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Strain energy maximization approach to the design of fully compliant mechanisms using topology optimization

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Abstract

The paper presents an alternative formulation for the design of flexible structures and fully compliant mechanisms using topology optimization. The key to this approach is the maximization of a function of the strain energy stored in the mechanism. The proposed formulation reduces the appearance of common problems like intermediate densities, checkerboard and 1-node hinges, and can be extended to multiphysics and non-linear problems. The kinematic behavior of the fully compliant mechanism is imposed by a set of displacement constraints. The sensitivities are derived using an adjoint method and the optimization problem is solved using mathematical programming. The properties of the proposed formulation are shown with the aid of some examples.

Keywords: Compliant mechanism, Topology optimization, Strain Energy, Hinges, Checkerboard

1 Introduction

Compliant mechanisms are elastic structures that gains mobility from relative compliance of its members [1]. There are two categories of compliant mechanisms: partially compliant mechanisms and fully compliant mechanisms. This classification is presented in [2], where partially compliant mechanisms consist of rigid links and flexible parts such as flexible couplers. Fully compliant mechanisms are composed of only flexible members or joints, and can be classified in two categories: lumped and distributed compliant mechanisms. In lumped compliant mechanisms, the flexibility is provided in localized areas, and in distributed compliant mechanisms, the compliance is distributed through most of the mechanism.

A brief review of the existing approaches to the design of compliant mechanisms is presented in [1]. Basically, it is possible to distinguish between two main approaches: the maximization of some functional related to the force or displacement delivered by the mechanism [3–5] or

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reciprocal formulations, where reciprocal theorems, like Betti's theorem [6] or the mutual transdutance [6,7] are used. Within the first approach, there are formulations such the maximization of the mechanical advantage [3] and the maximization of the output displacement [8]. The maximization of the output displacement is very general, easily extended to multi-physics and non-linear problems [9]. The second approach is based in reciprocal relations between the input and output ports, using concepts like the mutual potential energy and mutual transdutance. It is extensible to multi-physics problems [6], but it is not directly extensible to non-linear problems, since it resorts to the superimposition of the reciprocal load cases.

A common problem of both formulations is the appearance of one-node connection hinges, that is, solid elements connected diagonally by just one node. A simple energetic balance provides an explanation to its appearance: as the optimization problem is trying to maximize the amount of energy delivered by the output port, the least energy should be stored in the mechanism, that is, the least elastic energy must be stored. As those hinges maximize displacement without storing elastic energy, they are used by the optimization solver. There are some ways to overcome for this problem, like the filter proposed in [10] as very simple way to avoid checkerboards and those hinges.

One question remaining about the existing formulations is: are they really trying to design a fully compliant mechanism? If the objective is to maximize the amount of energy delivered by the output port, than the answer is no, because a fully compliant mechanism must spend part of the energy received in the input port in the form of elastic deformation. As those formulations try to maximize the energy transfer between the input and the output ports, they are trying to minimize the amount of energy stored in the elastic medium, such that the result tends to be a partially compliant mechanism. Reciprocal formulations are based in compromise relations between flexibility and stiffness [6, 11], and the results can range from very flexible to very stiff structures, depending on the weights chosen by the designer. The results obtained with these approaches also show some hinges, indicating that the problem is not posed as the design of fully compliant mechanisms.

Analyzing the results obtained with the existing formulations, one can get a hint about the specifications for a formulation that addresses the design of fully compliant mechanisms. First, one should impose that part of the input energy must be stored in the form of strain energy in the regions with the base material. Second, it must be easy to extend to multi-physics and nonlinear problems. Third, the characteristics of the external medium must be considered during the design of the compliant mechanism, since the topology of the optimized mechanism also depends on it. With those objectives in mind, we study the design of compliant topologies as the design of topologies with maximum strain energy absorption. Using the objective function proposed, we impose the kinematic behavior of the compliant mechanism as a set of displacement constraints. The characteristics of the external medium are simulated by the use of the spring model [12].

2 Design of Compliant Structures

The methodology used in this work to designing of compliant structures is the topology optimization [13]. The topology optimization of continuum domains deals with the material distribution that leads to extreme values of a functional associated to the behavior or a given property of the structure (objective function), while respecting a set of functional and equilibrium constraints. One of the fundamental aspects of the topology optimization is the material parameterization. The most used material parameterizations are based on the homogenization theory [13] and artificial material models, like SIMP [14] and Voigt-Reuss [15]. In this work, the linear parameterization

$$\mathbf{E}(x) = \rho(x)\mathbf{E}^0,\ \rho \in [0,1] \tag{1}$$

is used to represent the dependency of the effective constitutive tensor $\mathbf{E}$, in each point of the design domain, with respect to the spatial distribution of the material (pseudo) density $\rho(x)$ and the fourth order constitutive tensor of the base material $\mathbf{E}^0$. After the finite element discretization, a constant (centroidal) density value is assumed for each finite element, reducing the number of design variables to a finite value. As a result of such material parameterization, the effective stiffness matrix of an element i in the mesh is given by $\mathbf{K}_i = \rho_i \mathbf{K}_i^0$, where $\mathbf{K}_i^0$ is the stiffness matrix considering the base material.

Using this parameterization, we can evaluate the relation between the strain energy density with the material density. As an illustrative example, we consider a linear elastic structure, with a uniform material density. We also assume density independent external forces. In this case, the displacements and strains are inversely proportional to the density, and it is possible to verify that the strain energy density W is given by

$$W = \frac{1}{2}\,\varepsilon : \mathbf{E} : \varepsilon = \frac{1}{\rho}\varepsilon^0 : \rho\mathbf{E}^0 : \frac{1}{\rho}\varepsilon^0 = \frac{1}{\rho}W^0, \tag{2}$$

where W^0 is the strain energy density corresponding to $\rho = 1$. As the density goes to zero, the effective strain energy goes to infinity. Thus, if the objective is to maximize the stored strain energy in the domain Ω, given by $U = \int_\Omega W d\Omega$, it is clear that the resulting design will show very low density values.

3 Maximization of a function of the strain energy

Directly maximizing the strain energy has little practical use, since we are interested in building the part using the base isotropic material and void, that is, we want to store the strain energy in the elements with maximum values of density. We instead propose to maximize a different objective function, related to the strain energy, crafted in such a way to penalize the storage of strain energy in intermediate density elements. This alternative to Eq. (2) is to scale the strain energy density with powers of the material density, defining a new function

$$W^* = \rho^n W = \rho^{n-1} W^0 \ . \tag{3}$$

Figure 1 shows the behavior of this function for different values of the exponent n, where the original expression (Eq. 2) is attained for $n = 0$. For $n \geq 2$, we obtain a remarkable different behavior, maximizing the value of the new function for large values of the density while penalizing its value for intermediate values of density.

The objective objective function Φ is then defined as the integral of this function

$$\Phi = \int_\Omega \rho^n W^* d\Omega \ . \tag{4}$$

Extending Eq. (4) for a mesh with N constant density finite elements, we can write

$$\Phi = \sum_e^N \rho_e^n \mathbf{u}_e^T \mathbf{K}_e \mathbf{u}_e, \tag{5}$$

where $\mathbf{u}_e$ is the displacement vector of the element e.

Without an upper bound constraint in the amount of material fraction, the maximization of this function would lead to a design domain full of material. Thus, the formulation

$$\begin{array}{rll} Max & \Phi & \\ S.t. & \int_\Omega \rho d\Omega & \leq V_{max} \end{array} \tag{6}$$

where V_{max} is the amount of base material avaliable, is proposed to the design of structures with maximum strain energy for a given amount of material.

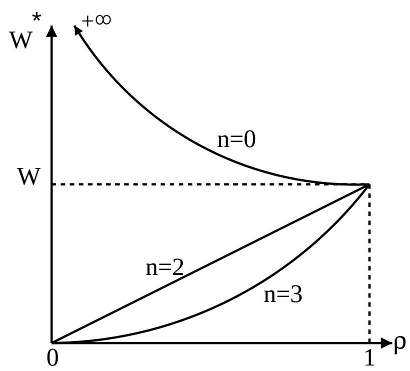

Figure 1: Behavior of the proposed function of strain energy for one element in the mesh.

4 Design of compliant mechanisms

A compliant mechanism is an elastic structure that uses its flexibility to transform the energy provided in the input ports into movement of the output ports. The actuation pattern is a function of the topology of the mechanism and the characteristics of the external medium, which are represented by springs attached to the output ports. The kinematic behavior of the mechanism is imposed with the use of an additional set of displacement constraints in Eq. (6), in the form

$$\begin{array}{lll} Max & \Phi & \\ S.t. & \int_\Omega \rho d\Omega & \leq V_{max} \\ & u_j & \leq u_{j_{max}} \\ or & u_j & \geq u_{j_{min}} \end{array} \tag{7}$$

where u_j is the displacement in the j^th degree of freedom of the finite element mesh. Equation (7) means that we are searching for a structure which stores the maximum amount of strain energy in areas with maximum density and respects, at least point wise, a set of displacement constraints.

It is important to emphasize that it may be impossible to find a fully compliant mechanism for given values of the volume fraction, dimensions of the design domain and the value of the displacement constraints. In this case, the designer should evaluate if the constraint values are realistic.

The proposed formulation is non-convex, so there is no assurance of finding the same minimum starting from different density distributions.

This formulation respects all the desired characteristics for the design of a fully compliant mechanisms discussed above. As it is based in energy, it is easy to extend to multi-physics and as it is not based in reciprocity, it can be extended to non-linear problems.

5 Sensitivity analysis

In this work, the sequential linear programming [16] method is used to solve the optimization problem. This problem consist of nonlinear and non-convex equations, but the linear programming is chosen due to its simplicity. To use the linear programming, the linearization of the objective function and the displacement constraints is carried on by the use of Taylor series, retaining just the linear terms. To perform this linearization, we must evaluate only the first derivatives of the objective functions and displacement constraints with respect to the design variables. The volume is linear with respect to the design variables, because

$$V = \sum_{e}^{N} \rho_e V_e \tag{8}$$

and the sensitivity is simply

$$\frac{dV}{d\rho_i} = V_i \tag{9}$$

where V_i is the volume of the $i-th$ element in the mesh.

5.1 Sensitivity of the objective function

To evaluate the sensitivity of the objective function with respect to the design variables, we use an adjoint approach. First, Eq. (5) is re-written as

$$\Phi = \sum_{e=1}^{N} \left[\rho^n \left(\mathbf{H}_e \mathbf{u}_g\right)^T \mathbf{K}_e \left(\mathbf{H}_e \mathbf{u}_g\right)\right] + \lambda_g^T \left(\mathbf{K}_g \mathbf{u}_g - \mathbf{f}_g\right), \tag{10}$$

where $\mathbf{H}_e$ is a localization operator, mapping the element displacement vector $\mathbf{u}_e$ from the global displacement vector $\mathbf{u}_g$, $\mathbf{K}_g$ is the global stiffness matrix, $\mathbf{f}_g$ is the global vector of external forces and λ_g is the global adjoint vector. We add the equilibrium equation multiplied by the pseudo-load vector λ, which must be zero and do not interfere in the original equation. In this work, design independent loads are assumed to simplify the presentation.

Proceeding with the derivatives with respect to the $i^t h$ design variable, we obtain

$$\begin{aligned} \frac{d\Phi}{d\rho_i} &= \sum_{e=1}^{N} \left[\frac{d\rho_e^n}{d\rho_i} \left(\mathbf{H}_e \mathbf{u}_g\right)^T \mathbf{K}_e \left(\mathbf{H}_e \mathbf{u}_g\right) + 2\rho_e^n \left(\mathbf{H}_e \mathbf{u}_g\right)^T \mathbf{K}_e \mathbf{H}_e \frac{d\mathbf{u}_g}{d\rho_i}\right. \\ &\quad \left. + \rho_e^n \left(\mathbf{H}_e \mathbf{u}_g\right)^T \frac{d\mathbf{K}_e}{d\rho_i} \left(\mathbf{H}_e \mathbf{u}_g\right)\right] + \lambda_g^T \frac{d\mathbf{K}_g}{d\rho_i} \mathbf{u}_g + \lambda_g^T \mathbf{K}_g \frac{d\mathbf{u}_g}{d\rho_i}, \end{aligned} \tag{11}$$

where the symmetry of the element stiffness matrix is used. Grouping the common terms, we have

$$\begin{aligned} \frac{d\Phi}{d\rho_i} &= \sum_{e}^{N} \frac{d\rho_e^n}{d\rho_i} \left(\mathbf{H}_e \mathbf{u}_g\right)^T \mathbf{K}_e \left(\mathbf{H}_e \mathbf{u}_g\right) \\ &\quad + \left[\sum_{e}^{N} 2\rho_e^n \left(\mathbf{H}_e \mathbf{u}_g\right)^T \mathbf{K}_e \mathbf{H}_e + \lambda_g^T \mathbf{K}_g\right] \frac{d\mathbf{u}_g}{d\rho_i} \\ &\quad + \sum_{e}^{N} \rho_e^n \left(\mathbf{H}_e \mathbf{u}_g\right)^T \frac{d\mathbf{K}_e}{d\rho_i} \left(\mathbf{H}_e \mathbf{u}_g\right) + \lambda_g^T \frac{d\mathbf{K}_g}{d\rho_i} \mathbf{u_g}, \end{aligned} \tag{12}$$

and, to avoid the evaluation of the derivative of the displacement vector with respect to ρ_i, we must turn the second term in the r.h.s. of Eq. (12) into zero. This is obtained by the use of the relation

$$\lambda_g^T \mathbf{K}_g = \sum_{e}^{N} \left\{-2\rho_e^n \left(\mathbf{H}_e \mathbf{u}_g\right)^T \mathbf{K}_e \mathbf{H}_e\right\} = \sum_{e}^{N} \left\{-\left(2\rho_e^n \mathbf{u}_e^T \mathbf{K}_e\right) \mathbf{H}_e\right\}. \tag{13}$$

It must be stressed that the product in the r.h.s. of Eq. (13) is evaluated at the element level, and that the local-global operator is used to turn it into a global pseudo-force vector. As the stiffness matrix is already factored, the cost of this operation is equivalent to a back substitution.

Analyzing Eq. (12), we verify that

$$\sum_e \frac{d\rho_e^n}{d\rho_i} (\mathbf{H}_e \mathbf{u}_g)^T \mathbf{K}_e (\mathbf{H}_e \mathbf{u}_g) \tag{14}$$

is nonzero only when $e = i$, resulting in

$$n\rho_i^{n-1} \mathbf{u}_i^T \mathbf{K}_i \mathbf{u}_i, \tag{15}$$

which is a local term. The term,

$$\sum_e \rho_e^n (\mathbf{H}_e \mathbf{u}_g)^T \frac{d\mathbf{K}_e}{d\rho_i} (\mathbf{H}_e \mathbf{u}_g) \tag{16}$$

is also local for linear problems, as the derivative of the local stiffness matrix is nonzero only when $e = i$, resulting in

$$\rho_i^n \mathbf{u}_i^T \mathbf{K}_i^0 \mathbf{u}_i. \tag{17}$$

The last term in Eq. (13) is also local, resulting in

$$(\mathbf{H}_i \lambda_g)^T \mathbf{K}_i^0 \mathbf{u}_i \tag{18}$$

Grouping all those results, we finally obtain the desired expression

$$\frac{d\Phi}{d\rho_i} = (n+1)\rho_i^n \mathbf{u}_i^T \mathbf{K}_i^0 \mathbf{u}_i + (\mathbf{H}_i \lambda_g)^T \mathbf{K}_i^0 \mathbf{u}_i \tag{19}$$

where λ_g is evaluated using Eq. (13).

5.2 Sensitivity of a displacement component

To evaluate the sensitivity of the $j^t h$ component of the global displacement vector with respect to the $i^t h$ design variable, we make use of a localization vector $\mathbf{L}$, having 1 in the $j-th$ position and zero on the other positions. Using this vector and an adjoint approach [17], we can write

$$u_j = \mathbf{L}^T \mathbf{u}_g + \gamma_g^T (\mathbf{K}_g \mathbf{u}_g - \mathbf{f}_g) \tag{20}$$

and the objective is to evaluate

$$\frac{du_j}{d\rho_i} = \mathbf{L}^T \frac{d\mathbf{u}_g}{d\rho_i} + \gamma_g^T \frac{d\mathbf{K_g}}{d\rho_i} \mathbf{u}_g + \gamma_g^T \mathbf{K}_g \frac{d\mathbf{u}_g}{d\rho_i}. \tag{21}$$

Grouping the common terms, we obtain

$$\frac{du_j}{d\rho_i} = \left(\mathbf{L}^T + \gamma_g^T \mathbf{K}_g\right) \frac{d\mathbf{u}_g}{d\rho_i} + \gamma_g^T \frac{d\mathbf{K}_g}{d\rho_i} \mathbf{u}_g. \tag{22}$$

and the derivative of the global displacement vector with respect to the i^th design variable can be avoided by the solution of the following linear system of equations

$$\mathbf{K}_g \gamma_g = -\mathbf{L}. \tag{23}$$

This equation must be solved for each displacement constraint, and its cost is equivalent to a back-substitution.

Thus, for linear problems, the desired sensitivity is

$$\frac{du_j}{d\rho_i} = \gamma_g^T \frac{\partial \mathbf{K}_g}{\partial \rho_i} \mathbf{u}_g = \left(\mathbf{H}_i \gamma_g\right)^T \mathbf{K}_i^0 \mathbf{u}_i, \tag{24}$$

since the derivative of the global stiffness matrix with respect to the i^th design variable is local. The extension of this sensitivity procedure to nonlinear problems is discussed by [5].

6 Solution Procedure

The proposed formulation is non-convex, and numerical experiments indeed found many local maxima, and some of them containing hinges. Therefore, it is advisable to use a spatial density gradient control to guide the optimizer to a local maxima with smooth inter-element density variation. It is important to emphasize that the use of filters with traditional objective functions does not prevent the appearance of hinges, as most of the objective functions try to minimize the amount of elastic energy stored in solid parts of the structure. This is the main point of the proposed objective function, since it favors the presence of solid material in regions with elastic deformation. Hinges are avoided during the optimization process unless they are necessary to satisfy the displacement constraints. The objective function also penalizes the appearance of intermediate densities, such that results with few "grey" areas are obtained, even for low values of the exponent n in Eq. (5).

One of the main concerns in the topology optimization of continuum structures is the appearance of the checkerboard instability [18]. The use of the proposed objective function is able to alleviate this numerical instability, as checkerboard is a minimum compliance artifact. Checkerboard areas were observed only when the volume fraction prescribed was insufficient to satisfy the displacement constraints, and vanished when volume fractions increased.

In this work, the mesh independent spatial filter proposed by [19] is used, but the methodology proposed should work with other gradient control techniques [13]. The main objective of the filter is to control the complexity of the mechanism by maintaining the inter-element density variation bounded. The main advantage of the filter used in this work is to bound the variation of the move-limits of the linear programming, avoiding artificial modification of the sensitivity or density fields.

Different values of the exponent n where tested during numerical experiments, and it was found that its value has little effect in the results. Since the degree of nonlinearity is proportional to the value of the exponent, it is advisable to use a low value. In this work all the results were obtained with $n = 2$.

7 Results

Two examples commonly found in the literature [6, 13] are selected to show the behavior of the proposed formulation. In both examples, the base material is Nylon, with $E = 3 * 10^9 Pa$ and $\nu = 0.4$ and the design domain is a square of dimensions $100 \times 100 \times 5\, mm$.

The first example is the design of an inverter mechanism, Fig. 2. The force in the input port is $200N$ and its displacement (u_{in}) is constrained to be smaller than $2mm$. The external medium is simulated by a spring (K_s) with stiffness of $1 * 10^5 N/m$ and the desired output displacement (u_{out}) is $-1mm$, as shown in Fig. 2. A simple energy balance shows that there exists enough energy to be stored in the mechanism, as the external work provided in the input port is $W_{in} \leq 0.2Nm$ and the energy delivered in the output port is $E_{out} \leq 0.05Nm$. Half design domain is discretized using 9800 four node bilinear isoparametric finite elements (each element is $1 \times 1mm$) and radius of the the spatial filter is equal to $1mm$.

If an exponent $n = 2$ is used in Eq. (5), with a volume fraction of 25%, the topology of Fig. 3 is obtained. The topology shown in Fig. 3 corresponds to a fully compliant mechanism with lumped compliance, as large amount of material is distributed in "rigid" regions, and the lumped flexible regions have less base material. The topology obtained has no hinges or checkerboard, and its shape is smoother than the topologies obtained with other formulations presented in the literature. The existence of areas with intermediate densities is a consequence of the use of the filter, as the proposed formulation would normally prevent this phenomenon. Fig. 4 shows the topology obtained for $n = 2$, 15% of volume fraction and a spring with stiffness of $1 * 10^5 N/m$ in the output port. This topology corresponds to a fully compliant mechanism with distributed compliance.

Other classical example is the grasper mechanism, as shown in Fig. 5. The objective is to transform an horizontal movement into vertical movement in the output ports. The force applied in the input port is $200N$, and its displacement is constrained to be smaller than $2mm$. The external medium is represented by a stiffness of $1 * 10^5 N/m$ and the desired output displacement is $1mm$ as shown in Fig. 5. Half design domain is discretized using 9604 four node bilinear isoparametric finite elements (each element is $0.72 \times 0.72mm$) and radius of the the spatial filter is equal to $0.75mm$. Fig. 6 shows the topology obtained with the formulation discussed in this work, corresponding to a fully compliant mechanism, without hinges or checkerboard.

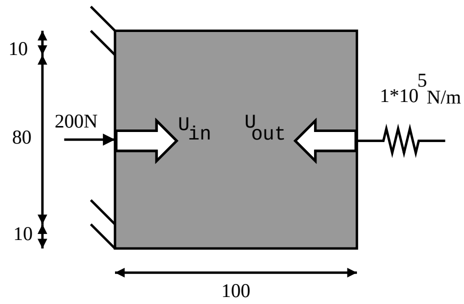

Figure 2: Geometry and boundary conditions of the first example. Dimensions in mm.

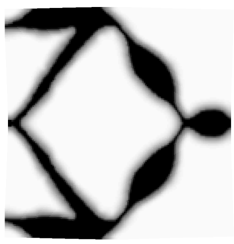

Figure 3: Topology obtained for the first example, $n = 2$ and 25% of volume fraction.

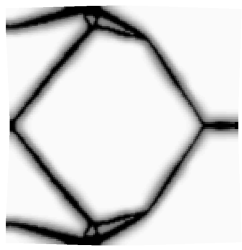

Figure 4: Topology obtained for the first example, $n = 2$ and 15% of volume fraction.

To show the influence of the value of the output displacement in the topology, the same example is considered for $u_{out} = 0.5mm$. It can be verified that the decrease in the energy delivered in the output port changes the topology from a lumped fully complied topology, Fig. 6, to a distributed fully compliant topology, Fig. 7.

Figure 8 shows the topology obtained for $K_s = 1 * 10^2 N/m$ and $u_{out} = 1mm$. Since the energy delivered in the output port decreases, more energy is available to be stored in the form of strain energy, changing the topology of the compliant part.

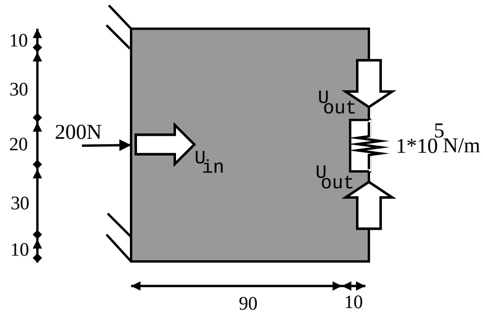

Figure 5: Geometry and boundary conditions of the second example. Dimensions in mm.

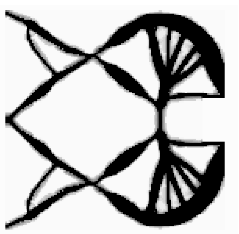

Figure 6: Topology obtained for the second example, $n = 2$ and 30% of volume fraction.

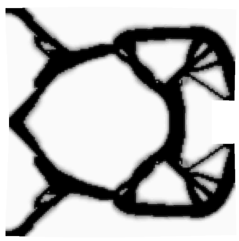

Figure 7: Topology obtained for the second example, $n = 2$, 30% of volume fraction and $u_{out} = 0.5mm$.

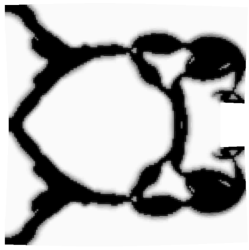

Figure 8: Topology obtained for the second example, $n = 2$, 30% of volume fraction, $u_{out} = 1mm$ and $K_s = 1 * 10^2 N/m$.

8 Conclusions

This paper presents an alternative formulation to the design of compliant mechanisms. It consists in the maximization of a function of the strain energy stored in the compliant part, while satisfying prescribed displacement constraints. It is shown that when there is enough energy to be stored in the form of strain energy, fully compliant topologies can be obtained with the use of this formulation. The formulation proposed also alleviates the appearance of checkerboard, intermediate densities and hinges, common problems in the design of compliant mechanisms using topology optimization.

The main drawback of the formulation is the possibility of specifying over-stringent constraints, thereby reducing or eliminating set of admissible solutions. This may occur in case of specifying output energy exceeding the input work, or the volume of material available to be inadequated to generate a good mechanism design.

The objective function proposed can be extended to non-linear and coupled problems, and the sensitivities for linear elastic problems are derived in this work by the adjoint method.

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