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Static and free vibration analysis of carbon nano wires based on Timoshenko beam theory using differential quadrature method

Abstract

Static and free vibration analysis of carbon nano wires with rectangular cross section based on Timoshenko beam theory is studied in this research. Differential quadrature method (DQM) is employed to solve the governing equations. From the knowledge of author, it is the first time that free vibration of nano wires is investigated. It is also the first time that differential quadrature method is used for bending analysis of nano wires.

Keywords

carbon nano wires, Timoshenko beam theory, differential quadrature method, free vibration, static analysis.

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1 INTRODUCTION

A nano wire (NW) is an extremely thin wire with a diameter on the order of a few nanometers (nm) or less, where 1 nm = 10^{-9} meters. Nano wires have many interesting properties that are not seen in bulk or 3-D materials. Many different types of nano wires exist, including metallic (e.g., Ni, Pt, Au), semiconducting (e.g., InP, Si, GaN, etc.), and insulating (e.g., SiO_2, TiO_2). Nano structures such as single and double walled carbon nanotubes (CNTs) [3], nano plates [10, 12, 14] and nano rods [1] are widely studied in past decade but static and dynamic analysis of nano wires are rarely investigated. Surface effects on the elastic behavior of static bending nano wires were studied by using a comprehensive Timoshenko beam model by Jiang and Yan [9]. Fu and Zhang [7] were established a continuum elastic model for core–shell nano wires with weak interfacial bonding. Critical buckling loads and resonant frequencies of simply supported nano wires are obtained by using the Ritz method. Song and Huang [19] presented a model of surface stress effects on bending behavior of nano wires based on the incremental deformation theory. Differential quadrature (DQ) was introduced by Bellman et al [2] as a simple and highly efficient numerical technique in 1971. This method generalized and simplified further by Quan and Chang [15] and Shu and Richards [18] through explicit expressions of the weighting coefficients associated with their derivatives. This method was used several times for investigation of micro and nano structures. Civalek et al [5] applied the differential quadrature method to the equations of motion and bending of Euler-Bernoulli beam using the nonlocal elasticity theory for cantilever microtubules. Wang et al [22] concerned with the use of the Timoshenko beam model for free vibration analysis of multi-walled carbon nanotubes by using differential quadrature method. Pradhan and Murmu [13] developed a single nonlocal beam model and applied to investigate the flapwise bending–vibration characteristics of a rotating nano cantilever using DQ method. Murmu and Pradhan [11] also presented the buckling analysis of a single-walled carbon nanotube embedded in elastic medium based on nonlocal elasticity and Timoshenko beam theory and using DQM. In this paper, static and free vibration analysis of carbon nano wires based on Timoshenko beam theory using differential quadrature method is investigated. From the knowledge of author, two kinds of nano wire are exits, one with rectangular cross section and the other with circular cross section. In this study, nano wires with rectangular cross sections are studied.

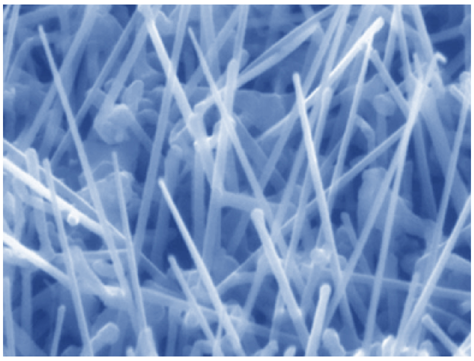

Figure 1 Structure of nano wires [21].

2 DIFFERENTIAL QUADRATURE METHOD

The numerical solution of partial differential equations plays a considerable role in the areas of engineering. The differential quadrature method is one which can satisfy the above purpose. Differential quadrature is characterized by approximating the derivatives of a function using a weighted sum of functions values on a set of selected discrete points. In order to show the mathematical representations of DQM, consider a function $f(x, y)$ having its field on a rectangular domain $0 \le x \le a$ and $0 \le y \le b$. let, in the given domain, the function values be known or desired on a grid of sampling points. According to DQ method, the r^{th} derivative of a function $f(x, y)$ can be approximated as [2],

$$\left.\frac{\partial^r f(x,y)}{\partial x^r}\right|_{(x,y)=(x_i,y_j)} = \sum_{m=1}^{Nx} A_{im}^{x(r)} f(x_m, y_j) = \sum_{m=1}^{Nx} A_{im}^{x(r)} f_{mj} \tag{1}$$

where $i = 1, 2, \ldots, N_x$, $j = 1, 2, \ldots, N_y$ and $r = 1, 2, \ldots, N_x - 1$.

From this equation one can deduce that the important components of DQ approximations are weighting coefficients and the choice of sampling points. In order to determine the weighting coefficients a set of test functions should be used in Eq. (1). For polynomial basis functions DQ, a set of Lagrange polynomials are employed as the test functions. The weighting coefficients for the first-order derivatives in ξ-direction are thus determined as

$$A_{ij}^{x} = \begin{cases} \frac{1}{L_\xi}\frac{M(x_i)}{(x_i - x_j)M(x_j)} & \text{for } i \neq j \\ -\sum\limits_{\substack{j=1 \\ i \neq j}}^{N_x} A_{ij}^{x} & \text{for } i = j \end{cases} \quad ; \quad i,j = 1,2\ldots,N_x \tag{2}$$

where L_x is the length of domain along the x–direction and $M(x_i) = \prod_{k=1,i\neq k}^{N_x}(x_i - x_k)$.

The weighting coefficients of second order derivative can be obtained as,

$$[B_{ij}^{x}] = [A_{ij}^{x}][A_{ij}^{x}] = [A_{ij}^{x}]^2 \tag{3}$$

In a similar way, the weighting coefficients for y-direction can be obtained. The weighting coefficient of the third and fourth order derivatives (C_{ij}, D_{ij}) can be computed easily from (B_{ij}) by

$$C_{ij} = \sum_{j=1}^{N} A_{ij}A_{ij}A_{ij}, \quad D_{ij} = \sum_{j=1}^{N} B_{ij}B_{ij} \tag{4}$$

3 GOVERNING EQUATIONS

The importance of shear deformation and rotary inertia in the description of the dynamic response of beams is well documented and an improved theory was given by Timoshenko already in 1921 [20]. Consider a constant thickness nano wire with rectangular cross section with length L, thickness h and width b. In order to calculate the deflections and natural frequencies of carbon nano wires, Timoshenko beam theory is used. The Timoshenko model is an extension of the Euler-Bernoulli model by taking into account two additional effects: shearing force effect and rotary motion effect. For Timoshenko beam theory, the displacement field is assumed to be as follow,

$$u_1 = u(x,t) + z\theta(x,t), \quad u_2 = 0, \quad u_3 = w(x,t) \tag{5}$$

The nonzero strains of the Timoshenko beam theory are [16],

$$\begin{aligned} \varepsilon_{xx} &= \partial u/\partial x + z\partial\theta/\partial x \\ 2\varepsilon_{xz} &= \partial w/\partial x + \theta \end{aligned} \tag{6}$$

The equations for a Timoshenko beam theory are given by,

$$\begin{aligned} \partial M/\partial x + V = \rho I \partial^2\theta/\partial t^2 \\ \partial V/\partial x = q(x) + \rho A \partial^2 W/\partial t^2 \end{aligned} \tag{7}$$

Where W is the deflection of beam, θ is the rotation of the normal line. We also have,

$$
\begin{aligned}
M &= EI\partial\theta/\partial x \\
V &= kGA\left(\partial W/\partial x - \theta\right)
\end{aligned}
\tag{8}
$$

Assuming the sinusoidal motion in time, Eq. (4) yields

$$
\begin{aligned}
&KGA\left(d^2W/dx^2\right) - KGA\left(d\theta/dx\right) + \rho A\omega^2 W = q\left(x\right) \\
&EI\left(d^2\theta/dx^2\right) + KGA\left(dW/dx\right) - KGA\theta + \rho I\omega^2\theta = 0
\end{aligned}
\tag{9}
$$

Contrary to most other publications, here, these two differential equations of second order are used instead of rearranging these to one differential equation of fourth order. Using the DQ-rules for the spatial derivatives, the DQ-analogs of the governing Eqs. (9) become

$$
\begin{aligned}
&KGA\left(\sum_{\mathbf{j=1}}^{\mathbf{N}} B_{ij}W_i\right) - KGA\left(\sum_{j=1}^{N} A_{ij}\theta_i\right) + \rho A\omega^2 W_i = q\left(x_i\right) \\
&EI\left(\sum_{\mathbf{j=1}}^{\mathbf{N}} B_{ij}\theta_i\right) + KGA\left(\sum_{j=1}^{N} A_{ij}W_i\right) - KGA\theta_i + \rho I\omega^2\theta_i = 0
\end{aligned}
\tag{10}
$$

Based on Eq. (7) a computer code is developed in MATLAB for static and free vibration analysis. For free vibration analysis, the DQ discretized form of the equations of motion and the related boundary conditions can be expressed in the matrix form as,

$$
\text{Equations of motion:} \quad \left[K_{db}\right]\{b\} + \left[K_{dd}\right]\{d\} + [M]\left\{\ddot{d}\right\} = \{0\} \tag{11}
$$

$$
\text{Boundary conditions:} \quad \left[K_{bb}\right]\{b\} + \left[K_{bd}\right]\{d\} = \{0\} \tag{12}
$$

In the above equations, K_{dd}, K_{db}, K_{bd} and K_{bb} refers to domain-domain, domain-boundary, boundary-domain and boundary-boundary stiffness matrix, respectively. The mass matrix $[M]$ is obtained from equations of motion and those of the stiffness matrices $[K_{ii}]$ $(i = b, d)$ are obtained from the domain and boundary conditions. Solving the eigenvalue system of below equation, the natural frequencies will be obtained.

$$
\left([KK] - \omega^2 [M]\right)\{X\} = \{0\} \tag{13}
$$

Where $[KK] = [K_{dd}] - [K_{db}][K_{bb}]^{-1}[K_{bd}]$. For static analysis, deflections can easily calculated from the equation below,

$$
\left([KK]\right)\{X\} = \{F\} \tag{14}
$$

where $\{F\}$ is the force vector. It is mentioned that in this case, the matrix $[KK]$ can be written directly and we do not need to use $[K_{ii}]$ $(i = b, d)$.

The boundary condition used in this study is defined as,

$$\text{Simply supported, (at both ends)}: \quad W = 0, \partial^2 W/\partial x^2 = 0 \tag{15}$$

The discretized form of boundary condition can be obtained by,

$$\text{Simply supported, (at both ends)}: \quad W_i = 0, \quad \sum_{j=1}^{N} B_{ij} W_j = 0 \tag{16}$$

Table 1 Comparison of deflections for S-S single walled carbon nanotube with constant thickness under uniformly distributed load ($D_o/L = .1$, $\mathbf{w} = wEI/qL^4$).

[4]	[17]	Present			
		$N_x = 8^*$	$N_x = 10$	$N_x = 12$	$N_x = 14$
1.3134	1.3152	1.2562	1.2867	1.3024	1.3115

*N_x is the number of grid points in x direction.

Table 2 Comparison of natural frequencies for S-S single walled carbon nanotube with constant thickness ($D_o/L = .1$, ρ=2300 Kg/m^3, $E = 1000$ GPa, $D_o = 33$nm, $\lambda^2 = \omega L^2 \sqrt{\rho A/EI}$).

Modes	Present	Demir [6]	Heireche et al [8]	Reddy [16]
1	3.1123	3.1405	3.0929	3.1217
2	6.0676	6.2747	5.9399	-
3	8.7784	9.3963	8.4444	-
4	11.2290	10.7218	10.6260	-
5	13.4415	-	-	-
6	15.4493	-	-	-
7	17.2847	-	-	-
8	18.9754	-	-	-

4 NUMERICAL RESULTS

Let us consider a carbon nano wire having a length $L = 400$ nm, thickness h=40 nm, width b= 2h, Young modulus $E = 76$ GPa [9], density $\rho = 1220$ Kg/m^3 and shear correction factor for rectangular cross section $k = 5(1+v)/(6+5v)$ [9]. The moments of inertia and the areas are calculated using $I = (1/12)(bh^3)$ and $A = bh$. The numerical results can be shown in the dimensionless quantities defined as follow:

$$\mathbf{w} = w/h, \quad \lambda^2 = \omega L^2 \sqrt{\rho A/EI} \tag{17}$$

In order to show the accuracy of present work, our results are compared with other works for single walled carbon nano tubes and then the results for carbon nano wires are presented.

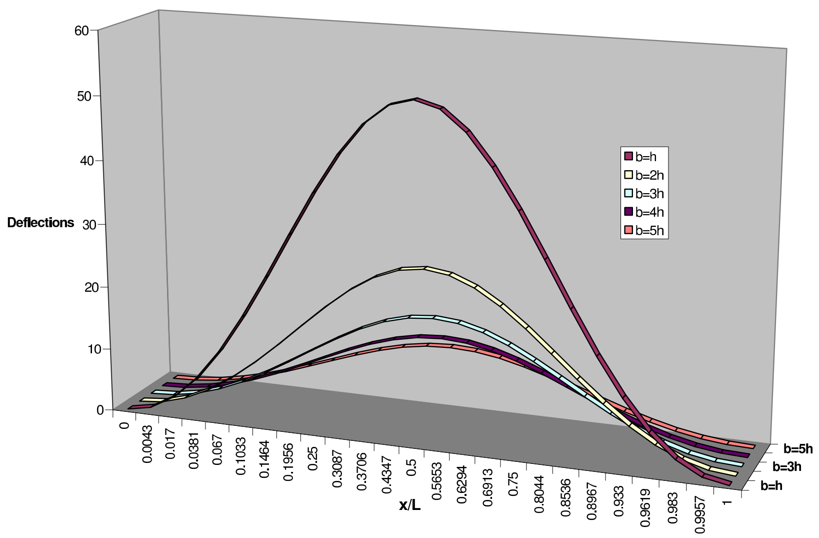

Figure 2 The effects of width to thickness ratio on the deflections of simply supported carbon nano wires subjected to uniformly distributed loading.

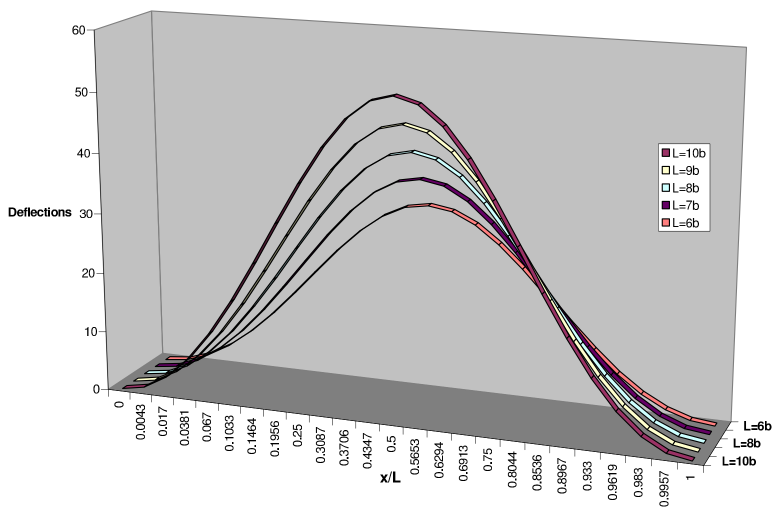

Figure 3 The effects of length to width ratio on the deflections of simply supported carbon nano wires subjected to uniformly distributed loading.

For static analysis, In Table 1, the results are compared with those of other numerical solutions for a simply supported single walled carbon nanotube. The convergency of the method is also investigated, too. One can easily find that even 12 grid points is enough to have accurate results. For free vibration analysis, to validate the present formulation and the computer program developed by the author, using the DQ method, our natural frequencies are compared with Reddy [16], Heireche et al [8] and Demir et al [6] in Table 2. Very good agreement is achieved between the results of present work and other methodologies. For carbon nano wires, the effects of width to thickness, length to width and length to thickness ratios on the

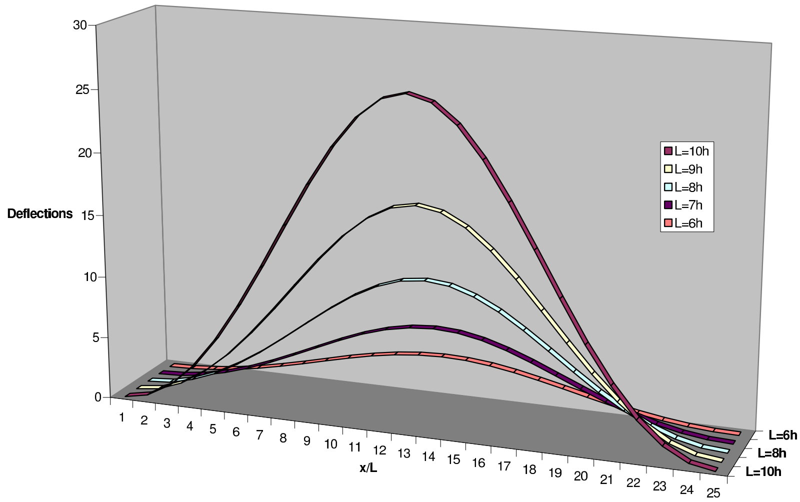

Figure 4 The effects of length to thickness ratio on the deflections of simply supported carbon nano wires subjected to uniformly distributed loading.

deflections of carbon nano wires subjected to uniformly distributed loading are shown in Figs. 2, 3 and 4. One can see that increasing the length to thickness, length to width and thickness to width ratios will increase the deflections of carbon nano wires. The shapes of carbon nano wires under mechanical loading are as we expected and the maximum deflections appear at the middle of nanowire. The convergency of free vibration analysis is studied in Fig. 5. It seems that only nine grid points are enough for investigation of carbon nano wires. It is also shown that for higher modes, more grid points are needed.

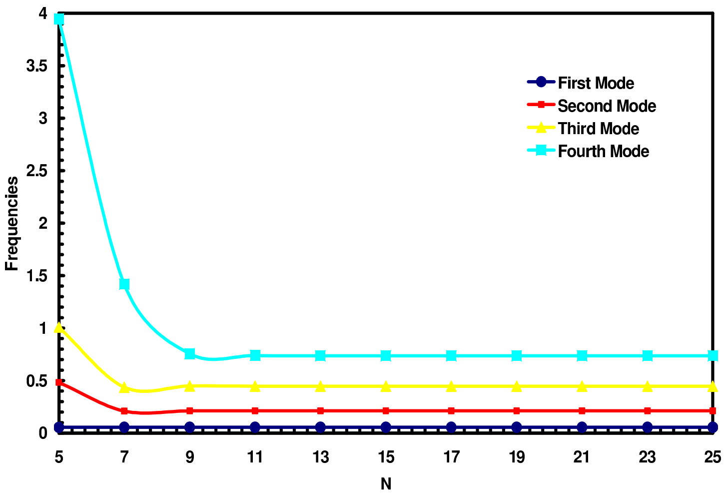

Figure 5 Convergency study for natural frequencies of simply supported carbon nano wires.

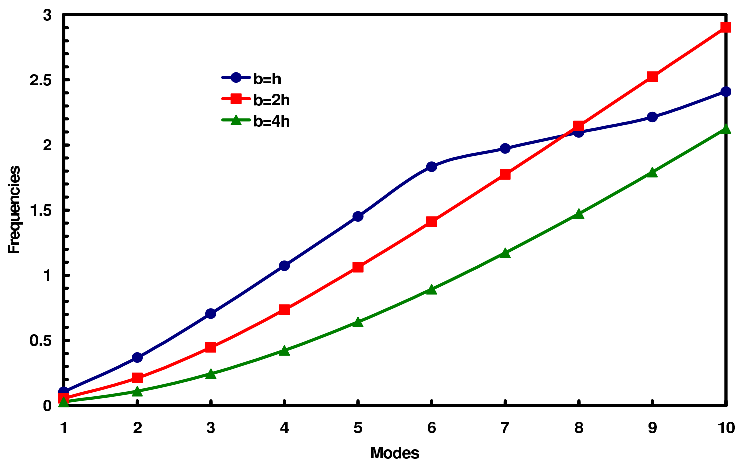

Figure 6 The effects of width to thickness ratio on the natural frequencies of simply supported carbon nano wires.

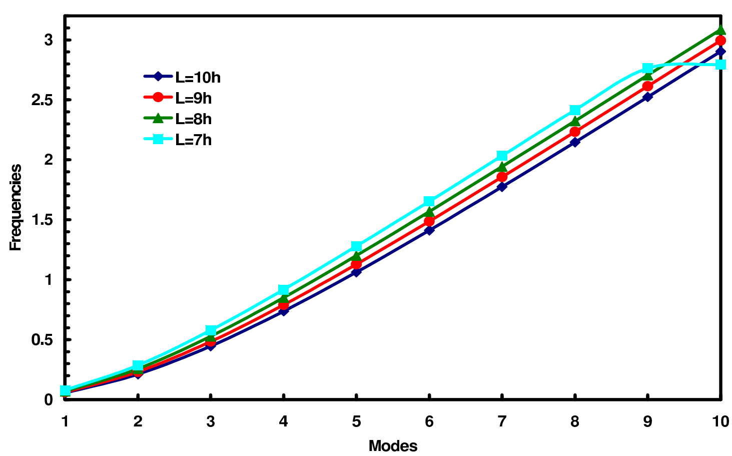

Figure 7 The effects of length to thickness ratio on the natural frequencies of simply supported carbon nano wires.

From the Tables 1 and 2 and Fig. 5, it may be observed that the differential quadrature method is one of the fastest numerical tools for solving governing equations. The effects of width to thickness and length to thickness ratios on natural frequencies are investigated in Figs. 6 and 7. It is shown that for increasing the natural frequencies of carbon nano wires, one may increase the length to thickness and thickness to width ratios. The increase in the values of the deflections and natural frequencies as the width to thickness and length to thickness ratios are varied is evident of its importance contribution to the mathematical model.

5 CONCLUSION

Deflections and natural frequencies of carbon nano wires with constant thickness have been obtained by using a Timoshenko beam theory. The differential quadrature method was employed to convert the governing differential equations into a linear system. The accuracy and convergency of present work were investigated. The effects of different parameters are studied, too. Some of the results of present study is,

1. Increasing the length to thickness, length to width and thickness to width ratios will increase the deflections of carbon nano wires.

2. Increase the length to thickness and thickness to width ratios will increase the deflections of carbon nano wires

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