Original Article

Latin American Journal of Solids and Structures
www.lajss.org

Influence of Stress on Propagation of Shear Wave in Piezoelectric-Piezoelectric (PE-PE) Composite Layered Structure

Abstract
Shear waves (SH) propagation in piezoelectric composite under the influence of initial stress is investigated analytically and numerically. The dispersion equation of shear waves propagation in direction normal to the layering is obtained in presence of initial stress. Numerical solutions were obtained for evaluating the effect of stress on dimensionless frequency and phase velocity. The effect of stress on stop band is discussed in this study. It can be concluded from the results that initial stress has significant effect on propagation characteristics of shear waves. The variation of initial stress has small effect on the phase velocity of shear waves. This study provides insight for development of piezoelectric composite structure under the influence of initial stress.

Keywords
Shear wave, Initial stress, Dispersion relation, Phase velocity, piezoelectric composite structure.

Anshu Mli Gaur [a]

[a] Electrical and Instrumentation Engineering Department, Thapar University, Patiala, Punjab, India-147004
E-mail: gauranshu10@gmail.com

http://dx.doi.org/10.1590/1679-78254032

Received 16.05.2017
In revised form 14.09.2017
Accepted 23.09.2017
Available online 30.09.2017

1 INTRODUCTION

The propagation behavior of shear horizontal wave is most important phenomena to be understood due to its invariably application in surface acoustic wave (SAW) sensors. It is well known fact that when mechanical stress is applied to piezoelectric material, an electric voltage is generated and application of electric field across material mechanical deformation is observed. It is important to know the propagation properties of shear wave in composite material for designing the SAW sensor [Burkov and Piliposian 2011]. The propagation characteristics depend upon the properties of piezoelectric materials [Danoyan et al. 2008]. Due to these properties, piezoelectric materials found potential applications in sensors, valves, actuators and smart structures [Yuan et al. 2014, Zhanga et al. 2014, Li et al. 2012]. SAW sensor essentially comprises of alternative layers of piezoelectric materials bonded together to form a composite structure. These composite structures of two materials have better properties as compare to single material structure [Levin et al. 2011].

Numerous investigations have been carried out in past which mainly focuses on propagation characteristics of shear horizontal waves in composite layered structure. Liu et al. (2003) discussed the propagation of B-G waves in pre-stressed piezoelectric structure. It is revealed in literature that, thickness ratio is affected by presence of initial stress in composite structure [Son and Kang 2011]. Mahmoud (2013) found that velocity of shear waves is influenced by presence of stress in multi layered structure. The effect of initial stress on propagation behavior of shear waves in functionally graded half space was discussed by Qin et al. (2010) extensively and reveals the influence of initial stress on dispersion relation and group velocity. Kayestha et al. (2011) investigated the wave propagation in pre-stressed elastic layers. Qian et al. (2004) investigated the effect of initial stress on characteristics of shear waves in polymer-piezoelectric structure. Du et al. (2008) discussed the Love waves propagation in pre-stressed piezoelectric structure and concluded that intimal stress has considerable effect on phase velocity.

The presence of initial stress in piezoelectric composite structure introduces number of problems such as frequency shift, variation in phase velocity of SH waves that degrades the response of SAW sensors [Jin et al. 2005, Su et al. 2005, Du et al. 2007]. Zhang et al. (2013) investigated the effect of stress on lateral modes frequencies and the stop band in piezo composites structures. Recently some of researchers studied the propagation of a Love wave in an initially stressed fluid-saturated anisotropic porous layer and concluded that presence of stress significantly affect the porosity [Chattraj et al. 2012]. But till now, no work has focused to consider the effect of initial stress in piezoelectric-piezoelectric composite structure and consideration of stress is important issue in designing and manufacturing of SAW sensors.

In this study, we have made an attempt to consider the influence of initial stress on propagation behavior of shear wave in piezoelectric layered structure. In section 2, we have formulated the constitutive equations for PE-PE interface and these equations are utilized to derive the dispersion relation under the presence of initial stress. In section 3, the solutions of dispersion equation were obtained by considering the bulk shear velocities for alternative layer of piezoelectric material respectively. In section 4, the numerical computation was carried out by considering two different piezoelectric materials i.e. PVDF and PZT-8. Finally the effect of initial stress on phase velocity with variation in wave number is observed for perfectly bonded piezoelectric layers.

2 PROBLEM FORMULATION

Consider a two layer composite structure with alternative layer of piezoelectric materials of thickness h_1 and h_2 bonded perfectly together as shown in figure 1. The shear wave propagates along the positive direction of x axis with poling of piezoelectric material taken along the z axis.

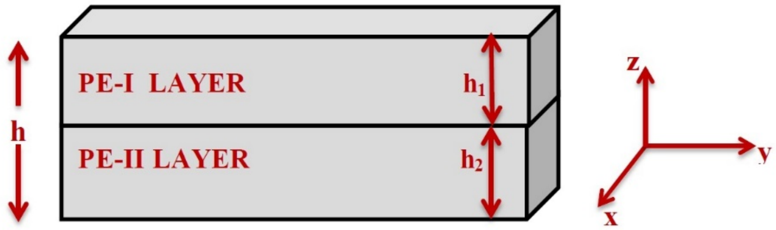

Figure 1: Schematic of piezoelectric layered structure.

The wave motion equation for piezoelectric material with initial stress can be described by following equation [Mahmoud 2013, Qian et al. 2012]

$$\sigma_{ij,j} + (u_{i,k}\sigma^0_{kj})_{,j} = \rho \ddot{u}_i \tag{1}$$

$$D_{i,i} + (u_{i,j}D_j)_{,j} = 0 \tag{2}$$

Where ρ is mass density, $\sigma_{ij,j}$ is stress tensor, σ^0_{kj} is initial stress tensor, u_i and D_i represents the mechanical and electrical displacements in the ith direction. D_j is the initial electrical displacement. For the piezoelectric layer I media, the constitutive equations of (1) can be represented as [Gaur and Rana 2015]

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{zy} \\ \tau_{zx} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{44} \end{bmatrix} \begin{Bmatrix} s_x \\ s_y \\ s_z \\ s_{zy} \\ s_{zx} \\ s_{xy} \end{Bmatrix} - \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{31} \\ 0 & 0 & e_{33} \\ 0 & e_{15} & 0 \\ e_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix} \tag{3}$$

$$\begin{Bmatrix} D_x \\ D_y \\ D_z \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{15} & 0 & 0 \\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} s_x \\ s_y \\ s_z \\ s_{zy} \\ s_{zx} \\ s_{xy} \end{Bmatrix} + \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{11} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix} \begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix} \tag{4}$$

where c_{11}, c_{12}, c_{13} and c_{14} are elastic constants, e_{15}, e_{31} and e_{33} are piezoelectric constants and ε_{11} and ε_{33} are dielectric constants. For the piezoelectric layer II media, the constitutive equations of (1) can be represented as follows

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{zy} \\ \tau_{zx} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} c'_{11} & c'_{12} & c'_{13} & 0 & 0 & 0 \\ c'_{12} & c'_{11} & c'_{13} & 0 & 0 & 0 \\ c'_{13} & c'_{13} & c'_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c'_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c'_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c'_{44} \end{bmatrix} \begin{Bmatrix} s_x \\ s_y \\ s_z \\ s_{zy} \\ s_{zx} \\ s_{xy} \end{Bmatrix} - \begin{bmatrix} 0 & 0 & e'_{31} \\ 0 & 0 & e'_{31} \\ 0 & 0 & e'_{33} \\ 0 & e'_{15} & 0 \\ e'_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix} \tag{5}$$

$$\begin{Bmatrix} D_x \\ D_y \\ D_z \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & e'_{15} & 0 \\ 0 & 0 & 0 & e'_{15} & 0 & 0 \\ e'_{31} & e'_{31} & e'_{33} & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} s_x \\ s_y \\ s_y \\ s_{zy} \\ s_{zx} \\ s_{xy} \end{Bmatrix} + \begin{bmatrix} \varepsilon'_{11} & 0 & 0 \\ 0 & \varepsilon'_{11} & 0 \\ 0 & 0 & \varepsilon'_{33} \end{bmatrix} \begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix} \tag{6}$$

where c'_{11}, c'_{12}, c'_{13} and c'_{14} are elastic constants, e'_{15}, e'_{31} and e'_{33} are piezoelectric constants and ε'_{11} and ε'_{33} are dielectric constants.

The strain tensor s_{ij} and electrical intensity E_k can be represented as follows

$$s_x = \frac{\partial u}{\partial x}, s_y = \frac{\partial v}{\partial y}, s_z = \frac{\partial w}{\partial z} \tag{7}$$

$$s_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, s_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}, s_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}. \tag{8}$$

$$E_x = -\frac{\partial \varphi}{\partial x}, E_y = -\frac{\partial \varphi}{\partial y}, E_z = -\frac{\partial \varphi}{\partial z} \tag{9}$$

We consider the Shear wave propagates along the direction normal to layering in presence of initial stress σ^0_x. The mechanical displacement component W and electrical function φ can be described by following equations, and z is the polarization direction [Gaur and Rana 2014]

$$u = \nu = 0, \omega = \omega(x,t), \varphi = \varphi(x,t) \tag{10}$$

Substituting equations (10) into (1)-(2) and (7)-(9), we have

$$\frac{\partial \tau_{zx}}{\partial x} + \sigma^0{}_x \frac{\partial^2 \omega}{\partial x^2} = \rho \frac{\partial^2 w}{\partial t^2} \tag{11}$$

$$\frac{\partial D_x}{\partial x} = 0 \tag{12}$$

$$s_x = s_y = s_z = 0 \tag{13}$$

$$s_{yz} = 0, s_{zx} = \frac{\partial w}{\partial x}, s_{xy} = 0 \tag{14}$$

$$E_x = -\frac{\partial \varphi}{\partial x}, E_y = 0, E_z = 0 \tag{15}$$

Substituting equations (13)-(15) into (3)-(4) results into

$$\sigma_x = \sigma_y = \sigma_z = 0 \tag{16}$$

$$\sigma_{xy} = 0, \sigma_{yz} = 0, \sigma_{zx} = c_{44} \frac{\partial w}{\partial x} + e_{15} \frac{\partial \varphi}{\partial x} \tag{17}$$

$$D_x = e_{15}\frac{\partial w}{\partial x} - \varepsilon_{11}\frac{\partial \varphi_x}{\partial x}, D_y = 0, D_z = 0 \tag{18}$$

Let us assume w_1 represents mechanical displacement and φ_1 electrical potential function in P.E-I layer. From equation (11), (12) and (16)-(18) the following equations can be obtained

$$(c_{44} + \sigma_x^0)\frac{\partial^2 w_1}{\partial x^2} + e_{15}\frac{\partial^2 \varphi_1}{\partial x^2} = \rho\frac{\partial^2 w_1}{\partial t^2} \tag{19}$$

$$e_{15}\frac{\partial^2 w_1}{\partial x^2} - \varepsilon_{11}\frac{\partial^2 \varphi_1}{\partial x^2} = 0 \tag{20}$$

Similarly w_2 represents mechanical displacement and φ_2 electrical potential function in P.E-II layer.

$$(c'_{44} + \sigma_x^0)\frac{\partial^2 w_2}{\partial x^2} + e'_{15}\frac{\partial^2 \varphi_2}{\partial x^2} = \rho'\frac{\partial^2 w_2}{\partial t^2} \tag{21}$$

$$e'_{15}\frac{\partial^2 w_2}{\partial x^2} - \varepsilon'_{11}\frac{\partial^2 \varphi_2}{\partial x^2} = 0 \tag{22}$$

The continuity conditions at x=0 in direction normal to the PE-PE interface i.e.

$$w_1(0,t) = w_2(0,t), \varphi_1(0,t) = \varphi_2(0,t) \tag{23}$$

$$\tau_{zx1}(0,t) = \tau_{zx2}(0,t), D_{x1}(0,t) = D_{x2}(0,t) \tag{24}$$

The following conditions must be also satisfied i.e.

$$w_1(h_1,t) = w_2(-h_2,t), \varphi_1(h_1,t) = \varphi_2(-h_2,t) \tag{25}$$

$$\tau_{zx1}(h_1,t) = \tau_{zx2}(-h_2,t), D_{x1}(h_1,t) = D_{x2}(-h_2,t) \tag{26}$$

For shear waves propagating in positive direction of x axis, solutions of the mechanical displacement and electrical potential function can be expressed as follows,

$$w_1(x,t) = W_1(x)exp[ik(x - ct)] \tag{27}$$

$$\varphi_1(x,t) = \phi_1(x)exp[ik(x - ct)] \tag{28}$$

$$w_2(x,t) = W_2(x)exp[ik(x - ct)] \tag{29}$$

$$\varphi_2(x,t) = \phi_2(x)exp[ik(x - ct)] \tag{30}$$

Where $k = 2\pi/\lambda$ is the wave number, λ is the wavelength, $i^2=-1$, and c is the phase velocity of wave propagation. Let $W_1(x)$, $W_2(x)$, $\Phi_1(x)$, and $\Phi_2(x)$ are the undetermined functions, respectively. Substituting equation (27)-(28) into equation (19)-(20) leads to,

$$(c_{44} + \sigma_x^0)[W_1'' + 2ikW_1' - k^2W_1] + e_{15}[\phi_1'' + 2ik\phi_1' - k^2\phi_1] = -\rho k^2c^2W_1 \tag{31}$$

$$e_{15}[W_1'' + 2ikW_1' - k^2W_1] - \varepsilon_{11}[\phi_1'' + 2ik\phi_1' - k^2\phi_1] = 0 \tag{32}$$

Inserting the value of equation (29)-(30) into (21)-(22) yields,

$$(c'_{44} + \sigma_x^0)[W_2'' + 2ikW_2' - k^2W_2] + e'_{15}[\phi_2'' + 2ik\phi_2' - k^2\phi_2] = -\rho' k^2c^2W_2 \tag{33}$$

$$e'_{15}[W_2'' + 2ikW_2' - k^2W_2] - \varepsilon'_{11}[\phi_2'' + 2ik\phi_2' - k^2\phi_2] = 0 \tag{34}$$

Where bulk shear velocity can be represented

$$c_{sh} = \sqrt{\frac{c_{44}\varepsilon_{11} + e_{15}^2}{\rho\varepsilon_{11}}}, c'_{sh} = \sqrt{\frac{c'_{44}\varepsilon'_{11} + e'^2_{15}}{\rho'\varepsilon'_{11}}}$$

By inserting the value of $\Phi_1(x)$ from equation (32), the solution of equation (31) can be found

$$W_1 = A_1 e^{(-1+b_1)ikx} + B_1 e^{(-1-b_1)ikx} \tag{35}$$

The solution of equation (32) is sum of particular and homogeneous solution. So the particular solution of equation is obtained as

$$\phi_1^p = \frac{e_{15}}{\varepsilon_{11}} W_1 \tag{36}$$

The homogeneous solution of equation (32) can be represented

$$\phi_1^h = (A_1' + B_1' x) e^{-ikx} \tag{37}$$

The complete solution of equation (32) leads to

$$\phi_1 = \phi_1^p + \phi_1^h \tag{38}$$

$$\phi_1 = (A_1' + B_1' x) e^{-ikx} + \frac{e_{15}}{\varepsilon_{11}} [A_1 e^{(-1+b_1)ikx} + B_1 e^{(-1-b_1)ikx}] \tag{39}$$

Similarly the solution of equation (33) is obtained

$$W_2 = A_2 e^{(-1+b_2)ikx} + B_2 e^{(-1-b_2)ikx} \tag{40}$$

The solution of equation (34) is sum of particular and homogeneous solution. So the particular solution is found as

$$\phi_2^p = \frac{e'_{15}}{\varepsilon'_{11}} W_2 \tag{41}$$

The homogeneous solution of equation (34) yields

$$\phi_2^h = (A_2' + B_2' x) e^{-ikx} \tag{42}$$

So complete solution of equation (34) is represented

$$\phi_2 = \phi_2^p + \phi_2^h \tag{43}$$

$$\phi_2 = (A_2' + B_2' x) e^{-ikx} + \frac{e'_{15}}{\varepsilon'_{11}} [A_2 e^{(-1+b_2)ikx} + B_2 e^{(-1-b_2)ikx}] \tag{44}$$

Substituting equations (35) and (39) into equations (27) and (28) yields

$$w_1(x,t) = [A_1 e^{(-1+b_1)ikx} + B_1 e^{(-1-b_1)ikx}] exp[ik(x - ct)] \tag{45}$$

$$\varphi_1 = \{(A_1' + B_1'x)e^{-ikx} + \frac{e_{15}}{\varepsilon_{11}}[A_1 e^{(-1+b_1)ikx} + B_1 e^{(-1-b_1)ikx}]\}exp[ik(x - ct)] \tag{46}$$

Now inserting the equations (40) and (44) into equations (29) and (30) leads to

$$w_2(x,t) = [A_2 e^{(-1+b_2)ikx} + B_2 e^{(-1-b_2)ikx}]exp[ik(x - ct)] \tag{47}$$

$$\varphi_2 = \{(A_2' + B_2'x)e^{-ikx} + \frac{e_{15}'}{\varepsilon_{11}'}[A_2 e^{(-1+b_2)ikx} + B_2 e^{(-1-b_2)ikx}]\}exp[ik(x - ct) \tag{48}$$

The components of stress and electrical displacement can be obtained as

$$\tau_{zx1} = \{e_{15}B_1'e^{-ikx} + Pikb_1[A_1 e^{(-1+b_1)ikx} - B_1 e^{(-1-b_1)ikx}]\}exp[ik(x - ct)] \tag{49}$$

$$D_{x1} = -\varepsilon_{11}B_1'e^{-ikx}exp[ik(x - ct)] \tag{50}$$

$$\tau_{zx2} = \{e_{15}'B_2'e^{-ikx} + P_1 ikb_2[A_2 e^{(-1+b_2)ikx} - B_2 e^{(-1-b_2)ikx}]\}exp[ik(x - ct)] \tag{51}$$

$$D_{x2} = -\varepsilon_{11}'B_2'e^{-ikx}exp[ik(x - ct)] \tag{52}$$

Where

$$b_1 = \sqrt{\frac{c^2}{(c_{sh}^2 + \frac{\sigma_x^0}{\rho})}}, b_2 = \sqrt{\frac{c^2}{(c_{sh}'^2 + \frac{\sigma_x^0}{\rho'})}}$$

$$P = c_{44} + \frac{e_{15}^2}{\varepsilon_{11}}, P_1 = c_{44}' + \frac{e_{15}'^2}{\varepsilon_{11}'}$$

3 SHEAR WAVE SOLUTIONS AND DISPERSION RELATIONS

Substituting the continuity conditions of Equations (23)- (26) in equations (45)-(52) yields the following set of linear algebraic equations with unknown coefficients $A_1,\ B_1,\ A_1',\ B_1',\ A_2,\ B_2,\ A_2',\ B_2'$.

$$A_1 + B_1 - A_2 - B_2 = 0 \tag{53}$$

$$\frac{e_{15}}{\varepsilon_{11}}A_1 + \frac{e_{15}}{\varepsilon_{11}}B_1 - \frac{e_{15'}'}{\varepsilon_{11'}'}A_2 - \frac{e_{15}'}{\varepsilon_{11}'}B_2 + A_1' - A_2' = 0 \tag{54}$$

$$ikb_1PA_1 - ikb_1PB_1 - ikb_2P_1A_2 + ikb_2P_1B_2 + e_{15}B_1' - e_{15}'B_2' = 0 \tag{55}$$

$$-\varepsilon_{11}B_1' + \varepsilon_{11'}'B_2' = 0 \tag{56}$$

$$e^{(-1+b_1)ikh_1}A_1 + e^{(-1-b_1)ikh_1}B_1 - e^{(1-b_2)ikh_2}A_2 - e^{(1+b_2)ikh_2}B_2 = 0 \tag{57}$$

$$\frac{e_{15}}{\varepsilon_{11}}e^{(-1+b_1)ikh_1}A_1 + \frac{e_{15}}{\varepsilon_{11}}e^{(-1-b_1)ikh_1}B_1 - \frac{e_{15}'}{\varepsilon_{11}'}e^{(1-b_2)ikh_2}A_2 - \frac{e_{15}'}{\varepsilon_{11}'}e^{(1+b_2)ikh_2}B_2 \tag{58}$$

$$+e^{-ikh_1}A_1' + h_1e^{-ikh_1}B_1' - e^{ikh_2}A_2' - h_2e^{ikh_2}B_2' = 0$$

$$ikb_1Pe^{(-1+b_1)ikh_1}A_1 - ikb_1Pe^{(-1-b_1)ikh_1}B_1 - ikb_2P_1e^{(1-b_2)ikh_2}A_2 + ikb_2P_1e^{(1+b_2)ikh_2}B_2 \\ +e_{15}e^{-ikh_1}B_1' - e_{15}'e^{-ikh_2}B_2' = 0 \quad (59)$$

$$-\varepsilon_{11}e^{-ikh_1}B_1' + \varepsilon_{11}'e^{ikh_2}B_2' = 0 \quad (60)$$

In order to obtain the nontrivial solution of equations (53)-(60), the determinant of the coefficient matrix must be equate to zero [Jin et al. 2005] i.e.

$$\cos(kh) = \cos(\alpha_1)\cos(\alpha_2) - \frac{1+Q^2}{2Q}\sin(\alpha_1)\sin(\alpha_2) \quad (61)$$

$$\cos[k(h_1 + h_2)] = \cos(kh_1b_1)\cos(kh_2b_2) - \frac{1+Q^2}{2Q}\sin(kh_1b_1)\sin(kh_2b_2) \quad (62)$$

Where

$$\alpha_1 = kh_1b_1, \alpha_2 = kh_2b_2, Q = \frac{b_2P_1}{b_1P}$$

The equation (61) and (62) represents the dispersion equation for shear wave propagating in direction normal to *x* axis. The equation provides relation between phase velocity *c* and wave number *k* under the influence of initial stress σ^0_x.

4 NUMERICAL EXAMPLES AND RESULTS

In above section, we have obtained analytically computation solution of the dispersion relation for propagation of shear wave in PE-PE layered structure. The thickness of PVDF is h_1 and thickness of PZT-8H is taken as h_2 respectively. In this section numerical computation is utilized to illustrate graphically the variation of initial stress on dispersion relation. For numerical calculation, the Table 1 lists material properties of PVDF and PZT-8.

Materials	Elastic constants $c_{44}(10^{10}N/m^2)$	Piezoelectric constants $e_{15}(C/m^2)$	Dielectric constants $\varepsilon_{11}(10^{-10}F/m)$	Mass density $\rho(kg/m^3)$
PVDF	0.91	- 0.16	1.6662	1.78
PZT-8	3.13	10.4	80	7.60

Table 1: Material constants of PVDF and PZT-8.

Let volume fraction η be defined as $h_1/(h_1+h_2)$. $\omega h_2/c_{sh}'$ and kh_2 regarded as two independent variable. With inclusion of these two variables, the dispersion relation transformed consequently into dimensionless equation. In order to predict the influence of initial stress on stop band, the variation of $\omega h_2/c_{sh}'$ and kh_2 is plotted for different values of stress in range of $\sigma_x^0 = 0$ to $\sigma_x^0 = 10^4$ MPa with volume fraction at constant value of η =0.4. Figure 2(a) - 2(d) shows the variation of stop band for initial stress $\sigma_x^0 = 0$ and $\sigma_x^0 = 10$ MPa. It can be observed from plots with subsequent increase in value of initial stress, the number of stop bands increases but width of each stop band decreases considerably. Further increase in value of stress beyond $\sigma_x^0 = 10^4$ MPa, shear wave propagation restricted to single mode only and wave filter effect can be clearly observed from propagation pattern of the curve. The stop band effect was not appreciable initially. As the value of stress increased beyond 10^2, the stop band effect become more pronounced and filter effect was observed which is similar to the results obtained in Refs. [Son and Kang 2011, Qian et al. 2004].

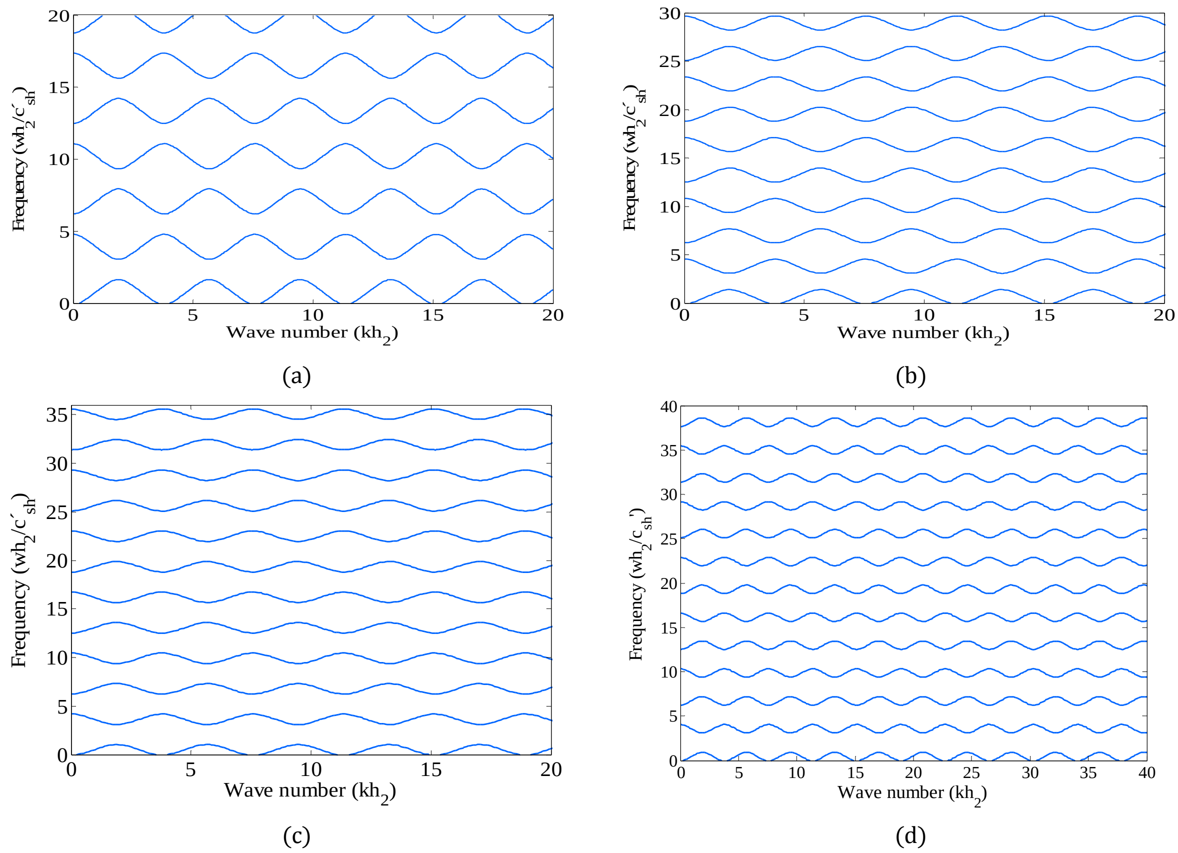

Figure 2: Stop band effects of shear waves in presence of initial stress of propagation direction normal to interface $(a)\sigma_x^0 = 0 (b)\sigma_x^0 = 10 (c)\sigma_x^0 = 10^2 (d)\sigma_x^0 = 10^4 MPa.$

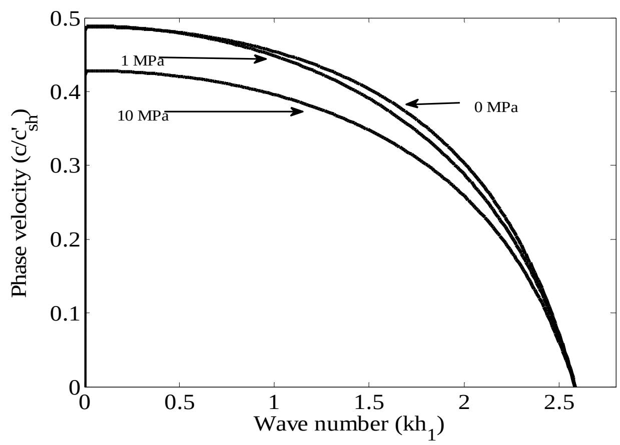

Figure 3: Phase velocity vs. wave number for different value of σ_x^0 .

The effect of initial stress on phase velocity is shown in figure 3. To consider the effect of initial stress on dispersion relation, $c\,/\,c'_{sh}$ and kh_1 regarded two independent variables transforming the dispersion relation into dimensionless equation. The curve is plotted for different value of initial stresses σ_x^0 i.e. 0, 1 and 10 MPa. It can observe from curve the initial stress has very small effect on phase velocity. With increase in value of initial stress, the phase velocity curve becomes straight line. Further increase in value of initial stress above 10 MPa, sharp increase in phase velocity was observed. The different combination of materials selected for two layer laminates broadens the applicability of this technique to the surface acoustic sensors and devices.

5 CONCLUDING REMARKS

In this study, we have considered the effect of initial stress on propagation characteristics of shear wave propagation in piezoelectric- piezoelectric multi-layer structure. Analytical and numerical solutions were found to investigate the effect of stress on the interface of two different piezoelectric materials i.e PVDF and PZT-8. The numerical results indicates that the initial stress up to 10^2 MPa has very small effect on dimensionless frequency of shear waves propagating in the direction normal to layering. But effect of stress becomes more pronounced when stress increased upto 10^4 MPa. Further increase in the value of initial stress may cause obstruction in propagation of shear waves. So variation in value of stress can be utilized for designing the filter for wireless applications. It is also observed that low value of initial stress has very little impact on phase velocity. This investigation is limited to two layer laminates which restricts the applicability to only some practical areas. Other numerical tools such as finite element method can be utilize further to investigate the shear wave propagation in multilayered laminates. The present analysis provides insight for design and manufacturing of new class of surface acoustic wave sensors. Further the imperfection at the layer interface can be also considered to study the effect of stress on the wave number and phase velocity.

References

Burkov, S.I., Zolotova, O.P., Sorokin, B.P. (2011), Influence of the external electric field on propagation of lamb waves in piezoelectric plates, IEEE transactions on ultrasonic, ferroelectrics, and frequency control, 58, 239-243.

Chattaraj, R., Samal, S.K., Mahanti, N.C. (2012), Dispersion of love wave propagating in irregular anisotropic porous stratum under initial stress, International Journal of Geomechanics, 13: 402-408.

Danoyan, Z.N., Piliposian, G.T. (2008), Surface electro-elastic shear horizontal waves in a layered structure with a piezoelectric substrate and a hard dielectric layer, International Journal of Solids and Structures, 45, 431-441.

Du, J., Jin, X., Wang, J., Zhou, Y. (2007), SH wave propagation in a cylindrically layered piezoelectric structure with initial stress, Acta Mechanica, 191, 59-74.

Du, J., Xian, K., Wang, J., Yong, Y.K. (2008), Propagation of love waves in pre-stressed piezoelectric layered structures loaded with viscous liquid, Acta Mech. Solida Sinica, 21, 542-548.

Gaur, A.M., Rana, D.S. (2014), Shear wave propagation in piezoelectric- piezoelectric composite layered structure, Latin American Journal of Solids and Structures, 11, 2483-2496.

Gaur, A.M., Rana, D.S. (2015), Dispersion relations for SH waves propagation in a porous piezoelectric (pzt-pvdf) composite structure, Acta Mechanica, 226, 4017-4029.

Jin, F., Qian, Z., Wang, Z., Kishimoto, K. (2005), Propagation behavior of love waves in a piezoelectric layered structure with inhomogeneous initial stress, Smart Materials and Structures, 14, 515-523.

Kayestha, P., Wijeyewickrema, A.C., Kishimoto, K. (2011), Wave propagation along a non-principal direction in a compressible prestressed elastic layer, International Journal of Solids and Structures, 48.

Levin, V.M., Valdiviezo-Mijangos, V., Sabina, F.J. (2011), Propagation of electroacoustic axial shear waves in a piezoelectric medium reinforced by continuous fibers, International Journal of Engineering Science, 49, 1232-1243.

Li, P., Jin, F., Lu, T.J. (2012), A three-layer structure model for the effect of a soft middle layer on love waves propagating in layered piezoelectric systems, Acta Mechanica Sinica, 28, 1087-1097.

Liu, H., Kuang, Z.B., Cai, Z.M. (2003), Propagation of Bleustein Gulyaev waves in a pre-stressed layered piezoelectric structure, Ultrasonics, 41,397-405.

Mahmoud, S.R. (2013), On problem of shear waves in magneto-elastic half-space of initially stressed nonhomogeneous anisotropic material under influence of the rotation, International Journal of Mechanical Sciences, 77.

Qian, Z.H., Jin, F., Kishimoto, K., Wang, Z. (2004), Effect of initial stress on the propagation behavior of SH-waves in multilayered piezoelectric composite structures, Sensors and Actuators A, 112, 368-375.

Qian, Z.H., Jin, F., Kishimoto, K., Lu, T. (2009), Propagation behavior of love waves in a functionally graded half-space with initial stress, International Journal of Solids and Structures, 46, 1354-1361.

Son, M.S., Kang, Y.J. (2011), The effect of initial stress on the propagation behavior of SH waves in piezoelectric coupled plates, Ultrasonics, 51,489-495.

Su, J., Kuang, Z.B., Liu, H. (2005), Love wave in ZnO/SiO2/si structure with initial stresses. Journal of Sound and Vibration, 286, 981-999.

Yuan, L., Du, J., Ma, T, Wang, J. (2014), Study on SH-saw in imperfectly bonded piezoelectric structures loaded with viscous liquid, Acta Mechanica, 225,1-11.

Zhang, H., Shen, Y., Li, Y., Mei, X. (2013), Effect of initial stress on the lateral modes in 1-3 piezocomposites, Mechanics of Advanced Materials and Structures, 20, 122-129.

Zhang, L.L., Liu J.X., Fang, X.Q., Nie, G.Q. (2014), Effects of surface piezoelectricity and nonlocal scale on wave propagation in piezoelectric nanoplates. European Journal of Mechanics - A/Solids, 46, 22-29.