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Field investigation on variation of rail support modulus in ballasted railway tracks

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Abstract

Rail support modulus is an important factor in safety of railway track. This parameter is defined as support force imposing on rail length unit per rail unit displacement in vertical direction. Rail support modulus is important because it affects track performance and maintenance requirements. Both low and large modulus is undesirable. Low modulus of the rail support has been shown to cause differential settlement that then increases maintenance needs. On the other hand, higher value of rail support causes axle load to be distributed on fewer sleepers and therefore received dynamic loads for any sleeper and rail-sleeper fastening will be increased. Sandy desert areas are critical regions about the contamination of ballast. In these areas, flowing sand grains influence between ballast aggregates and increase the stiffness of ballast layer and the rail support modulus. In this paper, the results of a field investigation about the effect of ballast contamination on the values of the rail support modulus in sandy desert areas are presented.

Keywords

Rail support modulus, Railway in sandy areas, ballast contamination

1 INTRODUCTION

Track quality plays an important role in railway safety. One of the affecting factors of track quality is the track stiffness. Track stiffness is defined as proportion factor between rail vertical displacement and vertical contact pressure between rail base and track foundation. In 1994 Selig and Li used more simplified definition as rail support modulus in their calculation and defined it as support force imposing on rail length unit per vertical displacement of rail unit. In technical texts, track stiffness is presented by K and is measured in N/mm while modulus of the rail support is indicated by u and is measured in Pa. In addition to above mentioned difference, the main difference between track stiffness and rail support modulus is that track modulus takes effects of flexural stiffness of rail, EI (rail dimensions and material determine EI), while u depends on other components of superstructure such as sleeper and rail-sleeper fastening system and infrastructure such as ballast, sub-ballast and soil subgrade layer. Thus rail support modulus is independent from rail type [2, 7, 15].

Rail support modulus is a major parameter because it affects track performance and maintenance requirements. Both low and large modulus is undesirable. Low modulus of the rail support has been shown to cause differential settlement that then increases maintenance needs [6]. On the other hand, if the variation of rail support modulus is too high, as in bridge vicinity and slab tracks, dynamic forces imposed on track increase. Higher value of rail support modulus leads to reduction of deflections and stresses in the track but on the other hand, this issue causes axle load to be distributed over fewer sleepers and therefore received share of axle load for any sleeper and rail-sleeper fastening force will increase [19, 21]. Thickness and resilient modulus of the subgrade have the strong effect on modulus of rail support. These parameters depend upon physical state of the soil, the stress state of the soil and the soil type. Ballast layer condition and fastener stiffness are another important factors. Increasing the width of ballast shoulder and or increasing fastener stiffness will increase rail support modulus [11].

Another numerous factors affect on resilient modulus of ballast and consequently on the rail support modulus such as maximum grain size, aggregate type, particle shape, moisture content and ballast breakage index (BBI) [8].

Ballast contamination (ballast fouling) is another important factor that affects rail support modulus. Sandy desert regions are critical areas about this issue. In these areas, flowing sand grains penetrate between ballast aggregates and increase ballast layer stiffness and consequently rail support modulus increases. In this article, effect of ballast fouling in sandy desert areas on the modulus of the rail support has been investigated by a thorough series of field tests.

2 THE METHODS FOR MEASUREMENT OF THE RAIL SUPPORT MODULUS

Different methods have been proposed for measurement and calculation of rail support modulus and track stiffness. These methods can be classified generally in 3 major groups: theoretical, theoretical-experimental and experimental. Fig. 1 shows the summary of this classification. Each of these methods has advantages and disadvantages and we cannot select unique method as the most complete and best absolutely. Theoretical methods are questionable about validity and on the other hand, generally, there are many problems with experimental methods about their field tests and they lead to very high time and costs consuming. Therefore, the widespread use of these experimental methods is not possible for all railroads and in all places [10].

2.1 Talbot-Wasiutynski method

Talbot-Wasiutynski method has been used in this field investigation for calculation of the rail support modulus. This method was proposed and used by Talbot committee [3] and by Wasiutynski [16]. In this method a car is moved to the track location of test and the caused vertical rail support deflection at each sleeper are measured, as shown in Fig. 2 [6].

According to this method, the rail support modulus, u, is calculated by dividing the sum

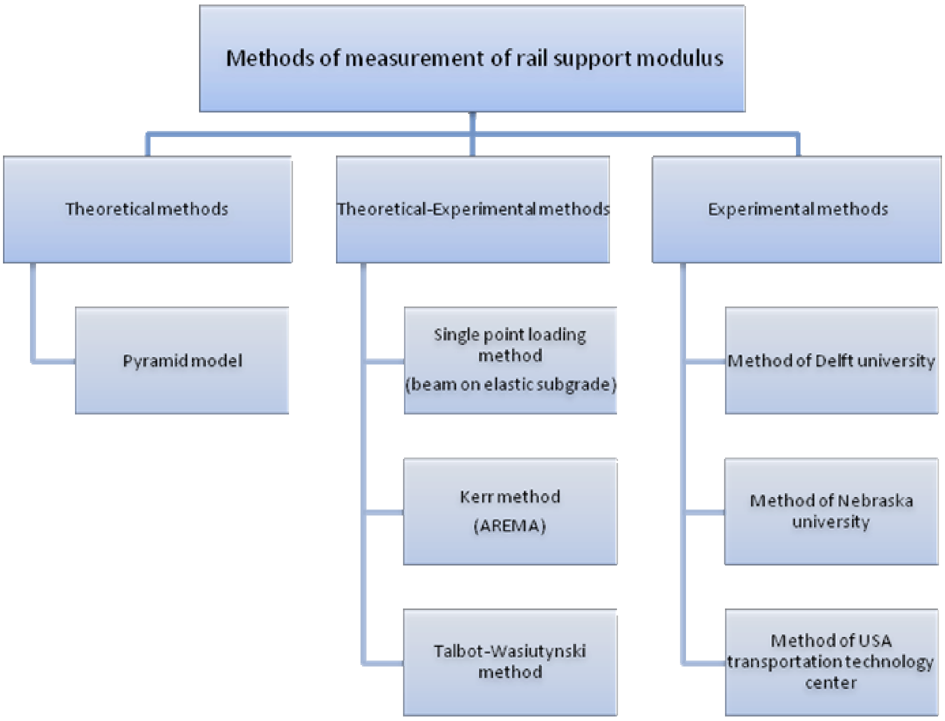


Figure 1 Classification of methods for measurement of rail support modulus and track stiffness

of the wheel loads P that act on the rail, by the area formed between the undeformed and the deformed rail, A_R. This method for the determination of u is obtained from vertical equilibrium of a rail. Noting that *p(x)* is the pressure which acts on the rail base it follows that:

$$\Sigma P - \int_{-\infty}^{+\infty} p(x)dx = 0 \tag{1}$$

Considering *p(x)=uw(x)*, where u is constant along the track, above equation becomes:

$$\Sigma P - u\int_{-\infty}^{+\infty} w(x)dx = 0 \tag{2}$$

Therefore:

$$u = \frac{\Sigma p}{\int_{-\infty}^{+\infty} w(x)dx} \tag{3}$$

Since the integral in the denominator is A_R, the above u expression proves that the prescription by the Talbot Committee satisfies vertical equilibrium. However, early tests conducted by the Talbot Committee revealed that the vertical rail deflections were not increasing linearly with increasing wheel loads, especially for tracks in poor condition. A similar type of non-linear response was recorded more recently by Zarembski and Chords for track in good condition but for larger wheel loads [20].

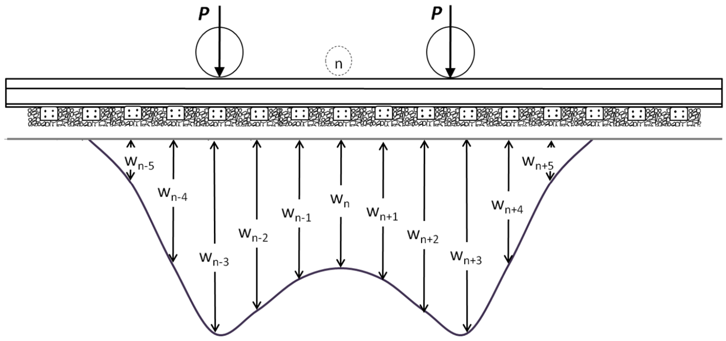


Figure 2 Recorded deflection of rail support under vertical loading

The observed non-linearity for relatively light wheel loads was attributed mainly to the play between the rails and the sleepers, the play between the sleepers and ballast, and the bending of the sleeper while they take full bearing in the ballast. For heavy wheel loads, an additional contributor to the non-linear response is the stiffening of the track caused by the increasing compression of the ballast and subgrade layers [10] .

To take into consideration this non-linearity, in a later paper the Talbot Committee [4] recommended to retain the linear analysis, but to determine the rail support modulus, u, using the difference between the vertical deflections from a heavy and a light weight car (Fig. 3).

For the determination of u, they proposed the formula:

$$u = \frac{\Sigma\left(p_h - p_i\right)}{a\,\Sigma_{i=1}^{n}\left(W_i^h - W_i^i\right)} \tag{4}$$

Where a is the sleeper spacing and h corresponds to heavy and l to light wheels. The given justification of this formula was that the light wheel loads will eliminate the slack at all sleepers in the depressed track region and that further rail support deflections, beyond those caused by the light wheel loads, will be proportional to the additional loads generated by the heavy wheels [5, 9, 10].

3 BALLAST CONTAMINATION

Ballast layer is a layer of broken stone materials with 20 to 60 mm in diameter that sleepers and rails are placed on it. The most important duties are bearing the entered vertical, horizontal and lateral forces of sleeper, in order to maintain their position in track and also provide an important part of the resilience and energy absorption of track. In addition to the ballast layer, using sub-ballast layer is necessary. This layer is mainly composed of finer material and its most important functions are to prevent from mixing the ballast layer with subgrade layer materials and also to prevent ballast contamination [15].

There are several standard tests to check the proper performance of ballast. These index tests include single particle crushing tests, oedometer tests, attrition tests in a revolving drum

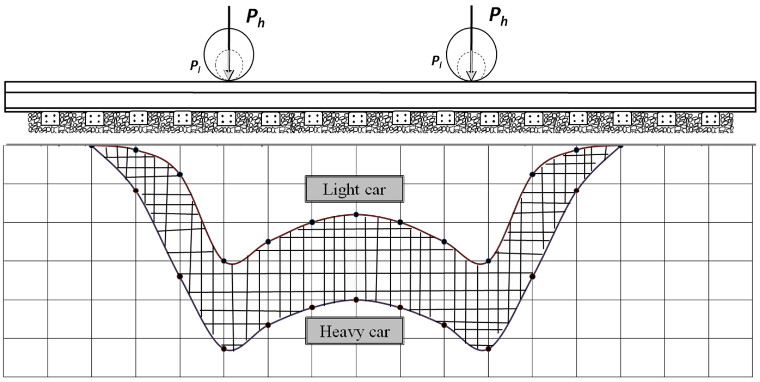


Figure 3 Reduced deflection area

(WAV, MDA and LAA tests), water absorption, particle shape characteristics and determination of ballast fouling index [12].

For the fouled ballast samples, the particles smaller than 9.5 mm (3/8 in.) are assumed to present the fouling components. The coarse fouling component consists of particles between 9.5 mm (3/8 in.) and 0.075 mm (No. 200 sieve) diameter. These are mainly sand size. The fine fouling component consists of those particles finer than 0.075 mm, which represent the silt and clay sizes. Over a period of time, the amount of fouling material will increase, the rate of increase varying widely with circumstances. The most important sources of ballast fouling include: Ballast breakdown (caused by handling, thermal stress from heating, freezing of water in particles, chemical weathering, tamping damage, traffic damage and from compaction machines), Infiltration from ballast surface (delivered with ballast, dropped from trains, wind blown, water borne and splashing from adjacent wet spots), Sleeper wear, Infiltration from underlying granular layers (sub-ballast particle migration from inadequate gradation and old track bed breakdown) and Subgrade infiltration. The effect of ballast fouling is to prevent the ballast from fulfilling its functions. The specific adverse effect depends on the amount and the character of the fouling material. Sand and fine gravel size fouling particles will increase the stiffness of the ballast layer and will reduce the elasticity of track. Also surfacing and lining operations will become increasingly more difficult as the ballast voids become filled [15]. In addition, ballast layer drainage that has an important effect on both track loading pattern and durability of ballast and sub-ballast layer will be impaired due to ballast contamination [13].

3.1 Ballast fouling in sandy desert areas

Desert areas due to the flowing sands, is one of the most critical areas of the fine grain filling in the ballast layer. During construction and use, several problems arise for repair and maintenance of track, locomotives and wagons that pass the route of desert sands. In sandy desert regions, ballast layer takes away from the granular behavior, because ballast fouling is increasing and thus track elasticity is reducing. In addition to increased modulus in track, numerous other problems are produced in these areas. Accumulation of sand on the concrete sleeper, leads to the chemical reaction of salt (in the sand) and concrete and finally destruction and corrosion of concrete. Also in desert regions, rails and other metal equipment of track are placed in the vicinity of salty sand and this issue leads to corrosion of rails and metal instruments of track pavement. Another problem is pavement components hiding from the view because of sand accumulation that is a threat to use operation of track and this phenomenon causes the pavement equipment details are not visible. Other problems occurred for track in these areas include: Crushing rail crown and therefore being flattened rail head due to reduced track elasticity, Increase of track maintenance activities and consequently increase of maintenance costs, Rails buckling due to temperature fluctuations much higher than the neutral temperature, Switch function reduction caused by sand accumulation and Different alignment levels of rails due to unequal quenched of two rails [17].

4 FIELD MEASUREMENT METHODOLOGY

A thorough field investigation is conducted in this research. The main objectives of the field tests include:

- Determination of the ballast contamination in sandy desert areas (at four points).
- Determination of rail support modulus at test points using vertical deflection values of track and by the method of Talbot-Wasiutynski.
- Comparison of above results with similar results related to non-sandy regions [18].

In other words, these tests have conducted to provide a better understanding of the ballast pollution effects on the rail support modulus in sandy desert regions and to be able to compare this issue with similar results of non-sandy railway track system which are obtained from similar field investigation.

The test site is located in a block of east district of Iran railway networking that is a critical area of the flowing sand. The tests have been conducted in four different locations and with different granular contamination of ballast to be examined the effects of the percentage of pollution on the rail support modulus.

The properties of the track in the field are as follows. Rail is UIC60; sleeper is pre-stressed concrete sleeper type B70; sleeper distance is 60 cm; the fastening system is Vossloh; ballast and sub-ballast thicknesses are 30 and 15 centimeters, respectively; ballast aggregates are Granite with a size of 20 to 60 mm.

To measure the values of track vertical deflection in order to determine the rail support modulus, several LVDT equipments are installed at the end of the adjacent sleepers (according to method of Talbot-Wasiutynski). Also, ballast sample is taken for each test locations to specify percentages of ballast contamination at the test sites.

4.1 LVDT equipments installation

The linear variable differential transformer (LVDT) is a type of electrical transformer used for very accurate measuring of linear displacement. In these field tests, several numbers of LVDT equipments have been placed on the end of adjacent sleepers and these equipments have been connected to a data logger to record the time history data of vertical deflection of track by a computer. The used data logger is capable to record up to 2000 data per second.

LVDTs are very accurate and sensitive tools and for using, they must be firmly fixed in the desired place to measure the displacements accurately. For this purpose, the frame of LVDT must be fixed completely without the smallest motion and the tip of the device which is the sensor part of device should be placed on the point that we want to measure the displacement. This part of device has the freedom to move inside and outside and thus, positive and negative displacements can be measured.

To fix the LVDTs, the special steel bases have been prepared in the required number. As can be seen in the Fig. 4, these bases have a plate on their ends which is placed perpendicular to the arm. The frame of the LVDT is attached to this plate and its tip is placed on the end of concrete sleeper to measure the vertical deflection of track. The frame of LVDT is connected by special plastic bases with glue on the mentioned plate. To fix the steel bases, their vertical arm is buried inside the trench in the vicinity of ballast shoulder to a height of 0.5 meter approximately and around soil is compacted. The metal horns located at the end of vertical bases can help to further stabilize of the bases inside the soil.

Figure 4 How to put the LVDT devices in tests place, using steel bases

4.2 Ballast sampling at test locations

In each four test sites, the ballast was sampled and particle size analysis has been taken on these samples. Fig. 5 Shows image of the test locations. Ballast sampling was performed according to ASTM-C136 standard. Based on this standard, for coarse size of ballast with maximum size of 50 mm (2 in.), about 20 kg sampling should be done. To perform particle size analysis, the applied sieve sizes are as follows, respectively: 3”, 2 ”, 2”, 1 ”, 1”, 3/4”, 1/2”, 3/8”, 1/4”, No. 4, No. 6, No. 10, No. 20 and No. 40 [1].

Figure 5 Four selected locations based on ballast contamination

4.3 Track loading for tests

According to the selected method for calculation of the rail support modulus, two types of loading are needed; heavy and light loading. As the heavy loading, load of 111 tons from a 6-axle (two 3-axle bogies) GT26CW locomotive was applied to the track (Fig. 6). Therefore the axle load is 18.5 tons and the wheel load is 9.3 tons. The locomotive was run over the rail with the speed of 15 km/hr and therefore dynamic load coefficient considered to be equal to 1.07 and therefore the axle load is equal to 19.8 tons.

In the similar field investigation related to the non-sandy regions, the axle load of 13.33 tons from a 4-axle tamping machine was applied to the track (static load) [18].

As the light loading, load of 4 tons from a 2-axle draisine was applied to the track (Fig. 7). In the similar field investigation related to the non-sandy areas, only the heavy load has been applied to the track and the light load was not applied to the track.

Figure 6 Heavy load applied to the track

Figure 7 Light load applied to the track

5 FIELD TESTS RESULTS

Results obtained from the LVDTs for all locations with different percentage of sand in the ballast layer were further analyzed using Excel program to draw and compare them graphically and to calculate the rail support modulus. These results are presented as follows and they are compared with similar results related to non-sandy region that are obtained from another similar field investigation. All of these results are presented as follows.

5.1 Ballast fouling results

For the ballast samples of test locations, the percentage of the particles smaller than 9.5 mm (3/8 in.) that are the fouling components of ballast, are presented in Table 1. Also Fig. 8 shows the particle size distribution curves for all selected locations.

Table 1 Ballast fouling percentage at test locations.

Test location	**No. 1**	**No. 2**	**No. 3**	**No. 4**	**No. 5** (none-sandy area)
ballast contamination	62.7%	50.7%	27.5%	25.9%	no pollution

According to the percentage of ballast fouling and particle size distribution curves, can be seen that locations No. 3 and No. 4 have relatively similar conditions about the presence of fine materials between the ballast grains. Also, this case is true about locations No. 1 and No. 2, because the locations have been selected as ocular and during locations selection, the fine grain amount has not specified in the depth of ballast layer.

5.2 Measured values by LVDTs and calculation of rail support modulus

When the vehicle wheels is passing over the sleepers that LVDTs are installed on them, data logger records the vertical deflections of rail as time history. These data have been recorded for all test locations and for both heavy and light loadings. Therefore, using recorded data, the vertical deflection of track can be obtained for any sleeper at any particular moment. For example, Fig. 9 indicates two cases of deflection time history (time-deflection curve).

Then, using the rail support deflections at any location, the rail support modulus can be calculated according Talbot-Wasiutynski method. Fig. 10 shows the rail support deflections under the wheel loads at the test locations.

Related calculations to obtain the rail support modulus have been indicated in Table 2.

6 DISCUSSIONS OF THE RESULTS

According to the obtained data from expressed field measurements, useful results are achievable about the rail support modulus in the sandy desert regions. Table 3 and Fig. 12 indicate the values of the rail support modulus considering the amount of ballast fouling in sandy desert areas. Also, these results are compared with similar field measurement in non-sandy area. According to Fig. 11, the rail support modulus can be set to any fouling percentage of ballast.

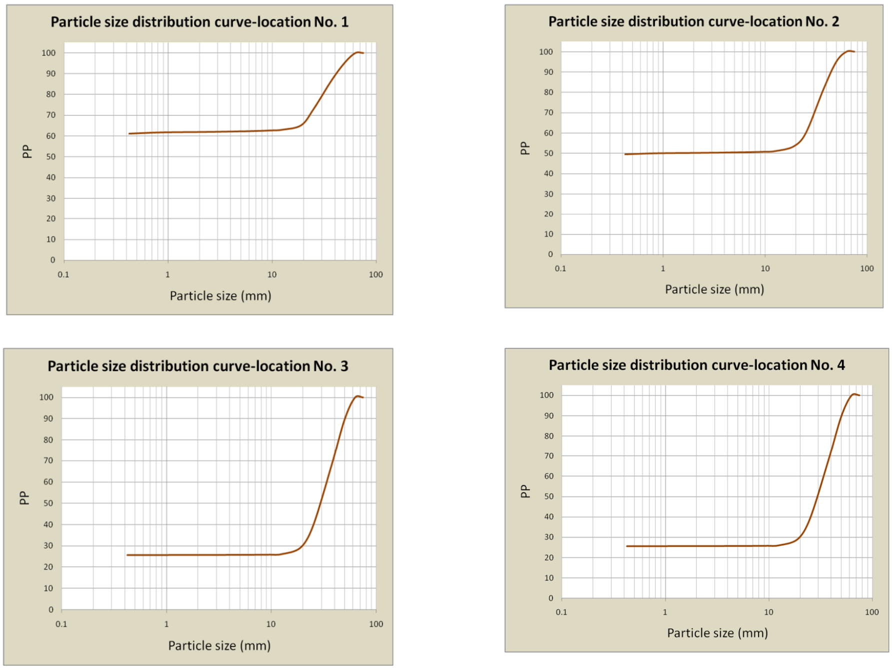


Figure 8 Particle size distribution curve for test locations

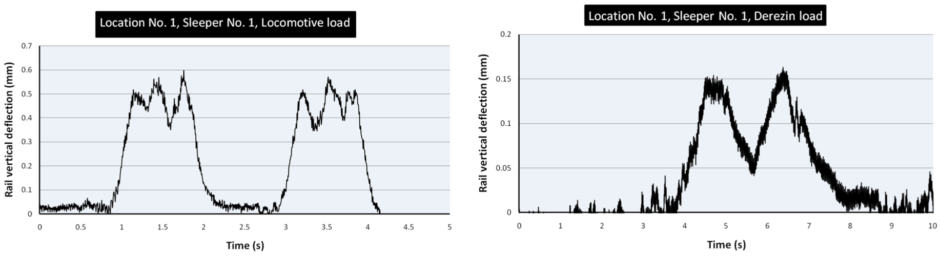
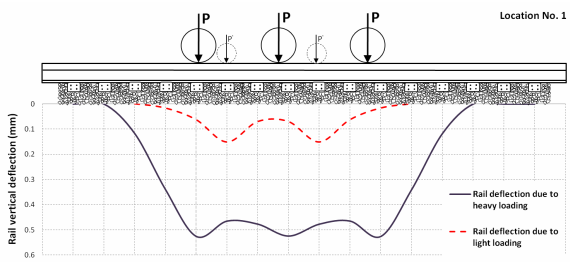


Figure 9 Two examples of time history of rail support deflection recorded by LVDTs

Table 2 The rail support modulus calculation for test locations

Test location	**No. 1**	**No. 2**	**No. 3**	**No. 4**	**No. 5 (non-sandy region)**
Heavy applied load (KN)	93	93	93	93	75
Light applied load (KN)	10	10	10	10	-
A_R due to heavy load (mm^2)	2628	2741	3049	3265	8100
A_R due to light load (mm^2)	361	287	359	343	-
calculated due to heavy load (MPa)	106	101	92	85	18
calculated due to heavy load (MPa)					
Rail support modulus calculated due to light load (MPa)	55	70	56	58	-
Rail support modulus calculated due to both heavy and light loads (MPa)	116	108	96	90	-



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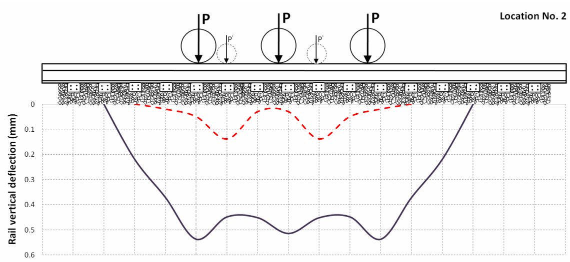

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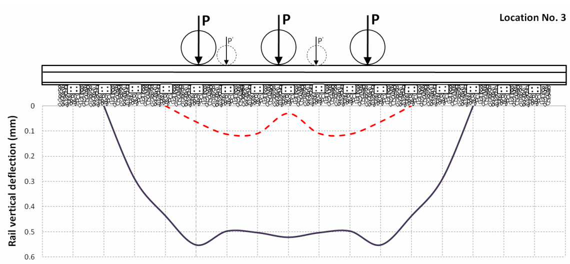

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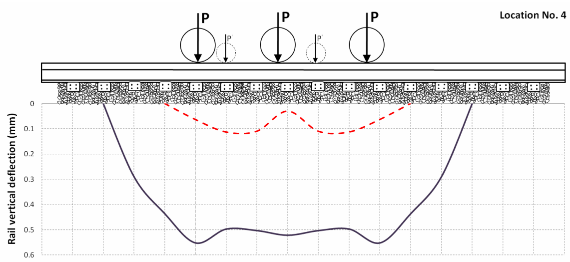

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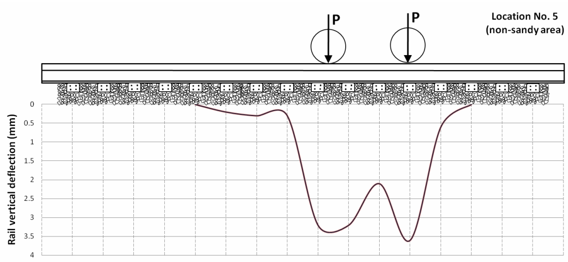

(e)

Figure 10 Vertical deflections of rail support for test locations, due to heavy and light loadings.

Table 3 Rail support modulus considering the ballast contamination

Ballast contamination percentage	62.7%	50.7%	27.5%	25.9%	trace
Rail support modulus (MPa)	116	108	96	90	18

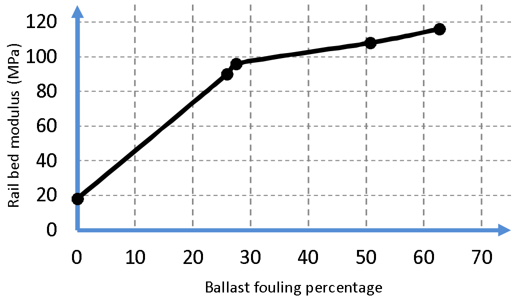

Figure 11 Relationship between ballast contamination percentage and rail support modulus

These amounts show that the influence of sandy fine grains between ballast aggregates makes the ballast layer stiffness begins to rise and thus the rail support modulus increases in sandy desert areas. Therefore, more accurate planning to eliminate the ballast layer pollution and other maintenance operations are necessary in such areas, because increasing of this modulus causes more dynamic loads to the sleeper and thus more and faster damage of sleeper is expected in such regions.

The following suggestions may be introduced about the maintenance operations in sandy desert areas:

1. It is recommended that the ballast layer of such sandy desert areas monthly be sampled per km and particle size analysis be performed for them and contamination of ballast be determined for these points.

2. The certain amount can be determined as the maximum allowable amount of rail support modulus and corresponding percentage of ballast contamination can be obtained according to the Fig. 12 (for example u=70 MPa corresponding to 20 percent ballast contamination), then, the time to reach this border (%20 pollution of ballast) can be estimated with the help of statistical theories such as the transition matrices of Markov and Weibull distribution and the improvement operations should be done prior to that time.

3. It is better to use ballast cleaning operation only if the interval between two rehabilitation operations is less than three mouths, because the serious damage will not be entered to other components of the track in this short interval.

4. The flowing sands should not be poured beside the track during the cleaning operation of ballast and these fine grains must be transmitted by the conveyor to the remote areas. This issue can help in increase of interval between periods of ballast cleaning operations effectively and thus, costs of track maintenance operations will be less, consequently.

5. Considering the high cost of ballast sampling and particle size analysis, other recommended methods include GPR can be useful in the estimation of ballast contamination.

6. Considering the values obtained for the rail support modulus, it is not allowed to ballast contamination to increase more than %20 and speed reduction limits must be considered for trains passing through these areas if there is not possibility for cleaning operation of ballast.

7. Tests results show that despite previous assumptions about changes in the modulus of the rail support, the increase in this parameter even for not too much contamination of ballast layer is very high and thus special planning is needed for maintenance operations of track in such sandy desert areas.

7 CONCLUSION

In this paper the results of a field investigation about the ballast contamination percentage and the values of the rail support modulus in sandy desert areas presented. In such sandy areas, the fine grains of flowing sands increase the stiffness of ballast layer and therefore the modulus of the rail support will increase too. Considering the amount of ballast fouling, four different locations are selected for tests. One another point, related to non-sandy area in a similar field tests, is considered as the fifth location and the results of sandy desert areas are compared with this result of non-sandy area.

Ballast contaminations in four selected locations are %62.7, %50.7, %27.5 and %25.9, and there is no pollution for the fifth point.

Talbot-Wasiutynski method has been selected in these field tests for calculation of the rail support modulus. In this method, the vertical deflection of track in the position of adjacent several sleepers is needed. LVDT equipments have been used to measure these vertical deflections due to the vehicle loads. In Talbot-Wasiutynski method two types of loading must be used; heavy and light. Heavy loading is done by GT26CW locomotive and a draisine has been used for light loading. According to this method, the obtained values of the rail support modulus for test locations are 116, 108, 96 and 90 MPa respectively, and for non-sandy location this modulus value is 18 MPa.

These results indicate that despite previous assumptions about changes in the modulus of the rail support, the increase in this parameter even for not too much contamination of ballast layer is very high and therefore the special precision must be provided in planning of maintenance operations of such sandy areas, because increasing of the rail support modulus leads to increasing of applied dynamic load and thus more and faster damage is expected for sleepers in such areas.

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