Latin American Journal of
Solids and Structures
www.lajss.org

Harmonic differential quadrature method for static analysis of functionally graded single walled carbon nanotubes based on Euler-Bernoulli beam theory

Abstract
Bending analysis of functionally graded single walled carbon nano tubes is presented in this paper. Carbon nano tubes are modeled as Euler-Bernoulli beam theory in this study. Harmonic differential quadrature (HDQ) method is used to discretize the governing equations. In order to show the accuracy of present work, the results are compared with those of other existing results. Then the effects of different parameters such as power law index, inner and outer radius of nano tubes and length nano tubes of are studied, too.

Keywords
Functionally graded nanotubes, Euler-Bernoulli beam theory, Harmonic differential quadrature method, Static analysis

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Received 22 Mar 2012;
In revised form 08 May 2012

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1 INTRODUCTION

The concept of functionally graded materials (FGMs) was first proposed by Japanese material scientists in the early 1980s [17]. An advantage of functionally graded materials (FGMs) over laminated composites is that material properties vary continuously in an FGM as opposed to being discontinuous across adjoining layers in laminated composites. El-Abbassi and Meguid [10] presented a new thick shallow shell element to study the thermoelastic behavior of functionally graded structures made from shells and plates. Tutuncu and Ozturk [24] obtained closed-form solutions for stress and displacements in functionally graded pressure vessels subjected to internal pressure alone by using the infinitesimal theory of elasticity. Qian et al [22] studied the static and dynamic deformations of thick functionally graded elastic plates by using higher-order shear and normal deformable plate theory and meshless local Petrov–Galerkin method. Chen et al. [5] presented elasticity solution for bending and thermal deformations of FG beams with various end conditions, using the state space method coupled with differential quadrature method. Şimşek and Kocatürk [9] considered the free and forced vibration of a functionally graded beam subjected to a concentrated moving harmonic load. Malekzadeh et al [18] investigated the out-of-plane free vibration of functionally graded circular curved beams in thermal environment based on the first order shear deformation theory (FSDT), using differential quadrature method. Janghorban and Rostamsowlat

[13] studied the bi-directional functionally graded plate based on three-dimensional elasticity theory. Differential quadrature method was used to solve the governing equations. The advancements of micro technology and nanotechnology have enthused scientists and engineers in their pursuit of studying all sorts of micro/nanostructures such as carbon nanotubes (Fig. 1). Single walled nanotubes use a single sheath of graphite one atom thick, called "graphene" but multi walled nanotubes are either wrapped into multiple layers or are constructed of multiple cylinders, one inside the other. Nanotubes are members of the fullerene structural family, which also includes the spherical buckyballs. The ends of a nanotube may be capped with a hemisphere of the buckyball structure. Civalek and Akgöz [6] presented the static analysis of carbon nano tubes using the nonlocal Bernoulli-Euler beam theory by differential quadrature method. Demir et al [4] studied the free vibration analysis of carbon nanotubes based on Timoshenko beam theory using discrete singular convolution (DSC) method. Nonlocal longitudinal vibration of single-walled-carbon-nanotubes with attached buckyballs was considered by Murmu and Adhikari [20]. Hashemnia et al [12] studied the dynamical analysis of single walled carbon nanotubes conveying water considering carbon–water bond potential energy and nonlocal effects.Ansari and Hemmatnezhad [2] proposed the nonlinear vibrations of embedded multi-walled carbon nanotubes using a variational approach. In most recent years, functionally graded materials are finding increasing employments in micro-/nano-electro-mechanical systems [8, 11]. On the basis of the modified couple stress theory, the size-dependent static and vibration behavior of micro-beams made of functionally graded materials are analytically studied by Asghari et al [3]. Ke et al [16] investigated the nonlinear free vibration of functionally graded carbon nanotube-reinforced composite beams based on Timoshenko beam theory and von Kármán geometric nonlinearity. Ke et al [15] presented the nonlinear free vibration of size-dependent functionally graded micro beams based on the modified couple stress theory and von Kármán geometric nonlinearity. Mohammadi-Alasti [19] investigated the mechanical behavior of a functionally graded micro-beam subjected to a thermal moment and nonlinear electrostatic pressure using step-by-step linearization method and finite difference Method. Functionally graded carbon nanotubes/hydroxyapatite (CNTs/HA) composite coatings have been fabricated by laser cladding technique using CNTs/HA composite powders by Pei et al [21]. Recently, Janghorban and Zare [14] investigated the free vibration analysis of functionally graded single walled carbon nanotubes with variable thickness based on Timoshenko beam theory using differential quadrature method. In this paper, functionally graded single walled carbon nanotubes subjected to mechanical loading based on Euler-Bernoulli beam theory is investigated. Harmonic differential quadrature method as an efficient and numerical tool is used to solve the beam equation.

2 MATERIAL PROPERTIES

Young's modulus of the functionally graded single walled carbon nanotubes is assumed to vary across the longitude directional of nanotubes. In this case, in order to compute the results for single walled carbon nanotube subjected to mechanical loading, the Young's modulus can be

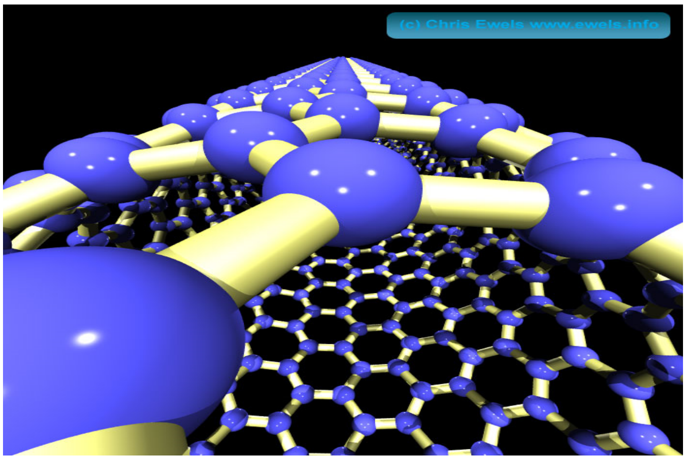

Figure 1 Nanotube [1]

defined as follow,

$$E(x) = E_2 + (E_1 - E_2)(x/L)^n \tag{1}$$

where $0 < x < L$ and n is the power law index. E_1 and E_2 refer to the Young's modulus at both ends of functionally graded carbon nanotubes.

3 HARMONIC DIFFERENTIAL QUADRATURE METHOD

Harmonic Differential quadrature (HDQ) method is a relatively new numerical technique in applied mechanics. The harmonic differential quadrature method is a development of the differential quadrature method, which has been used successfully to solve a variety of problems. The HDQ method chooses harmonic functions as its test functions instead of polynomials in the DQ method, i.e.,

$$f(x) = \{1, sin\pi x, cos\pi x, sin2\pi x, cos2\pi x, ..., sin(N-1)\pi x/2, cos(N-1)\pi x/2\} \tag{2}$$

where N is an odd number. The weighting coefficients of the first-order derivatives Aij can be obtained by using the following formula:

$$A_{ij} = \begin{cases} \frac{(\pi/2)P(x_i)}{(x_i - x_j)P(x_j)} & \text{for} \quad i \neq j \\ -\sum\limits_{\substack{j=1 \\ i \neq j}}^{N_x} A_{ij} & \text{for} \quad i = j \end{cases} \tag{3}$$

where $P(x_i) = \prod\limits_{k=1, i \neq k}^{N_x} \sin[(x_i - x_k)\pi/2]$

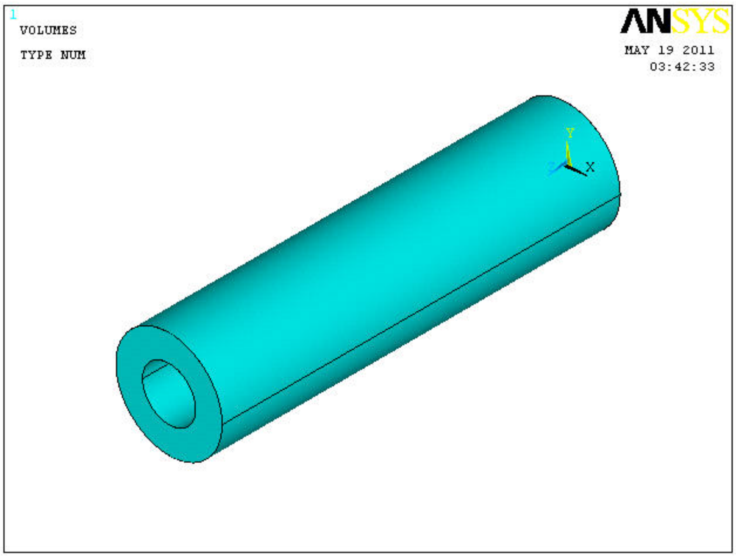

Figure 2 The effects of length to width ratio on the deflections of clamped

The weighting coefficients of second order derivative can be obtained as,

$$[B_{ij}] = A_{ij}[2A_{ii}^{(1)} - \pi ctg(x_i - x_k)\pi/2] \qquad i,j = 1,2,3,...,N, \tag{4}$$

The weighting coefficient of the fourth order derivatives (D_{ij}) can be computed easily from (B_{ij}) by

$$D_{ij} = \sum_{j=1}^{N} B_{ij} B_{ij} \tag{5}$$

The above equation is the same for differential quadrature and harmonic differential quadrature methods. In numerical computations, Chebyshev-Gauss-Lobatto quadrature points are used, that is,

$$\frac{x_i}{a} = \frac{1}{2}\{1 - \cos[\frac{(i-1)\pi}{(N_x-1)}]\}; \quad \text{for } i = 1,2,\ldots,N_x \tag{6}$$

4 GOVERNING EQUATIONS

This study is carried out on the basis of the Euler-Bernoulli beam model (Fig. 2). Consider a single walled functionally graded carbon nanotubes with length L, inner radius and outer radius . The beam model is subjected to centrally concentrated load. For single walled carbon nanotubes, the equation for Euler-Bernoulli beam model can be expressed as follow,

$$\partial V/\partial x + q(x) = pA\partial^2 W/\partial t^2 \tag{7}$$

where q(x) is the mechanical loading on single walled carbon nanotubes, p is the density, W is the vertical deflection of nanotube, A is the cross section of beam model and V is the shear force which is define as follow,

$$V = \partial M / \partial x \tag{8}$$

The bending moment in equation (6) can be define as,

$$M = \int y\sigma ds \tag{9}$$

where, $\sigma = E\varepsilon$. For small deflection, the axial strain is define by,

$$\varepsilon = -y\partial^2 W / \partial x^2 \tag{10}$$

Shear force and bending moment can be expressed as,

$$V = -EI\partial^2 W / \partial x^3 \qquad M = -EI\partial^2 W / \partial x^2 \tag{11}$$

From the classical Euler beam theory, the vertical deflection W that results from load distribution $d(x)$ satisfies the fourth-order ordinary differential equation:

$$q(x) = EI\partial^4 W / \partial x^4 + \partial^2 EI / \partial x^2 \partial^2 W / \partial x^2 + 2\partial EI / \partial x \partial x^3 W / \partial x^3 + pA\partial^2 W / \partial t^2 \tag{12}$$

In order to solve the bending equation, the harmonic differential quadrature method is used. The discretized form of equation above is,

$$q(x_i) = EI \sum_{j=1}^{N} D_{xij} W_j + pA \sum_{j=1}^{N} B_{tij} W_j + \partial^2 EI / \partial x^2 \sum_{j=1}^{N} B_{xij} W_j + \partial 2EI / \partial x \sum_{j=1}^{N} C_{xij} W_j \tag{13}$$

The weighting coefficient of the fourth order derivatives (D_{ij}) can be computed easily from (B_{ij}) by

$$D_{ij} = \sum_{j=1}^{N} B_{ij} B_{ij} \tag{14}$$

Two-types of boundary conditions are considered. These are,
Fully clamped, (at both ends)

$$W = 0, \ \partial W / \partial x = 0 \tag{15}$$

Simply supported, (at both ends)

$$W = 0, \ \partial^2 W / \partial x^2 = 0 \tag{16}$$

The discretized form of boundary condition can be obtained by, Fully clamped, (at both ends)

$$W_i = 0, \quad \sum_{j=1}^{N} A_{ij} W_j = 0 \tag{17}$$

Simply supported, (at both ends)

$$W_i = 0, \quad \sum_{j=1}^{N} B_{ij} W_j = 0 \tag{18}$$

5 NUMERICAL RESULTS

For functionally graded carbon nanotubes, different examples are investigated in this section. From the knowledge of author, there are not any results exist for static analysis FG nanotubes. So in order to show the validation of present results, a comparison is made for the isotropic case. The material properties used in the present study are as follows:

$$E_1 150GPa, \quad E_2 200GPa, \quad r = 15nm, \quad R = 20nm, \quad L = 600nm \tag{19}$$

where E_1 and E_2 refer to the Young's modulus at both ends of functionally graded carbon nanotubes, r and R are the inner and outer radius of carbon nanotubes and L is the length of the carbon nanotubes. The deflection of functionally graded carbon nanotubes subjected to mechanical loading can be defined as follow,

$$\boldsymbol{W} = 1000 * w * E_2 \tag{20}$$

In Table 1, a comparison is made between the present results and the results of Civalek et al [7] for the isotropic case (n = 0). Exact analytical solution is also obtained by the analytical formula given by

Reddy and Pang [23] for comparison. Excellent agreement has been achieved between the present results and the results obtained by analytical formula given by Reddy and Pang [23]. It is obviously shown that the harmonic differential quadrature method can achieve accurate results. In Fig. 3, the effects of power law index on the deflections of simply supported functionally graded single walled carbon nanotubes subjected to centrally concentrated load are shown. One can easily see that with the increase of power law index, the deflections of nanotube will increase. The effects of lenght of fully clamped single walled carbon nanotubes under centrally concentrated load are investigated in Fig. 4. As it is expected, increasing the length of nanotubes will also increase the deflections of nanotubes. In Figs. 5 and 6, the effects of inner and outer radius of functionally graded single walled carbon nanotubes are studied, as it can be seen in Fig. 2. The results for fully clamped functionally graded carbon nanotubes vs. different inner radius are figured in Fig. 5. It is obtained that increasing the inner radius will increase the deflections of nanotubes. The outer radius of carbon nanotube is considered in Fig. 6. One can understand that in order to decrease the deflections of single walled carbon nanotubes, the outer radius must increase.

Table 1 Comparison of maximum deflection (wEI/qL^4) under uniformly distributed loading, $(I = 105 * 10^{-34}, v = 0.3, E = 2 * 10^9)$

HDQ	Civalek et al [7]	Reddy et al [23]
0.1250	0.12501	0.1250

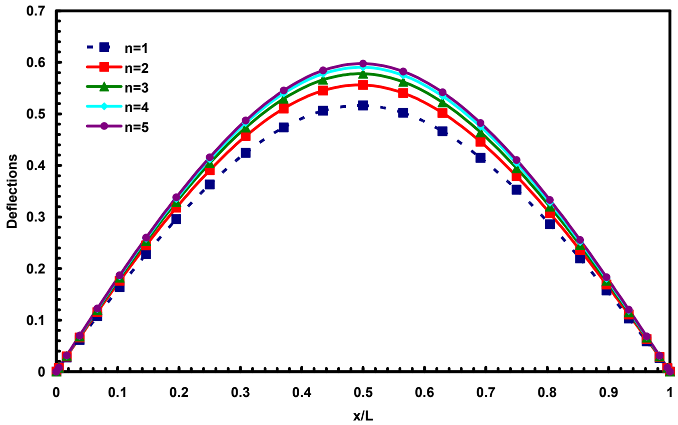

Figure 3 The effects of power law index on the deflections of simply supported functionally graded nano tubes $q = (200nN)$

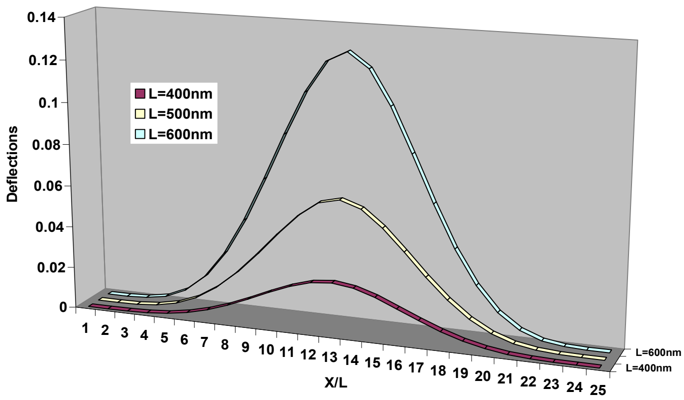

Figure 4 The effects of length of nanotubes on the deflections of fully clamped functionally graded nanotubes $(N = 4)$

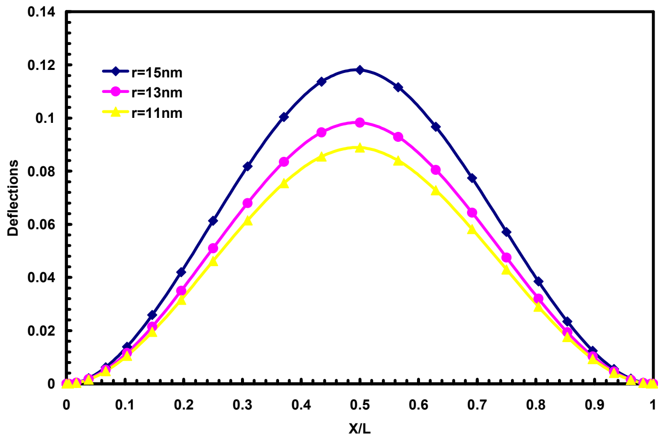

Figure 5 The effects of inner radius of nanotubes on the deflections of fully clamped functionally graded nanotubes ($N = 1$)

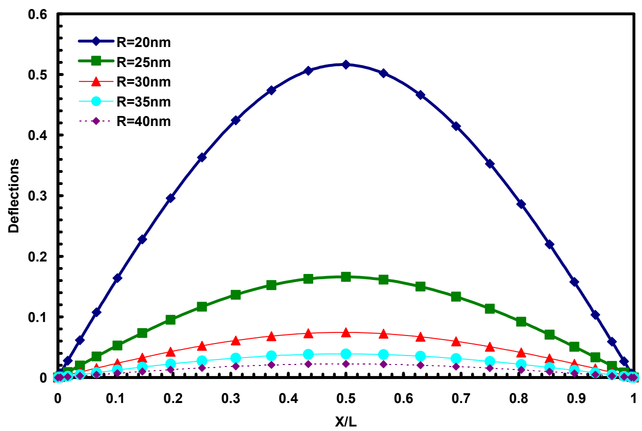

Figure 6 The effects of outer radius of nanotubes on the deflections of simply supported functionally graded nanotubes ($N = 1$)

6 CONCLUSION

Based on Euler-Bernoulli beam theory, static analysis of functionally graded carbon nanotubes subjected to mechanical loading was investigated. The harmonic differential quadrature method was employed to solve the governing equations. Various parameters were studied for functionally graded nanotubes in this paper. It was shown that the results for isotropic carbon nanotubes are very different from the results for functionally graded carbon nanotubes. It was also shown that in order to decrease the deflections of functionally graded carbon nanotubes, the power law index, the length and inner radius of nanotubes should decrease and the outer radius must increase.

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