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Static analysis of tapered nanowires based on nonlocal Euler-Bernoulli beam theory via differential quadrature method

Abstract

As a first endeavor, bending analysis of tapered nano wires with circular cross section is investigated. In this research, nonlocal elasticity theory based on Euler-Bernoulli beam theory is used to formulate the equations. Differential quadrature method (DQM) is employed to solve the governing equations. Different parameters such as nonlocal parameter, length and radius of tapered nano wires are also considered. The results of present work can be used as bench marks for future works.

Keywords

Tapered nanowires, Nonlocal Euler-Bernoulli beam theory, Differential quadrature method, Static analysis

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1 INTRODUCTION

Nonlocal theories were used for studying micro and nano structures for several times. Reddy [18] reformulated various available beam theories, including the Euler–Bernoulli, Timoshenko, Reddy, and Levinson beam theories, using the nonlocal differential constitutive relations of Eringen. Zhang et al [25] studied the free transverse vibrations of double-walled carbon nanotubes using a theory of nonlocal elasticity. Wang et al [21] investigated the buckling analysis of micro- and nano-rods/tubes based on nonlocal Timoshenko beam theory. The small scale effect on the vibration analysis of orthotropic single layered graphene sheets was studied by Pradhan and Kumar [16]. Elastic theory of the graphene sheets was reformulated using the nonlocal differential constitutive relations of Eringen. Pradhan [15] presented the buckling of single layer graphene sheet based on nonlocal elasticity and higher order shear deformation theory. Jomehzadeh and Saidi [7] decoupled the nonlocal elasticity equations for three dimensional vibration analysis of nano-plates. Malekzadeh et al [11] proposed the free vibration of orthotropic arbitrary straight-sided quadrilateral nanoplates using the nonlocal elasticity theory. The formulation was derived based on the first order shear deformation theory. Civalek et al [14] investigated the free vibration and bending analyses of cantilever microtubules based on nonlocal continuum model. Wang and Liew [23] studied the application of nonlocal continuum mechanics to static analysis of micro- and nano-structures. Aghababaei and Reddy [2] presented the nonlocal third-order shear deformation plate theory with application to bending and vibration of plates. Analytical solutions of bending and free vibration of a simply supported rectangular plate were presented using this theory to illustrate the effect of nonlocal theory. Wang [22] proposed the vibration and instability analysis of tubular nano- and micro-beams conveying fluid using nonlocal elastic theory. Kiani [9] studied the application of nonlocal beam models to double-walled carbon nanotubes under a moving nanoparticle. Hu et al [5] investigated the nonlocal shell model for elastic wave propagation in single- and double-walled carbon nanotubes. Nonlocal longitudinal vibration of single walled carbon nanotubes with attached buckyballs was considered by Murmu and Adhikari [12]. Closed-form nonlocal transcendental equation for vibrating system with arbitrary mass ratio i.e. mass of buckyball to mass of single-walled carbon nanotubes was derived. Yang et al [24] presented the nonlinear free vibration of single-walled carbon nanotubes using nonlocal Timoshenko beam theory. The differential quadrature method was employed to discretize the nonlinear governing equations which were then solved by a direct iterative method to obtain the nonlinear vibration frequencies of single-walled carbon nanotubes with different boundary conditions. Nonlinear free vibration of embedded double-walled carbon nanotubes was studied based on Eringen's nonlocal elasticity theory and von Karman geometric nonlinearity by Ke et al [8]. Mustapha and Zhong [13] studied the free transverse vibration of an axially loaded non-prismatic single-walled carbon nanotube embedded in a two-parameter elastic medium using nonlocal Rayleigh beam.

Nano wires have many interesting properties that are not seen in bulk or 3-D materials. Many different types of nano wires exist, including metallic (e.g., Ni, Pt, Au), semiconducting (e.g., InP, Si, GaN, etc.), and insulating (e.g., SiO_2, TiO_2. Surface effects on the elastic behavior of static bending nano wires were studied by using a comprehensive Timoshenko beam model by Jiang and Yan [6]. Fu and Zhang [4] established a continuum elastic model for core–shell nano wires with weak interfacial bonding. Critical buckling loads and resonant frequencies of simply supported nano wires were obtained by using the Ritz method. Song and Huang [20] presented a model of surface stress effects on bending behavior of nano wires based on the incremental deformation theory. The free longitudinal vibration of tapered nanowires was investigated in the context of nonlocal continuum theory by Kiani [10]. The problem was studied for the nanowires with linearly varied radii under fixed–fixed and fixed–free boundary conditions. From the above works, it can be seen that tapered nano wires are rarely studied. So, more discussions may be needed. From the knowledge of author, up to now, it is the first time that static analysis of tapered nano wires is studied. The governing equations are derived based on nonlocal elasticity theory using Euler-Bernoulli beam theory. In this study, the differential quadrature method as an accurate numerical tool is adopted to discretize the governing equations.

2 DIFFERENTIAL QUADRATURE METHOD

The basic idea of the differential quadrature method is that the derivative of a function, with respect to a space variable at a given sampling point, is approximated as the linear weighted sums of its values at all of the sampling points in the domain of that variable [3, 17, 19].

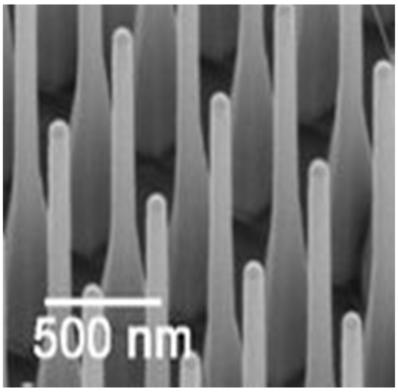

Figure 1 Nanowires [1]

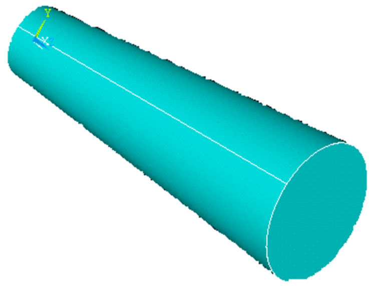

Figure 2 Tapered nanowire model

In order to illustrate the DQ approximation, consider a function $f(x,y)$ having its field on a rectangular domain$0 \le x \le a$ and$0 \le y \le b$. let, in the given domain, the function values be known or desired on a grid of sampling points. According to DQ method, the r^{th}derivative of a function $f(x,y)$ can be approximated as,

$$\left.\frac{\partial^r f(x,y)}{\partial x^r}\right|_{(x,y)=(x_i,y_j)} = \sum_{m=1}^{Nx} A_{im}^{x(r)} f(x_m, y_j) = \sum_{m=1}^{Nx} A_{im}^{x(r)} f_{mj} \tag{1}$$

where i=1,2,...,N_x, j=1,2,...,$N_y$and r=1,2,..., $N_x - 1$

From this equation one can deduce that the important components of DQ approximations are weighting coefficients and the choice of sampling points. In order to determine the weighting coefficients a set of test functions should be used in Eq. (1). For polynomial basis functions DQ, a set of Lagrange polynomials are employed as the test functions. The weighting coefficients for the first-order derivatives in ξ-direction are thus determined as

$$A_{ij}^x = \begin{cases} \frac{1}{L_\xi}\frac{M(x_i)}{(x_i-x_j)M(x_j)} & \text{for} \quad i \ne j \\ -\sum_{\substack{j=1 \\ i\ne j}}^{N_x} A_{ij}^x & \text{for} \quad i = j \end{cases} \quad ; \quad i,\ j = 1,2\ldots,N_x \tag{2}$$

where $L_x$is the length of domain along the $x-$direction and $M(x_i) = \prod_{k=1,i\ne k}^{N_x}(x_i - x_k)$

The weighting coefficients of second order derivative can be obtained as,

$$B_{ij}^x] = [A_{ij}^x][A_{ij}^x] = [A_{ij}^x]^2 \tag{3}$$

In a similar way, the weighting coefficients for y-direction can be obtained. The weighting coefficient of the third and fourth order derivatives $(C_i j, D_i j)$ can be computed easily from $(B_i j)$ by

$$C_{ij} = \sum_{j=1}^{N} A_{ij}A_{ij}A_{ij}, \quad D_{ij} = \sum_{j=1}^{N} B_{ij}B_{ij} \tag{4}$$

3 GOVERNING EQUATIONS

For an Euler-Bernoulli beam theory, the displacement field is assumed to be as follow,

$$u_1 = u(x,t) + z\partial w/\partial x, u_2 = 0, u_3 = w(x,t) \tag{5}$$

where are the axial and transverse displacements. The only nonzero strain of the Euler–Bernoulli beam theory is [1],

$$\varepsilon_{xx} = \partial u/\partial x + z\partial^2 w/\partial x^2 \tag{6}$$

The equations for an Euler–Bernoulli beam theory are given by,

$$\partial N/\partial x + f = m_0\partial^2 u/\partial t^2 \partial^2 M/\partial x^2 + q(x) - \partial(\bar{N}\partial w/\partial x)/\partial x = m_0\partial^2 w/\partial t^2 - m_2\partial^4 w/\partial x^2\partial t^2 \tag{7}$$

where $f(x)$ and $q(x)$ are the axial and transverse distributed forces. According to the nonlocal elasticity theory, the classic Hooke's law for a uniaxial stress state is given by,

$$\sigma - (e_0 a)^2\partial^2\sigma/\partial x^2 = E\varepsilon \tag{8}$$

where$\sigma(x)$ is the axial stress,e_0 is a beam constant and$\alpha$is an internal characteristic length. The constitutive relation for a nonlocal Euler–Bernoulli beam theory is given by,

$$M - (e_0 a)^2\partial^2 M/\partial x^2 = EI\partial^2 w/\partial x^2 \tag{9}$$

By performing the differentiation of this equation with respect to the variable x twice we obtain

$$\partial^2 M/\partial x^2 - (e_0 a)^2\partial^4 M/\partial x^4 = \partial^2(EI\partial^2 w/\partial x^2)/\partial x^2 \tag{10}$$

The equation of nonlocal Euler–Bernoulli beam theory now can be expressed in terms of the displacements as,

$$\begin{aligned}&\partial^2(-EI\partial^2 w/\partial x^2)/\partial x^2 + (e_0 a)^2\partial^2[\partial\left(\bar{N}\partial w/\partial x\right)/\partial x - q + m_0\partial^2 w/\partial t^2\\&-m_2\partial^4 w/\partial x^2\partial t^2]\partial x^2 + q - \partial(\bar{N}\partial w/\partial x)/\partial x = m_0\partial^2 w/\partial t^2 - m_2\partial^4 w/\partial x^2\partial t^2\end{aligned} \tag{11}$$

Using the DQ-rules for the spatial derivatives, the DQ-analogs of the governing Eqs. (11) become

$$\begin{aligned}&EI\left(\sum_{j=1}^{N} D_{ij}w_i\right) + 2\left(I\partial E/\partial x + E\partial I/\partial x\right)\left(\sum_{j=1}^{N} C_{ij}w_i\right) + q(x_i)\,.\\&\left(I\partial^2 E/\partial x^2 + 2(\partial E/\partial x)(\partial I/\partial x) + E\partial^2 I/\partial x^2\right)\left(\sum_{j=1}^{N} B_{ij}w_i\right) + (e_0 a)^2\left[-\partial^2 q(x_i)/\partial x^2\right] = 0.\end{aligned} \tag{12}$$

It should be mentioned that for $e_0a = 0$, the above equation will reduce to the classical Euler-Bernoulli beam theory. Two-types of boundary conditions are considered. These are,

Fully clamped, (at both ends)

$$W = 0\partial W/\partial x = 0 \tag{13}$$

Simply supported, (at both ends)

$$W = 0\partial^2 W/\partial x^2 = 0 \tag{14}$$

The discretized form of boundary condition can be obtained by,

Fully clamped, (at both ends)

$$W_i = 0, \quad \sum_{j=1}^{N} A_{ij}W_j = 0 \tag{15}$$

Simply supported,(at both ends)

$$W_i = 0, \quad \sum_{j=1}^{N} B_{ij}W_j = 0 \tag{16}$$

4 NUMERICAL RESULTS

Consider a tapered nano wire having a length L and bigger radius $R = r|_{x=0}$. The radius of nano wire is varying linearly according to *r=R-px*. In this section, on the basis of the above equations, different parameters such as length and radius of nano wire, nonlocal parameter and different boundary conditions are investigated. In these examples, a tapered nano wire is assumed to have a $80nm$ length and bigger radius $R = 10nm$, unless otherwise specified. In all of the examples, a tapered nano wire is subjected to sinusoidally mechanical loading defined as follow,

$$q = q_0 \sin(\pi x/L) \tag{17}$$

where q_0 is a constant parameter. In Table 1, the effects of nonlocal parameter on the deflections of simply supported tapered nano wires subjected to sinusoidally mechanical loading are shown. One can see that with the increase of nonlocal parameter, the deflections will increase. The increase in the values of the deflections as the nonlocal parameter is varied is evident of its importance contribution to the mathematical model. It is also shown that the maximum deflection of tapered nano wire is not occurring at the middle of the wire, as it is expected. In Fig. 3, the deflections of simply supported tapered nano wires subjected to sinusoidally mechanical loading for different geometries are figured. The nonlocal parameter is assumed to be $1nm$. It can be seen that increasing the constant parameter p will increase the deflections of nano wire. It is important to state that increase the constant parameter p is equal to decrease the radius of nano wire.

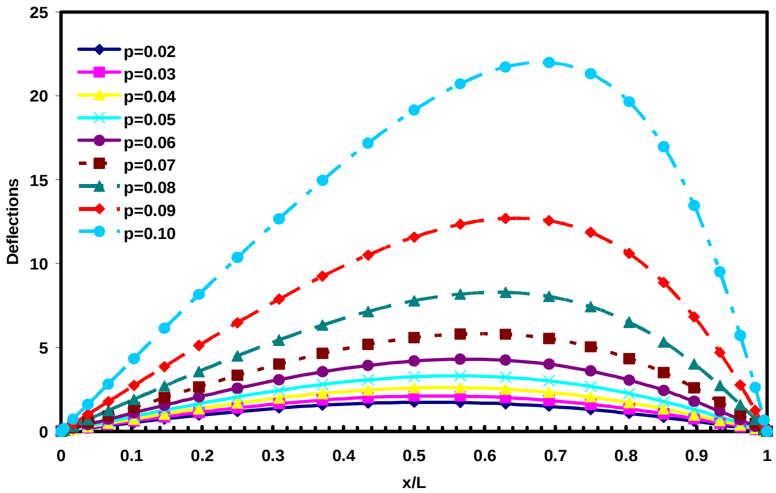

Figure 3 The deflections of simply supported tapered nano wires subjected to sinusoidally mechanical loading for different geometries $((e_0a)^2 = 1nm^2)$

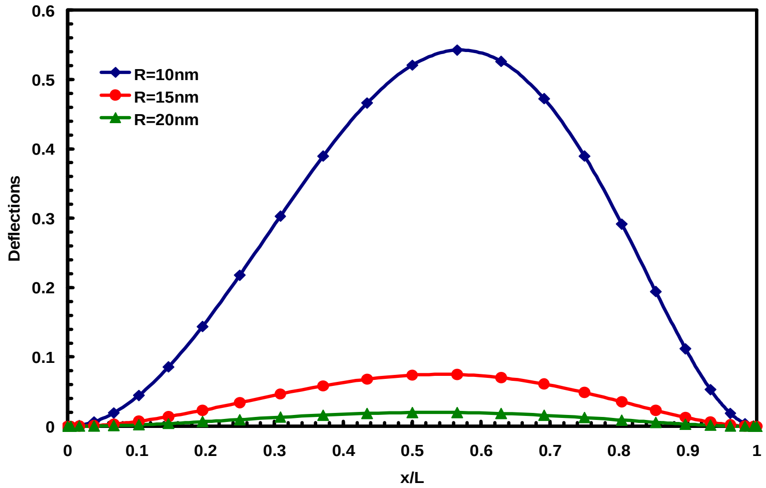

Figure 4 The effects of radius $(R = r|_{x=0})$ of fully clamped tapered nano wires on the deflections under sinusoidally mechanical loading $((e_0a)^2 = 1nm^2, p = 0.05)$

In this figure, it is obviously shown that the maximum deflections shift to the right side of nano wire as the constant p increase. The influences of radius $(R = r|_{x=0})$ of fully clamped tapered nano wires on the deflections under sinusoidally mechanical loading are figured in Fig. 4. The figure indicates that increase of radius R, leads to the decrease of the deflections of tapered nano wire. It is also shown that with less amount of parameter p and more amount of radius R, the results for tapered nano wires are close to the results for nano wires with constant cross section. In Fig. 5, the effects of length of fully clamped tapered nano wires are investigated. As it is expected, in order to decrease the deflections of tapered nano wires, one should decrease the length of them.

Table 1 The effects of nonlocal parameter on the deflections of simply supported tapered nano wires subjected to sinusoidally mechanical loading (*p=1*)

	$(e_0a)^2(nm)^2$		
x/L	**0.1**	**1**	**10**
0.0000	0.0000	0.0000	0.0000
0.0043	0.1376	0.1378	0.1397
0.0170	0.5481	0.5489	0.5565
0.0381	1.2243	1.2260	1.2430
0.0670	2.1536	2.1566	2.1865
0.1033	3.3178	3.3225	3.3685
0.1464	4.6918	4.6983	4.7635
0.1956	6.2422	6.2508	6.3374
0.2500	7.9251	7.9361	8.0461
0.3087	9.6844	9.6978	9.8322
0.3706	11.4483	11.4642	11.6230
0.4347	13.1275	13.1457	13.3279
0.5000	14.6139	14.6342	14.8370
0.5653	15.7816	15.8035	16.0225
0.6294	16.4927	16.5156	16.7444
0.6913	16.6095	16.6326	16.8630
0.7500	16.0165	16.0387	16.2610
0.8044	14.6523	14.6727	14.8760
0.8536	12.5501	12.5675	12.7417
0.8967	9.8752	9.8889	10.0260
0.9330	6.9368	6.9464	7.0427
0.9619	4.1442	4.1500	4.2075
0.9830	1.8999	1.9025	1.9289
0.9957	0.4802	0.4809	0.4875
1.0000	0.0000	0.0000	0.0000

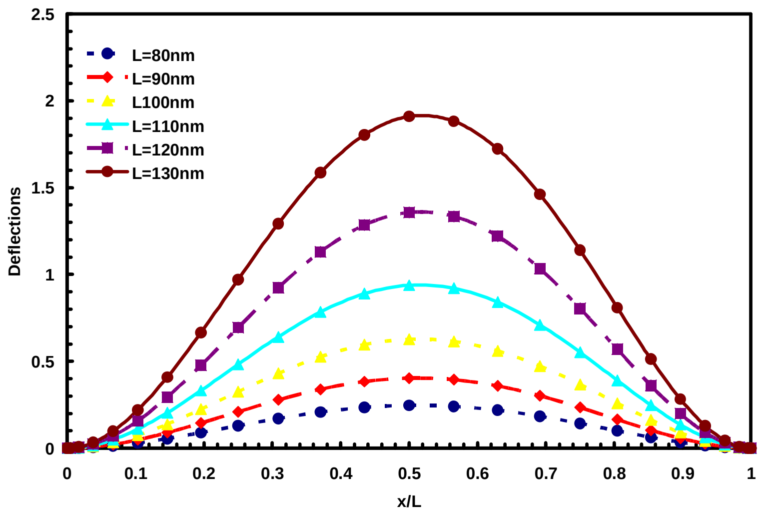

Figure 5 The effects of length of fully clamped tapered nano wires on the deflections under sinusoidally mechanical loading ($(e_0a)^2 = 1nm^2, p = 0.01$)

5 CONCLUSION

In this research, utilizing the nonlocal elasticity theory based on Euler-Bernoulli beam theory, the deflections of tapered nano wires with circular cross section have been obtained. The differential quadrature method was employed to convert the governing differential equations into a linear system. It was shown that with the increase of nonlocal parameter and length of tapered nano wires and with the decrease of radius of tapered nano wires, $(R = r|_{x=0})$ the deflections will increase.

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