\right)\cos 2\alpha - \left(\frac{1-\nu_1}{E_1} - \frac{1-\nu_2}{E_2}\right)\sin^2\alpha\right]\sin\alpha\cos\alpha \tag{21}$$

The equation $\Delta = 0$ falls into two

$$\sin\alpha\cos\alpha = 0 \tag{22}$$

$$\left(\frac{1}{E_2} - \frac{1}{E_1}\right)\cos 2\alpha - \left(\frac{1-\nu_1}{E_1} - \frac{1-\nu_2}{E_2}\right)\sin^2\alpha = 0 \tag{23}$$

Equation (22) in the variation range $\alpha\ (0 < \alpha < \pi / 2)$ has no roots. From the equation (23) follows

$$\sin^2\alpha = \frac{\kappa - 1}{\kappa(1+\nu_2) - (1+\nu_1)} \quad \kappa = \frac{E_1}{E_2} \tag{24}$$

The solution of the equation (24) exists under the condition (taking into account the variation range α)

$$0 < \frac{\kappa - 1}{\kappa(1+\nu_2) - (1+\nu_1)} < 1 \tag{25}$$

Inequality (25) is performed if all conditions from any one of the following groups are true:

$$\kappa > 1 \quad \kappa > \frac{\nu_1}{\nu_2} \quad \kappa > \frac{1+\nu_1}{1+\nu_2} \tag{26}$$

$$\kappa < 1 \quad \kappa < \frac{\nu_1}{\nu_2} \quad \kappa < \frac{1+\nu_1}{1+\nu_2} \tag{27}$$

At fulfillment of conditions (26) or (27) the equation (24) has one root within the area of admissible values α . This root determines the critical value of the angle of wedge bonding elements 1, 2, in the case when $(\omega_1 - \omega_2)\Delta T \neq 0$, as at the combination of material parameters, tending to fulfill the equality (24), the solution of equations (7) tends to infinity. If $(\omega_1 - \omega_2)\Delta T = 0$, from the equation (7) follows the dependence between components of stresses

$$\sigma_{11}^{(1)} = \sigma_{11}^{(2)} = ctg^2\alpha\sigma_{22} \qquad \sigma_{12} = -ctg\alpha\sigma_{22} \tag{28}$$

Equations (28) jointly with (4) and (5) impose six restrictions on components of stresses and displacements at the singular point. The same number of restrictions corresponds to the classical case.

3) **$\alpha + \beta = \pi$.**

The determinant (8) in this case coincides up to a sign with the determinant (21). Equation $\Delta = 0$ is equivalent to relations (22), (23). Equation (22) within the area of admissible values α $(0 < \alpha < \pi)$ has the single root $\alpha = \pi / 2$, therefore, $\beta = \pi / 2$. Such a case of the wedge elements connection is considered in item 2.1. Equation (23) reduces to the equation (24), which has real solutions under restrictions on parameters (26) or (27). In the case under consideration of parameter α critical values, determining the singular behavior of the solution at $(\omega_1 - \omega_2)\Delta T \neq 0$ in the singular point, turns to be two:

$$\begin{aligned} \sin\alpha_1 &= \sin\alpha_2 = \left\{\frac{\kappa - 1}{\kappa(1+\nu_2) - (1+\nu_1)}\right\}^{1/2} \\ \cos\alpha_1 &= -\cos\alpha_2 = \left\{\frac{\kappa\nu_2 - \nu_1}{\kappa(1+\nu_2) - (1+\nu_1)}\right\}^{1/2} \end{aligned} \tag{29}$$

In the case, when $(\omega_1 - \omega_2)\Delta T = 0$, from equations (7) follows the dependences between stresses at the points $\alpha = \alpha_i\ (i = 1,2)$:

$$\sigma_{11}^{(1)} = \sigma_{11}^{(2)} = ctg^2\alpha_i\sigma_{22} \qquad \sigma_{12} = -ctg\alpha_i\sigma_{22} \tag{30}$$

Dependences (30) jointly with (4) and (5) determine the restrictions on the state parameters at singular points. The number of restrictions is equal to six, that coincides with the number of restrictions in the classical problem.

4) **$\alpha + \beta = 2\pi$.**

The determinant (8) coincides with the expression (21). The condition of equality to zero of this determinant reduces to equations (22), (24). Equations (22) in the range α $(0 < \alpha < 2\pi)$ have roots.

$$\alpha = \pi / 2 \qquad \alpha = \pi \qquad \alpha = 3\pi / 2 \tag{31}$$

All these cases are considered in the item 2.1. The equation (24) at fulfillment of conditions (26) or (27) within the area of admissible values α has four roots, to which suit four pairs of angle values α, β

$$
\begin{aligned}
\sin\alpha_1 &= \sin\alpha_2 = -\sin\alpha_3 = -\sin\alpha_4 = \left\{\frac{\kappa - 1}{\kappa(1+\nu_2) - (1+\nu_1)}\right\}^{1/2} \\
\cos\alpha_1 &= -\cos\alpha_2 = -\cos\alpha_3 = \cos\alpha_4 = \left\{\frac{\kappa\nu_2 - \nu_1}{\kappa(1+\nu_2) - (1+\nu_1)}\right\}^{1/2}
\end{aligned}
\tag{32}
$$

In points α_i (i=1,2,3,4) (32) determinants (9) at $(\omega_1 - \omega_2)\Delta T \neq 0$ are different from zero. Therefore, the rank of the augmented matrix of the equation system (7) turns to be greater than the rank of the system. There is no solution of equations. Each of points (32) turns to be critical.

If condition $(\omega_1 - \omega_2)\Delta T = 0$ is performed, equations (7) turn to be compatible. Dependences (30), where (i = 1,2,3,4), are true between stresses at points (32). The number of restrictions on parameters $\sigma_{ij}^{(k)}$, u_k at the singular points is equal to six, which meets the classical case.

4 LOADING OF THE STRUCTURAL MEMBER WITH SURFACE FORCES

In the case under consideration there is no thermal load, therefore, in the column of free terms of the equation system (7) expressions containing the multiplier ΔT vanish. Determinants Δ_i $(i = 1,2,3,4)$, obtained through sequential substitution of matrix columns of the equation system (7) by the free term column, are determined by expressions

$$
\begin{aligned}
\Delta_1 &= -P_n\left[\left(\frac{\nu_1}{E_1} - \frac{1-\nu_2}{E_2}\right)\cos 2\alpha \sin\beta\cos\beta + \frac{1}{E_2}\cos 2\beta \sin\alpha\cos\alpha\right] - \\
&\quad - \tau_n\left[2\left(\frac{\nu_1}{E_1} - \frac{1-\nu_2}{E_2}\right)\sin\alpha\cos\alpha\sin\beta\cos\beta - \frac{1}{E_2}\cos 2\beta\cos^2\alpha\right] \\
\Delta_2 &= -P_n\left[\left(\frac{\nu_1}{E_1} - \frac{\nu_2}{E_2}\right)\cos 2\beta\sin\alpha\cos\alpha - \frac{1}{E_1}(\cos 2\alpha\sin\beta\cos\beta + \cos 2\beta\sin\alpha\cos\alpha)\right] + \\
&\quad + \tau_n\left[2\left(\frac{\nu_1}{E_1} - \frac{\nu_2}{E_2}\right)\sin^2\alpha\cos 2\beta - \frac{1}{E_1}(\sin 2\alpha\sin\beta\cos\beta - \cos 2\beta\cos^2\alpha)\right] \\
\Delta_3 &= -P_n\left(\frac{1-\nu_1}{E_1} - \frac{1-\nu_2}{E_2}\right)\sin\alpha\cos\alpha\sin\beta\cos\beta - \\
&\quad - \tau_n\left[\left(\frac{1-\nu_1}{E_1} - \frac{1-\nu_2}{E_2}\right)\sin^2\alpha\sin\beta\cos\beta - \frac{1}{E_1}\cos\beta\cos\beta\right]
\end{aligned}
\tag{33}
$$

$$\Delta_4 = -P_n\left(\frac{1}{E_1}\cos 2\alpha \sin\beta\cos\beta + \frac{1}{E_2}\cos 2\beta \sin\alpha\cos\alpha\right) - \\ -\tau_n\left(\frac{1}{E_1}\sin 2\alpha \sin\beta\cos\beta + \frac{1}{E_2}\cos 2\beta \sin^2\alpha\right)$$

Below is given the study of possible restrictions on state parameters at the wedge vertex for considered in the previous item cases of its elements connection.

4.1 Research of Solutions of Equations (7) in Cases when α,β is Determined by Equations (10)

At points $\alpha_i,\beta_i\,(i=1-5)$, determined by equations (10), the matrix determinant (8) of the equation system (7) vanishes. Equations (7) will be compatible in each of points (10), if the load $\overline{p}_n$ is restricted by the condition

$$\tau_n = 0 \tag{34}$$

Restriction (34) is due to the requirement of the stress tensor symmetry. In violation of the restriction (34) the matrix ranks of the equation system (7) and its augmented matrix in points (10) are different, so these points are points of the singular solution behavior. Further in this item the restrictions are built on state parameters at the wedge vertex under fulfillment restriction condition (34).

1. $(\alpha = \pi/2,\ \beta = \pi/2)$.

The matrix rank of system (7) and its augmented matrix is equal to three. Between stresses at the wedge vertex the following dependences are true

$$\sigma_{11}^{(1)} = p_n \qquad \sigma_{11}^{(2)} = \frac{E_2}{E_1}p_n - E_2\left(\frac{\nu_1}{E_1} - \frac{\nu_2}{E_2}\right)\sigma_{22} \qquad \sigma_{12} = 0 \tag{35}$$

Equations (35), (5) and (4) represent six restrictions on parameters in the singular points. This number of restrictions is redundant. Task is non-classical.

2. Similar results are obtained for points $(\alpha = \pi/2, \beta = \pi)$, $(\alpha = \pi/2, \beta = 3\pi/2)$, $(\alpha = 3\pi/2, \beta = \pi/2)$.

3. $(\alpha = \pi, \beta = \pi)$.

The matrix rank of the equations system (7) and its augmented matrix are equal to three. Restrictions in the singular point on the stress tensor components take the form

$$\sigma_{11}^{(1)}\frac{1}{E_1} - \sigma_{11}^{(2)}\frac{1}{E_2} = p_n\left(\frac{\nu_1}{E_1} - \frac{\nu_2}{E_2}\right) \qquad \sigma_{22} = p_n \qquad \sigma_{12} = 0$$

The total number of restrictions exceeds the number of restrictions considered in the classical approach.

4.2 Particular Cases of Bonding Elements of the Wedge

In this item the restrictions on state parameters at the wedge vertex are proposed for considered in the item 2.2 cases of its elements connection.

1. $\boldsymbol{\alpha} = \boldsymbol{\beta}$.

The determinant Δ of the equation system (7) reduces to the form (14) and vanishes in points (17), (18). Determinants Δ_i $(i = 1,2,3,4)$, corresponding to the augmented matrix of the equation system (7) are written in the form

$$\Delta_i = p_n \phi_i + \tau_n \psi_i, \quad (i = 1,2,3,4) \tag{36}$$

where

$$\begin{aligned}
\phi_1 &= \sin\alpha\cos\alpha(\sin^2\alpha - \cos^2\alpha)\left[\frac{2}{E_2} + \left(\frac{\nu_1}{E_1} - \frac{\nu_2}{E_2}\right)\right] \\
\phi_2 &= -\sin\alpha\cos\alpha(\sin^2\alpha - \cos^2\alpha)\left[\left(\frac{\nu_1}{E_1} - \frac{\nu_2}{E_2}\right) - \frac{2}{E_1}\right] \\
\phi_3 &= -\sin^2\alpha\cos^2\alpha\left[\frac{\nu_1 - 1}{E_1} - \frac{\nu_2 - 1}{E_2}\right] \\
\phi_4 &= \sin\alpha\cos\alpha(\sin^2\alpha - \cos^2\alpha)\left[\frac{1}{E_1} + \frac{1}{E_2}\right] \\
\psi_1 &= -\cos^2\alpha\left[\frac{2}{E_2}\sin^2\alpha + \frac{1}{E_2}(\sin^2\alpha - \cos^2\alpha) + 2\left(\frac{\nu_1}{E_1} - \frac{\nu_2}{E_2}\right)\sin^2\alpha\right] \\
\psi_2 &= -\sin^2\alpha(\sin^2\alpha - \cos^2\alpha)\left(\frac{\nu_1}{E_1} - \frac{\nu_2}{E_2}\right) - \frac{2}{E_1}\sin^2\alpha\cos^2\alpha - \frac{1}{E_1}\cos^2\alpha(\sin^2\alpha - \cos^2\alpha) \\
\psi_3 &= \sin\alpha\cos\alpha\left[\left(\frac{\nu_1}{E_1} - \frac{\nu_2}{E_2}\right)\sin^2\alpha + \frac{1}{E_1}\cos^2\alpha + \frac{1}{E_2}\sin^2\alpha\right] \\
\psi_4 &= \sin^2\alpha\left[\frac{1}{E_2}(\sin^2\alpha - \cos^2\alpha) - \frac{2}{E_1}\cos^2\alpha\right]
\end{aligned} \tag{37}$$

The following cases of the equation system (7) behavior are possible:

a) If the determinant $\Delta \neq 0$, the system of equations (7) has the single solution

$$\sigma_{11}^{(1)} = \frac{\Delta_1}{\Delta} \quad \sigma_{11}^{(2)} = \frac{\Delta_2}{\Delta} \quad \sigma_{12} = \frac{\Delta_3}{\Delta} \quad \sigma_{22} = \frac{\Delta_4}{\Delta} \tag{38}$$

The number of independent restrictions (38), (5) and (4) on the components of stresses and displacements at the wedge vertex is equal to seven. In the classical approach - six. Non-classical task.

b) Cases, when the angle α is determined by equations (17), are considered in the item 3.1.

c) The angle α is determined by the equations (18). The augmented matrix rank of the equation system (7) will be equal to the matrix rank of the system (equal to three), when all the determinants (36) vanish, i.e.

$$p_n\phi_i + \tau_n\psi_i = 0 \quad (i = 1,2,3,4) \tag{39}$$

Equations (39) form the system of four linear uniform equations relatively to the components of the stress vector p_n, τ_n. The matrix rank of this equation system at the values α_1, α_2, determined by relations (18), is equal to one.

Hence, equations (39) in this case turn to be linearly dependent by pairs.

Therefore, any of them can be considered as the restriction on the load stipulating the compatibility of equations (7). If this condition is performed, from equations (7) follow relationships between stress components at the wedge vertex

$$\begin{aligned} \sigma_{11}^{(1)} &= (p_n - \sigma_{22}\cos^2\alpha)(1 - ctg^2\alpha) - 2(\tau_n - \sigma_{22}\sin\alpha\cos\alpha)ctg\alpha \\ \sigma_{11}^{(2)} &= -(p_n - \sigma_{22}\cos^2\alpha)(1 - ctg^2\alpha) - 2(\tau_n - \sigma_{22}\sin\alpha\cos\alpha)(tg\alpha - ctg\alpha) \\ \sigma_{12} &= (p_n - \sigma_{22}\cos^2\alpha)ctg\alpha + \tau_n - \sigma_{22}\sin\alpha\cos\alpha \end{aligned} \tag{40}$$

Dependences (40), (5) and (4) - six restrictions on stress components and displacements at singular points. This number of set restrictions corresponds to the classical case. If components of the stress vector do not perform the equations (39), the system of equations (7) at values α_1, α_2 is incompatible. These angles should be regarded as critical angles of bonding elements, as when material parameters tend to fulfillment of equations (18), stresses in representative volumes close to the wedge vertex increase with no limit.

2. $\boldsymbol{\alpha + \beta = \pi / 2}$.

The determinant (8) in this case is expressed by formula (21) and at the fulfillment of conditions (26) or (27) vanishes at the single value of the angle α, equal to α^*, determined by the equation (24). Determinants $\Delta_i \; (i = 1,2,3,4)$, obtained by the sequential substitution of columns of the matrix determinant of the equation system (7) by columns of its free members are represented by equations of the form (36), wherein parameters ϕ_i, ψ_i are determined according to formulas

$$\begin{aligned} \phi_1 &= -\left(\frac{\nu_1}{E_1} - \frac{\nu_2}{E_2}\right)\cos 2\alpha\sin\alpha\cos\alpha, & \psi_1 &= -\cos^2\alpha\left[\frac{1}{\mathrm{E}_2} + 2\left(\frac{\nu_1}{E_1} - \frac{\nu_2}{E_2}\right)\sin^2\alpha\right] \\ \phi_2 &= \phi_1 & \psi_2 &= -\left[\frac{1}{\mathrm{E}_1}\cos^2\alpha + \left(\frac{\nu_1}{E_1} - \frac{\nu_2}{E_2}\right)\sin^2\alpha\cos 2\alpha\right] \end{aligned} \tag{41}$$

$$\phi_3 = -\left(\frac{1-\nu_1}{E_1} - \frac{1-\nu_2}{E_2}\right)\sin^2\alpha\cos^2\alpha, \qquad \psi_3 = -\sin\alpha\cos\alpha\left[\left(\frac{1-\nu_1}{E_1} - \frac{1-\nu_2}{E_2}\right)\sin^2\alpha - \frac{1}{E_1}\right]$$

$$\phi_4 = -\left(\frac{1}{E_1} - \frac{1}{E_2}\right)\cos 2\alpha\sin\alpha\cos\alpha, \qquad \psi_4 = -\sin^2\alpha\left[\frac{2\cos^2\alpha}{\mathrm{E}_1} - \frac{\cos 2\alpha}{E_2}\right]$$

The rank of the equation system (7) at the point α^* is equal to three. Hence, equations will be compatible at this α, if all the determinants Δ_i will vanish. That is equations (39) will be fulfilled, where ϕ_i, ψ_i are set by relations (41). Equations (39) represent the system of equations relatively to load parameters p_n, τ_n. The matrix rank of this system at $\alpha = \alpha^*$ is equal to one. Hence, equations (39) are in fact one condition imposed on parameters p_n, τ_n, ensuring compatibility of equations (7) at the point α^*. When fulfillment of conditions (39) between stresses at the singular point, the following dependences are performed

$$\begin{aligned} \sigma_{11}^{(1)} &= p_n(1 - ctg^2\alpha) - 2\tau_n ctg\alpha + \sigma_{22}ctg^2\alpha \\ \sigma_{11}^{(2)} &= p_n(1 - ctg^2\alpha) - \tau_n(ctg\alpha - tg\alpha) + \sigma_{22}ctg^2\alpha \\ \sigma_{12} &= p_n ctg\alpha + \tau_n - \sigma_{22}ctg \end{aligned} \tag{42}$$

The number of conditions (42), (5), (4) set at the singular point is equal to six, as in the classical task. If conditions (39) are not performed, equations (7) are incompatible. When material parameters of connected bodies lead to the fulfillment of the condition (24), stresses in adjacent to the wedge vertex representative volumes increase with no limit.

3. **$\alpha + \beta = \pi$.**

The determinant (8) coincides up to a sign with the determinant (21). The condition $\Delta = 0$ falls into equations (22) and (23). Equation (22) within the area of admissible values α $(0 < \alpha < \pi)$ has the single root $\alpha = \pi / 2$. Hence, the structure is set by angles $\alpha = \pi / 2$, $\beta = \pi / 2$. Such a case of elements connection is considered in item 3.1. The equation (23) under restrictions on material parameters (26) or (27) has two roots, calculated according to the formulas (29). Determinants (33) are represented in the form (36), where functions ϕ_i, ψ_i in the considered case are set by expressions

$$\begin{aligned} &\phi_1 = \left(\frac{\nu_1}{E_1} - \frac{\nu_2}{E_2}\right)\cos 2\alpha\sin\alpha\cos\alpha && \psi_1 = \cos^2\alpha\left[\frac{1}{\mathrm{E}_2} + 2\left(\frac{\nu_1}{E_1} - \frac{\nu_2}{E_2}\right)\sin^2\alpha\right] \\ &\phi_2 = \phi_1 && \psi_2 = \frac{1}{\mathrm{E}_1}\cos^2\alpha + \left(\frac{\nu_1}{E_1} - \frac{\nu_2}{E_2}\right)\sin^2\alpha\cos 2\alpha \\ &\phi_3 = \left(\frac{1-\nu_1}{E_1} - \frac{1-\nu_2}{E_2}\right)\sin^2\alpha\cos^2\alpha, && \psi_3 = \sin\alpha\cos\alpha\left[\left(\frac{1-\nu_1}{E_1} - \frac{1-\nu_2}{E_2}\right)\sin^2\alpha - \frac{1}{E_1}\right] \end{aligned} \tag{43}$$

$$\phi_4 = \left(\frac{1}{E_1} - \frac{1}{E_2}\right)\cos 2\alpha \sin\alpha\cos\alpha \qquad \psi_4 = \sin^2\alpha\left[\frac{2\cos^2\alpha}{\mathrm{E}_1} - \frac{\cos 2\alpha}{E_2}\right]$$

At points α_1, α_2 (29) equations

$$p_n\phi_i(\alpha_k) + \tau_n\psi_i(\alpha_k) = 0 \quad (i = 1,2,3,4, \quad k = 1,2) \tag{44}$$

represent the linearly dependent system of equations relative to parameters p_n, τ_n. This means, that at each α_k $(k = 1,2)$ all the equations (44) are the same and represent restrictions on the load p_n, τ_n, at which equations (7) are compatible. When fulfillment of such restriction between stresses at singular points, the following dependences (40) are performed.

The number of restrictions on stresses at singular points corresponds to the classical case. If parameters of the load p_n, τ_n do not perform the restriction (44), points α_k $(k = 1,2)$ are critical. There is no solution of the equation system (7) in them. Components of stresses in the singular points at values α, tending to points α_k, increase with no limit.

4. **$\boldsymbol{\alpha + \beta = 2\pi}$**

The determinant Δ of the equation system (7) vanishes in points (31) and (32). Points α , which are on the list (31), are considered in item 3.1. Functions ϕ_i, ψ_i in representation of determinants (33) in the form (36) coincide with expressions (43). In points α_k $(k = 1,2,3,4)$, determined by the equality (32), the system of equations (44) has the rank, equal to one. Therefore, in each of points α_k $(k = 1,2,3,4)$ equations (44) turn to be one restriction on parameters p_n, τ_n, ensuring the compatibility of the equation system (7). When fulfillment of this restriction at points α_k $(k = 1,2,3,4)$, from equations (7) follow dependences between stresses (40). The number of restrictions (40), (5) and (4) corresponds to the classical case. If restriction (44) is not performed, equations (7) are incompatible at points α_k $(k = 1,2,3,4)$. In this case at α tending to α_k $(k = 1,2,3,4)$, stresses in the singular points tend to infinity.

5 THE PROBLEM OF STATICS FOR THE COMPOSITE BODY COMPRISING SINGULAR POINTS, ITERATIVE METHOD OF ITS SOLUTION

5.1 Statement of the Static Problem of Elasticity for a Body Comprising Singular Points. Uniqueness Theorem

It was ascertained above that the task of SM comprising singular points becomes non-classical, when the number of restrictions on the state parameters in these points exceeds the number of restrictions prescribed by the classical statement. Here is given a formulation of the non-classical static problem for a linearly elastic body composed of two isotropic bodies by continuous bonding along the surface S_{12} (example Figure 1). Let V - volume of the composite body, S – its surface, x^* - set of singular

points. The body is subjected to mechanical and uniform thermal load. Mathematical model of this task includes:
– equilibrium equations

$$\nabla \cdot \Sigma^{(k)} + \overline{f} = 0 \quad (k = 1,2) \tag{45}$$

- dependences expressing the strain tensor through displacements

$$E^{(k)} = \frac{1}{2}\left(\nabla \overline{u}^{(k)} + \nabla \overline{u}^{(k)T}\right) \tag{46}$$

- physical equations of thermo elastic behavior

$$\left(E^{(k)} - \omega_k G \Delta T\right) = \lambda_k I^{(k)} G + 2\mu_k \Sigma^{(k)} \tag{47}$$

- boundary conditions in stresses

$$\overline{n} \cdot \Sigma^{(k)} = \overline{p}_n \quad on\ \overline{x} \in S_\Sigma \quad if\ x \neq x^* \tag{48}$$

- boundary conditions in displacements

$$\overline{u}^{(k)} = \overline{u}_0 \quad on\ \overline{x} \in S_U \quad if\ x \neq x^* \quad where\ S_\Sigma \cup S_U = S, \tag{49}$$

- conditions of the continuity of stresses and strains on the border S_{12} of connected bodies

$$g_l = g_l^0 \quad on\ \overline{x} \in S_{12} \quad if\ x \neq x^* \quad (l = 1,2,\dots,L) \tag{50}$$

- restrictions on state parameters in singular points

$$h_m = h_m^0 \qquad on\ x = x^* \quad (m = 1,2,\dots,M) \tag{51}$$

Designations: $\bar{x}$ – radius-vector of body points; ∇ – Hamiltonian operator; Σ – tensor of stresses; I – the first invariant of the stress tensor; $\bar{f}$ – density of the volumetric forces; λ_k, μ_k – material Lame's constants; $\overline{p}_n$ – specified vector of stresses; $\overline{u}^0$ – specified vector of displacements; g_l, h_m – linear functions relative to state parameters; g_l^0, h_m^0 – constants, depending on the external impacts, L, M – the number of restrictions. It is required to determine the vector of displacements, tensors of stresses and strains performing relations (45) - (51).

It is claimed that if the solution of the problem (45) - (51) exists, it is the only one. Indeed, let us assume that there are two different solutions performing all the equations of the task. For parameters, which are differences of these solutions ($\tilde{\Sigma}^{(k)}$, $\tilde{E}^{(k)}$, $\tilde{u}^{(k)}$), by subtracting equations (45) – (51) corresponding to different solutions, we obtain the relations

$$\begin{aligned}
&\nabla \cdot \tilde{\Sigma}^{(k)} = 0 \qquad \tilde{E}^{(k)} = \frac{1}{2}\left(\nabla \tilde{\tilde{u}}^{(k)} + \nabla \tilde{\tilde{u}}^{(k)T}\right)\\
&\tilde{E}^{(k)} = \lambda_k \tilde{I}^{(k)} G + 2\mu_k \tilde{\Sigma}^{(k)}\\
&\overline{n} \cdot \tilde{\Sigma}^{(k)} = 0 \quad in\ \overline{x} \in S_\Sigma \quad if\ x \neq x^* \\
&\tilde{\tilde{u}}^{(k)} = 0 \quad on\ \overline{x} \in S_U \quad if\ x \neq x^* \quad where\ S_\Sigma \cup S_U = S\\
&g_l = 0 \quad on\ \overline{x} \in S_{12} \quad if\ x \neq x^* \quad (l = 1,2,...,L)\\
&h_m = 0 \qquad in\ x = x^* \quad (m = 1,2,...,M)
\end{aligned} \tag{52}$$

Equations (52) are equations of the elasticity problem for the composite body V, comprising a set of singular points x^*. There is no external impacts in this problem, therefore the work of strains in the body V is equal to zero.

Due to the positive determinateness of the quadratic form of the strain work, tensor components $\tilde{E}^{(k)}$ vanish and according to physical equations tensor components $\tilde{\Sigma}^{(k)}$ are equal to zero. At that, all the function g_l, h_m vanishes too, since they are uniform within this problem. Thus, the problem (52) has only the zero solution, that proves the assertion.

5.2 Algorithm of the Problem (45) – (51) Solution

The fundamental difference of the problem (45) - (51) from problem for the elastic body with singular points in the classical setting lies in restrictions on state parameters (51). In the classical formulation the singular point it is not considered representative body volume, so any state parameters in it are not considered and, hence, restrictions for them can't be formulated (equations of the form (51) do not take part in the formulation of the problem).

Here the solution of the problem (45) - (51) for composite body V is constructed by iterative numerical-analytical method using mixed variation principle, wherein independent functions are displacements and strains (suggested by authors in 2015). Displacements are searched in the class of continuous functions throughout the entire body V. Their first derivatives are continuous in members which constitute the body V. The null approximation is built by solving the problem with the finite element method in the classical formulation (restrictions (51) are not taken into account). On the specified finite element mesh the conditions (45) - (51) are represented by the equation

$$\mathbf{AU} = \mathbf{b} \tag{53}$$

where $\mathbf{U}$– global vector of displacements, $\mathbf{b}$– vector of the set constants. Displacements in nodes, where conditions (48) - (51) are set, form the vector $\mathbf{U}_P$. The vector $\mathbf{U}$ is represented by combination of smaller dimensionality vectors $\mathbf{U} = \mathbf{U}_P \cup \mathbf{U}_l$. The matrix $\mathbf{A}$ is represented by the combination of two rectangular matrices $\mathbf{A} = \mathbf{A}_P \cup \mathbf{A}_l$, in such a way, that the equation

$$\mathbf{A}_P \mathbf{U}_P = \mathbf{b} - \mathbf{A}_l \mathbf{U}_l \tag{54}$$

becomes true. This equation is the base on which the iterative process $\mathbf{A}_P\mathbf{U}_P^{(n)} = \mathbf{b} - \mathbf{A}_l\mathbf{U}_l^{(n-1)}$, n=1,2,... is organized. The matrix $\mathbf{A}_P$ of this system turns to be rectangular, the number of lines in it exceeds the number of columns, therefore, the solution of equations (54) exists only in the generalized sense (pseudo solution). At each step of successive approximations it is searched by the method of singular value decomposition (Forsythe.G.,1980). Under pseudo solution of equations (54) is understood either their single solution (if it exists), or a decision in the sense of the least mean square value of the residual vector $(\mathbf{AU}^{(n)} - \mathbf{b})$. The vector $\mathbf{U}_P^{(n)}$ is used as the boundary condition at building n-th approximation of the thermo elasticity problem solution. The decrease of the residual vector value in successive iterations is evidence of the iterative process convergence. The value of the residual vector characterizes the error of the fulfillment of set conditions in singular points.

6 EXAMPLE. LOADING OF THE RECTANGULAR COMPOSITE PLATE

The considered plate is composed of two isotropic elements 1,2 so that sum angles at the vertex A $(\alpha + \beta = \pi / 2)$. Side OA of the plate slides without friction along the rigid surface (Figure 2). Geometric and material design parameters have the values: $E_1 = 1.05e5MPa$, $E_2 = 0.7e5MPa$, $\nu_1 = 0.3$, $\nu_2 = 0.4$, $\omega_1 = 0.11e-4c^{-1}$, $\omega_2 = 0.85e-5c^{-1}$, OA=50 mm, AA'= 80 mm. The plate can be loaded by distributed forces with intensity of q=100 MPa on the upper side $O'A'$ $(\Delta T = 0)$ or subjected to the temperature change $\Delta T = 100^o$ (item 3.2, the case 2).

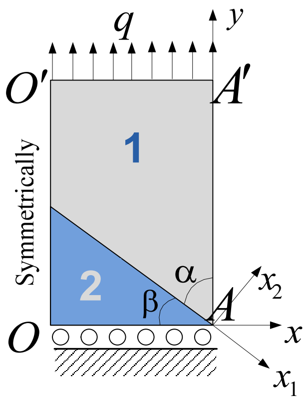


Figure 2: Composite plate.

The critical value of the angle α, calculated according to the formula (24), is $\alpha^* = 52.239^o$. When $\Delta T = 0$ and $\alpha \neq \alpha^*$, from equations (7) and conditions (5) follows the equality to zero of all stresses in singular points. In the classical approach, there is no such certainty, because the restriction (6) is not taken into account. The problem solution is constructed by the iterative numerical-analytical method as described in item 5. Figures 3 – 5 show the calculation results. The largest normal stresses at the plate tension are realized in the vicinity of the vertex A on the side AA'. These stresses for different angles α are shown in Figure 3.

When solving this problem by conventional methods, subjecting of the solution to all set restrictions fails. Figure 4 shows by the dashed line the solution obtained using the ANSYS package. It does not match restrictions which are imposed on state parameters in singular points. A significant difference of solutions is observed in the small neighborhoods of these points. Beyond small neighborhoods the solution obtained by the iterative method, coincides with the classical one.

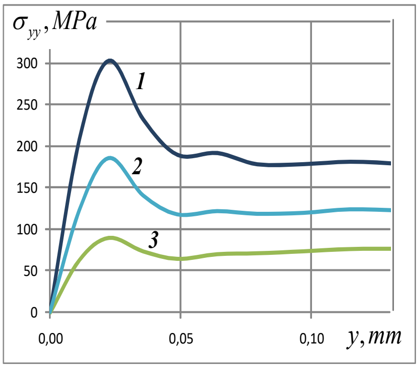


Figure 3: Normal stresses σ_{yy} on the side AA' close to the point A at different α: 1– 40°; 2 – 49°; 3 – 64°. q=100 MPa, ΔT=0.

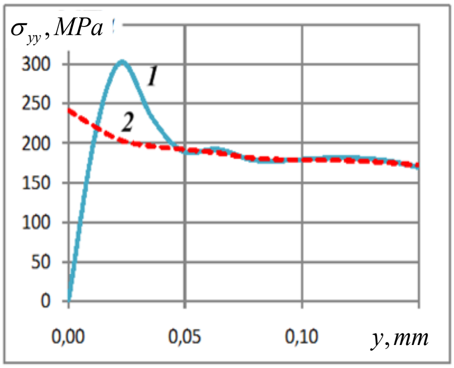


Figure 4: Normal stresses σ_{yy} on the side AA' close to the point A for α=40°: 1– solution by the iterative method; 2– solution in the classical setting.

The degree of non-fulfillment of restrictions, set in singular points, is evaluated by the root-mean-square deviation η of their values from the really set values. For the classical approach (FEM-ANSYS) this value was η =152.3% ($q \neq 0, \quad \Delta T = 0$), for the solution by the method of successive approximations – η =1.2%. Figure 5 shows at different values α stresses σ_{yy} nearby the point A on the side AA' at the thermal loading of the composite plate.

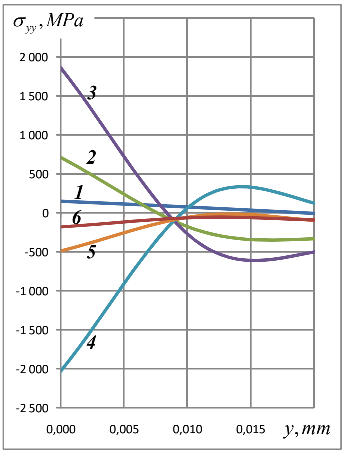
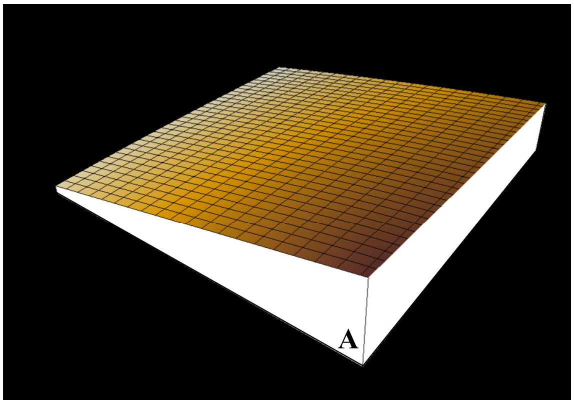


Figure 5: Normal stresses σ_{yy} on the side AA' close to the point A for different α:

1– 40°, 2 – 49°, 3 – 51°,4– 53°, 5 – 55°, 6 – 64°. q=0, $\Delta T = 100^o C$.

It is seen that with the approach of the angle α to the critical value α^* stresses σ_{yy} increase with no limit. In the case, if $\alpha=\alpha^*$, $q\neq0$, $\Delta T=0$, the task becomes classical. Its solution can be built, for example, with the use of ANSYS engineering package.

Figure 6a shows the surface of the stress σ_{yy} level nearby the wedge vertex for this case.



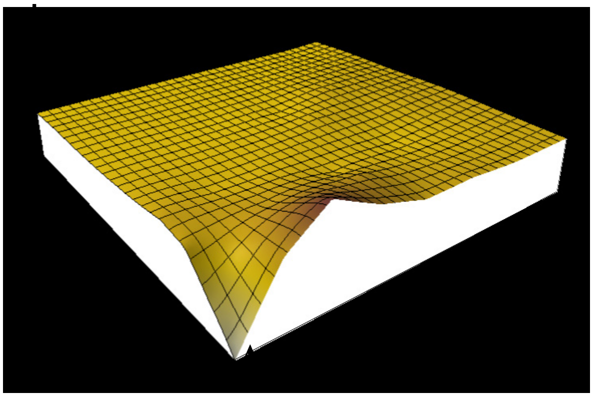


Figure 6: Surfaces of normal stresses σ_{yy} (MPa) in the neighborhood of the point A (wedge vertexes) ($q=100MPa, \Delta T=0$): a) $\alpha=\alpha^*$; b) $\alpha\neq\alpha^*$(α = 40°).

It is seen that there is no concentration of stresses in the neighborhood of the point A. For comparison, the solution for the case $\alpha\neq\alpha^*$ is given in the Figure 6b, when the stress concentration with the factor of 3 is realized in the neighborhood of the point A.

7 CONCLUSIONS

The new, consistent with postulates of continuum mechanics, the concept of SSS research nearby singular points is suggested. A singular point is interpreted as an elementary volume of a continuum with a characteristic size equal to the characteristic size of the real body representative volume. The feature of representative volumes shows itself in an excessive number (as compared to conventional boundary material points) of restrictions which are set in them. This circumstance makes tasks of mechanics for bodies with singular points non-classical. The formulation of the non-classical problem is given, the theorem of uniqueness of its solution is proved. The procedure of its numerical-analytical iterative solution is described. It is studied in detail the state parameters distribution nearby the plane wedge vertex, one of which sides slides without friction along the rigid surface. Combinations of material and geometric parameters, which meet the classical and non-classical settings of the problem, are detected. Criteria of the singular behavior of solutions in the neighborhood of the wedge vertex are formulated. The examples of SSS research in singular points and their vicinity of the rectangular composite wedge which is subjected to the thermal and mechanical load are showed.

Research results can be extended to other elements of constructs, containing singular points, and developed towards creating methods for solving non-classical tasks. Results of the study can be extended to other elements of the constructs containing the singular points, and advanced towards the establishment of methods for solving non-classical SM tasks.

Calculations were carried out using the software package, implemented on the supercomputer Tesla-PSU (Research and Education Center of parallel and distributed computing, Perm State National Research University).

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