

Fatigue Life and Residual Strength prediction of GFRP Composites: An Experimental and Theoretical approach

Abstract

This paper presents the fatigue behavior of Glass Fiber Reinforced Polymer (GFRP) composites at constant amplitude tension-tension loading conditions. A two parameter residual strength and fatigue life model has been proposed by accounting the effect of stress ratio when the structure undergoes continuous loading. A model is also developed to predict the fatigue life of GFRP composites based on fatigue endurance limit. Experiments were conducted on GFRP composite specimens to predict fatigue life and residual strength at various stress levels. Tests were also conducted to gain an understanding of the tensile behavior of GFRP composite specimens under different quasistatic strain rates. The lowest tensile strength resulting from strain rate studies has been used ultimately for conducting fatigue life and residual strength tests. Reliability of the proposed models has been verified with experimental results and with the models seen in literature.

Keywords

Fatigue life, Residual Strength, Endurance limit, Strain Rate, GFRP Composite.

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<http://dx.doi.org/10.1590/1679-78255095>

Received: May 24, 2018

In Revised Form: May 26, 2018

Accepted: July 04, 2018

Available Online: July 10, 2018

Nomenclature

X_r residual strength	ϵ_0 ultimate tensile strain
n number of loading cycles	$\dot{\epsilon}$ strain rate
a, b constants depending on material and loading conditions	R stress Ratio
ΔX stress amplitude	c, d, u, v, w, S, T regression constants
n_f number of cycles to failure	<i>Acronyms</i>
X_{\max} applied maximum stress	GFRP Glass Fiber Reinforced Polymer
X_{\min} minimum stress	
X_0 ultimate tensile strength	

1. INTRODUCTION

Fiber reinforced composites are extensively used in every sector ranging from aerospace to medical instruments due to their excellent properties such as high strength to weight ratio and high stiffness to weight ratio. Despite its crucial benefits over the conventional materials, composites are yet to be a primary choice for all high strength applications due to the complex failure mechanisms under different loading conditions because of their anisotropic characteristics (Jefferson Andrew et al., 2016). In various circumstances, the composite structures are made of higher weight and strength than required as a matter of safety, which is much more than with conventional metals or alloys. This problem with the composite materials occurs from the lack of prediction of damage and its propagation. The prediction of failure of composite materials, despite its high values of resistance, has equivalent significance (Rotem and Nelson, 1989). One of the merits of composites is their potential to distribute the stress all over the plane by the matrix. It is also a drawback, as this makes the damage propagation more rapid, where the composites dissolve, lacking notable creep or a disclaimer before failure, places them back from extensive applications (Anderson, 2005).

Fatigue is a phenomenon associated with the damage propagation in a material when loading cycles are applied with stresses below its ultimate strength for a prolonged time. Fatigue in homogeneous materials is simple; a defect, slip bands or dislocation, from which emerges inflating stress and time enhancing to fracture, whereas, in heterogeneous materials it is not that forthright, as the damage can be initiated from multiple sites and also the minor damages are not apparent until the occurrence of the final failure (Dick et al., 2009). Thus the study in this domain is not complete and the vital models in fatigue prediction are still not adequate.

Fatigue load, which should be combated by machines and structures, varies in the alternating stress amplitude and mean stress. Furthermore, the shape and configuration of the stress–time pattern during service take many different forms according to their actual operation (Harris, 2003a). While there were many approaches made to the determination of fatigue failure in composites, only a few were able to make a substantial hypothesis. Amongst many approaches made, the current and widely used metric for fatigue failure determination is the residual strength method. The initial models of material failure were based only on the change in stress state of the material. It ignored the concept of material state changes; thus the use of strength as a metric accomplishes this task by considering both of them simultaneously, as the strength depends on changes in both stress state and material state (Harris, 2003b). The application of residual strength for fracture is based on the fact that, when the number of loading cycle is zero, the residual strength of the material is equal to the static strength, while at failure ($n = N$), it is equal to the maximum applied stress (Post et al., 2008). While there are many other theories, where damage accumulation models proved effective, residual strength models have an advantage, as the damage is quantified in terms of strength and, therefore, can be analyzed or substantiated by experiments, at any time in the material's fatigue lifetime (Post et al., 2010).

The first known attempts at using the residual models were by Halpin et al. (1973), who suggested a power law relationship between the residual strength and the number of cycles. Yang and his co-workers have done significant work in modeling the fatigue behavior of composite materials (Yang and Jones, 1980; Yang and Liu, 1977; Yang and Cole, 1982; Yang and Du, 1983; Yang and Jones, 1981; Chiao et al., 1982). Initially, a three-parameter residual strength model was proposed by Yang and Liu (1977) based on rate type equation. It was upgraded by using strength life equal rank assumption. Later, the model was modified through the introduction of extra parameters which made the model tedious because of their dependency on Stress-Life curve (Chiao et al., 1982; Yang and Du, 1983).

Many researchers have followed the procedure adopted by Halphin and the summary of wear out models has been provided by Kedward and Beaumont (1992). Caprino and D'Amore (1998) have proposed a two-parameter model, which eventually proved insufficient due to its lack of feasibility in a relation between the strength and the number of the cycles. Spearing and Beaumont (1992a, 1992b) have developed a model based on interactive matrix cracking which explains the effect of fatigue loading by relating post-fatigue strength and stiffness of notched fiber composites. The model proposed by Broutman and Sahu (1972) is considered to be a strong competitor for many other models seen in the literature due to its simplicity. A summary of residual strength wear out models has been reviewed noticeably by Degrieck and Van Paepegem (2001) and Wicaksono and Chai (2013).

The objective of this research is to study the fatigue behavior of Glass fiber reinforced polymer (GFRP) composites based on residual strength degradation. The focus is on the development of the reliable residual strength model which include stress ratio, much simpler, inexpensive computationally and independent of experimental material parameters. A fatigue life model is also deduced from the proposed residual strength model by applying the condition that the material fails when residual strength equals the maximum applied stress. The problems seen in conducting fatigue experiments are long test times and a large number of specimens are to be tested for the determination of fatigue properties (Gornet et al., 2013; Mandell, 1997; Rosa and Risitano, 2000). Hence a model is also proposed for predicting the fatigue life without any experimental parameters but based on the fatigue endurance limit of GFRP composites. Reliability of the proposed models in finding residual strength and fatigue life has been verified with the models seen in literature and also with the experimental results.

The performance of composite materials varies under different loading conditions due to their heterogeneous nature and adverse failure mechanisms. The results of many intense researches about the strain rate effect on GFRP composites show the tensile strength and strain as less sensitive to the strain rate (Naresh et al., 2016; Naresh et al., 2017). But at the same time, conflicting results have been reported by some researchers about the linear increase in tensile strength of the GFRP composites with the strain rate (Armenàkas and Sciammarella, 1973; Okoli, 2001; Okoli and Smith, 1999). A detailed review of the strain rate effects on different materials was given in Jacob et al. (2004) and Ray and Rathore (2015). In this study, experiments were conducted under different quasistatic strain rates such as 0.5, 1, 1.5, 5 and 50 mm/min as per the ASTM Standard D 3039/D3039M-17 (ASTM D3039, 2017) to understand the tensile behavior of GFRP composites and the lowest tensile strength has been used for conducting fatigue and residual strength tests.

2. MODEL DESCRIPTION

A new residual strength model has been proposed by modifying the power law given by D'Amore et al. (1996). During fatigue loading, the strength of the material undergoes a continuous deterioration which follows the power law as given in Eq. (1),

$$\frac{dX_r}{dn} = -a(1+n)^{-b} \quad (1)$$

where,

X_r - Residual strength after 'n' number of cycles.

a, b - Constants depending on material and loading conditions.

The effect of stress level has been considered by introducing the constant 'a' which is assumed to increase linearly with the stress amplitude. Hence 'a' is given by the Eq. (2)

$$a = a_0 \Delta X \quad (2)$$

where,

a_0 - Constant

$$\Delta X = X_{\max} - X_{\min} = X_{\max}(1 - R) \quad (3)$$

X_{\max} - Applied maximum stress

X_{\min} - Minimum stress

$$R - \text{Stress ratio} = X_{\min} / X_{\max} \quad (4)$$

Substituting Eq. (2) and Eq. (3) in Eq. (1), the Eq. (5) is obtained,

$$\frac{dX_r}{dn} = -a_0 [X_{\max} - X_{\min}] (1+n)^{-b} \quad (5)$$

Integrating the Eq.(5), we get Eq. (6)

$$X_r = P - \frac{a_0}{1-b} X_{\max} [1-R] [1+n]^{(1-b)} \quad (6)$$

$$X_r = P - v X_{\max} [1-R] [1+n]^\varphi \quad (7)$$

where

$$v = \frac{a_0}{1-b} \quad \varphi = 1 - b \quad (8)$$

In fatigue, when the number of cycles is zero, residual strength (X_r) equals the ultimate tensile strength of the material (X_0) is considered as the boundary condition as given in Eq. (9),

$$X_r = X_0, n = 0 \quad (9)$$

Substituting the above boundary condition in Eq. (7) and rearranging, the residual strength can be expressed as given in Eq.(10),

$$X_r = X_0 + v X_{\max} [1-R] \left[1 - (1+n)^\varphi \right] \quad (10)$$

where

X_r - Residual strength after 'x' number of cycles,

X_0 - Ultimate tensile strength,

X_{max} - Applied maximum stress,

R - Stress ratio,

n - Number of cycles,

v, φ - Constants

Rearranging Eq. (10), the expression for 'n' is given in Eq. (11)

$$n = \left[1 - \frac{X_r - X_0}{vX_{max} [1 - R]} \right]^{\frac{1}{\varphi}} - 1 \tag{11}$$

It is apparent that failure will happen when the residual strength of the material is equal to the maximum applied stress during fatigue loading as given in Eq. (12),

$$\text{At } X_r = X_{max}, n = n_f \tag{12}$$

where n_f - Number of cycles to failure

Applying the above boundary condition, the number of cycles to failure ' n_f ' can be expressed as given in Eq. (13)

$$n_f = \left[1 + \frac{\left(\frac{X_0}{X_{max}} - 1 \right)}{vX_{max} [1 - R]} \right]^{\frac{1}{\varphi}} - 1 \tag{13}$$

Rearranging Eq.(13),

$$v = \frac{P}{(n_f + 1)^{\varphi} - 1} \tag{14}$$

where,

$$P = \left(\frac{X_0}{X_{max}} - 1 \right) \left[\frac{1}{1 - R} \right]$$

The constants φ and v were calculated by plotting ' P ' and $(n_f + 1)^{\varphi} - 1$ for various trial values of φ and fitted by a straight line using least square fit. The slope of the straight line indicates the value of ' v '. Fatigue life and residual strength can be predicted using Eq. (10) and Eq. (13) with the constraint of the constants depending on the experimental Stress-Life curve.

The fatigue studies in the literature, point out to the absence of true fatigue limit for composites (Harris, 2003b; Montesano et al., 2013). Literature (Post, 2005; Harris, 2003b; Demers, 1998; Kaminski et al., 2015; Colombo et al., 2011; Tomblin and Seneviratne, 2011) also indicates that, the GFRP specimens have not failed up to 50000 cycles when the stress levels range between 30% to 60% of the ultimate tensile strength. Hence, in this paper, the fatigue limit for GFRP composites is considered as 50000 cycles, when the applied stress is equal to 30% of the ultimate tensile strength as given in Eq. (15). Also, when the applied stress is equal to the ultimate tensile strength, the number of cycles to failure is zero.

$$\text{At } X_{max} = 0.3X_0, n_f = 50000 \text{cycles} \tag{15}$$

$$\text{At } X_{max} = X_0, n_f = 0 \tag{16}$$

Applying the above boundary conditions and assuming the minimum possible value of ' φ ' as 1, the value of ' v ' is found to be 9.33×10^{-5} . Therefore, the final equation for finding fatigue life can be expressed as given in Eq. (17),

$$n_f = \frac{\left(\frac{X_0}{X_{\max}} - 1 \right)}{v[1 - R]} \quad (17)$$

The above Eq. (17) can be used for predicting the fatigue life of GFRP composites at different stress levels and different stress ratios without any experimental parameters. Further, the above model can be used for predicting the residual strength at regular intervals of cycles. The proposed two-parameter model given in Eq. (10) for the prediction of the residual strength depends on the experimental Stress-Life relationship, that is, the parameters can be calculated only if the details of experimental fatigue life at different stress levels are known. But, by using the fatigue life model proposed in Eq.(17), the fatigue life cycles can be calculated for different stress ratios and stress levels without any experimental testing. This can be further used for predicting the residual strength at regular intervals of cycles using the residual strength model proposed in Eq. (10). The proposed model (Eq.17) can be considered for predicting the minimum fatigue life cycles (cycles to failure) which can be used for the safe design of a component or structure because the least stress level i.e. 30% of ultimate tensile strength has been adopted for deriving the fatigue life model. The difference between the proposed fatigue life models as seen in Eq. (13) and Eq. (17) is that the former model (Eq. 13) which deduced from the residual strength model (Eq. 10) depends on experimental parameters whereas the latter model (Eq.17) is based on the fatigue endurance limit of the material.

3. EXPERIMENTAL INVESTIGATION

The composite material utilized in this study was made up of the E-GLASS woven fabric with Araldite LY556 and Hardener HY951. The fabrication of the GFRP composites was carried out by hand lay-up technique followed by the vacuum bagging method. Specimens of dimension of 250 mm \times 25 mm \times 2 mm as per the ASTM standard D3039/D3039M-17 (ASTM D3039, 2017), were cut from the rectangular GFRP composite plates using the water-jet cutting technique. The ensuing plates having an average thickness of about 2 mm were cured at 100° C for two hours and post-cured at room temperature for 24 hours. Quasistatic tensile experiments were conducted to examine the behavior of the ultimate tensile strength under diverse crosshead rates viz 0.5, 1, 1.5, 5 and 50 mm/min. Four samples were tested in each condition and the average values are stated here. The lowest tensile strength ensuing from different strain rates has been used as the ultimate tensile strength for executing the fatigue and residual strength tests. Tension-tension Fatigue and residual strength experiments (Table 1) were conducted on GFRP specimens for three different stress levels such as 55%, 65% and 75% of ultimate tensile strength using MTS Landmark Servo Hydraulic 250 KN machine according to ASTM D3479/D3479M-12 standard (ASTM D3479, 2012). Residual strength tests were accomplished at regular intervals of cycles as shown in Table 1 at all the stress levels. The GFRP specimens were subjected to loading and unloading cycles of a certain range. Following this, decrease in strength from the original condition was calculated as the residual strength of the specimen. The stress ratio and the frequency for carrying out the fatigue life and residual strength tests were selected as 0.5 (Imad, 1997; Zuluaga, 2013) and 3 Hz (Zuluaga, 2013).

Table 1: Details of the experimental investigation.

Test type	Testing conditions	No. of successful tests	
Tensile strength	Strain rate 0.5, 1.0, 1.5, 5 and 50 mm/min	20	
Fatigue life	55%, 65% and 75% of ultimate tensile strength	18	
Residual strength	55% of the ultimate tensile strength	2000 cycles	12
		4000 cycles	
		8000 cycles	
		16000 cycles	
	65% of the ultimate tensile strength	2000 cycles	9
		4000 cycles	
		8000 cycles	
	75% of the ultimate tensile strength	1000 cycles	9
		2000 cycles	
4000 cycles			

4. RESULTS AND DISCUSSION

4.1. Tensile behavior under different strain rates

Quasistatic tensile tests were conducted on GFRP specimens under different strain rates such as 0.5 mm/min, 1 mm/min, 1.5 mm/min, 5 mm/min and 50 mm/min. Four specimens in each category were tested and the average values were taken. The ultimate tensile strength and ultimate strain of GFRP composite specimens are summarized in Table 2. The lowest ultimate tensile strength of GFRP composite is 330 N/mm² which occurs at the strain rate of 0.5 mm/min. Figure 1 and Figure 2 show the ultimate tensile strength and ultimate strain respectively for various strain rates in logarithmic scale.

Table 2: Quasistatic Tensile Test Results.

Material	GFRP	
Strain rate (mm/min)	Ultimate Tensile Strength (N/mm ²)	Ultimate Tensile Strain (mm/mm)
0.5	330.3±16.96	0.044±0.002
1.0	345.5±9.54	0.052±0.001
1.5	380.2±27.02	0.058±0.01
5.0	405.8±42.72	0.075±0.01
50	432.4±23.03	0.091±0.01

The ultimate tensile strength of GFRP composites increases with strain rate from 0.5 to 50 mm/min. The increase in ultimate tensile strength was 22.7% and 6.67% when the strain rate increased from 0.5 to 5mm/min and 5 to 50 mm/min respectively. The regression equation to find the ultimate tensile strength of GFRP composites for various strain rate is given by $X_0 = u + v\varepsilon^w$ and $\varepsilon_0 = u + v\varepsilon^w$ where X_0 , ε_0 and ε^* are the ultimate tensile strength, tensile strain and strain rate respectively. The values of u, v, w and the correlation coefficient R are given in Table 3.

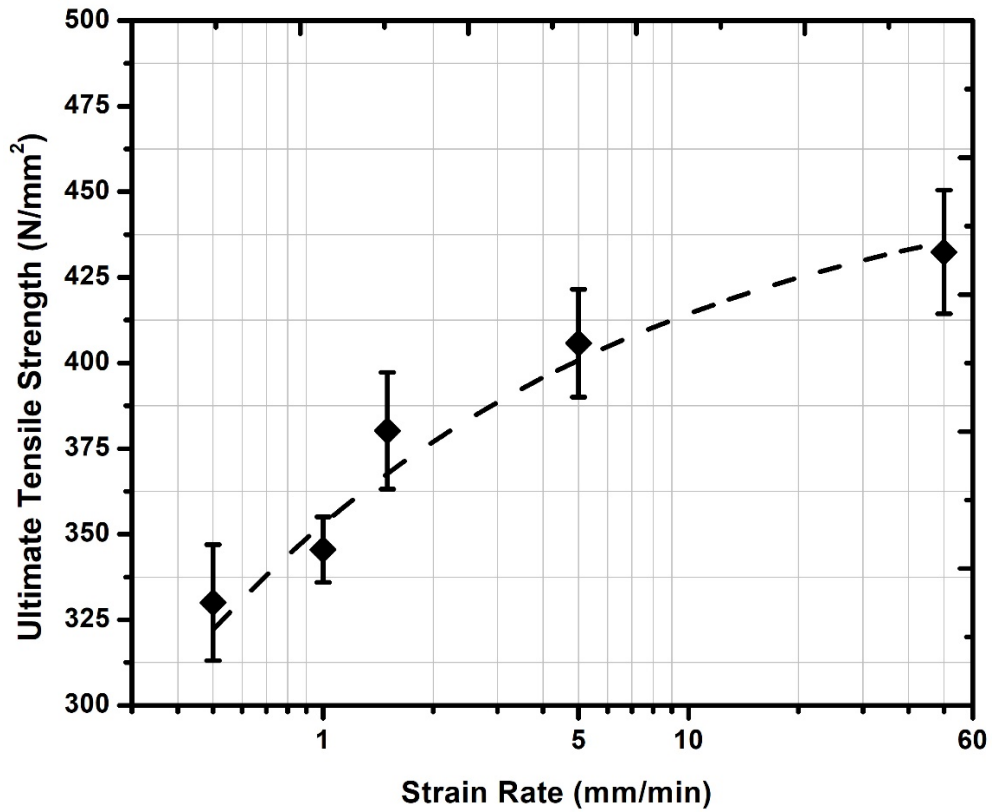


Figure 1: Ultimate Strength versus Strain Rate.

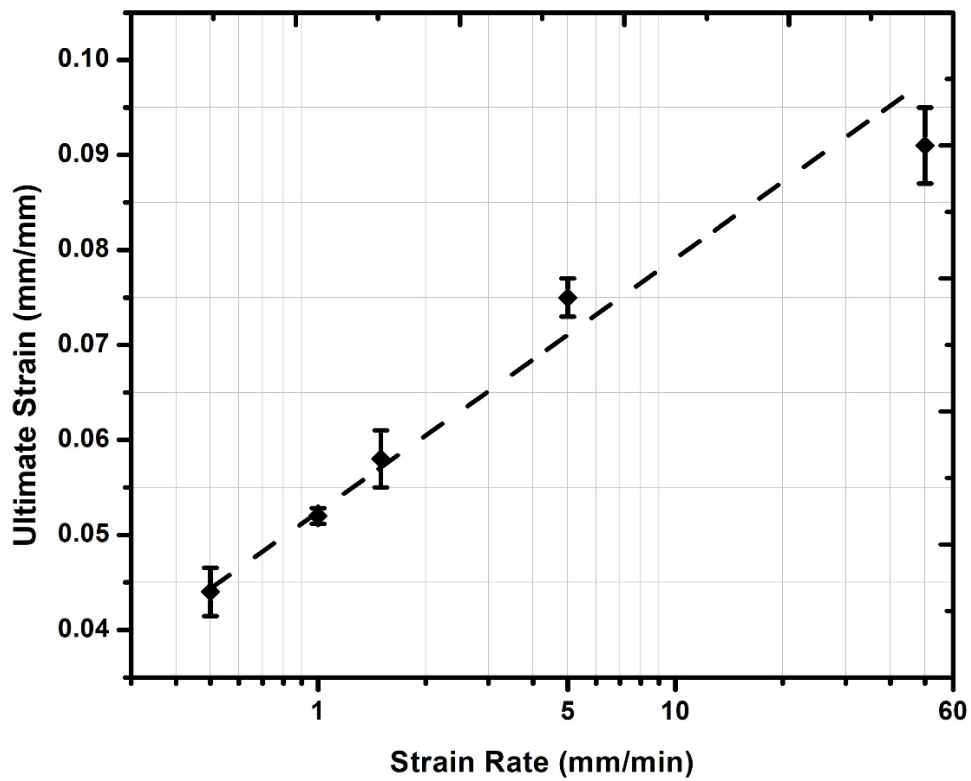


Figure 2: Ultimate Strain versus Strain Rate.

Table 3: Regression equation constants for Quasistatic tensile test results.

Material	u	v	w	R^2
Ultimate tensile strength	462.2	-109.27	-0.358	0.945
Ultimate tensile strain	-385.04	385	3.012×10^{-5}	0.933

4.2 Fatigue Results

Tension-Tension Fatigue tests were carried out at 55%, 65% and 75% of the ultimate tensile strength of the GFRP specimen with stress ratio ($R=0.5$) and the frequency 3 Hz. The lowest tensile strength of the GFRP composite was 330 N/mm² at 0.5 mm/min which was taken as the input for carrying out the fatigue life tests. Figure 3 shows the Stress-Life curve for GFRP specimens at three different stress levels. The number of cycles to failure is 19748, 8509 and 4252 when the stress levels are 55%, 65% and 75% of ultimate tensile strength respectively.

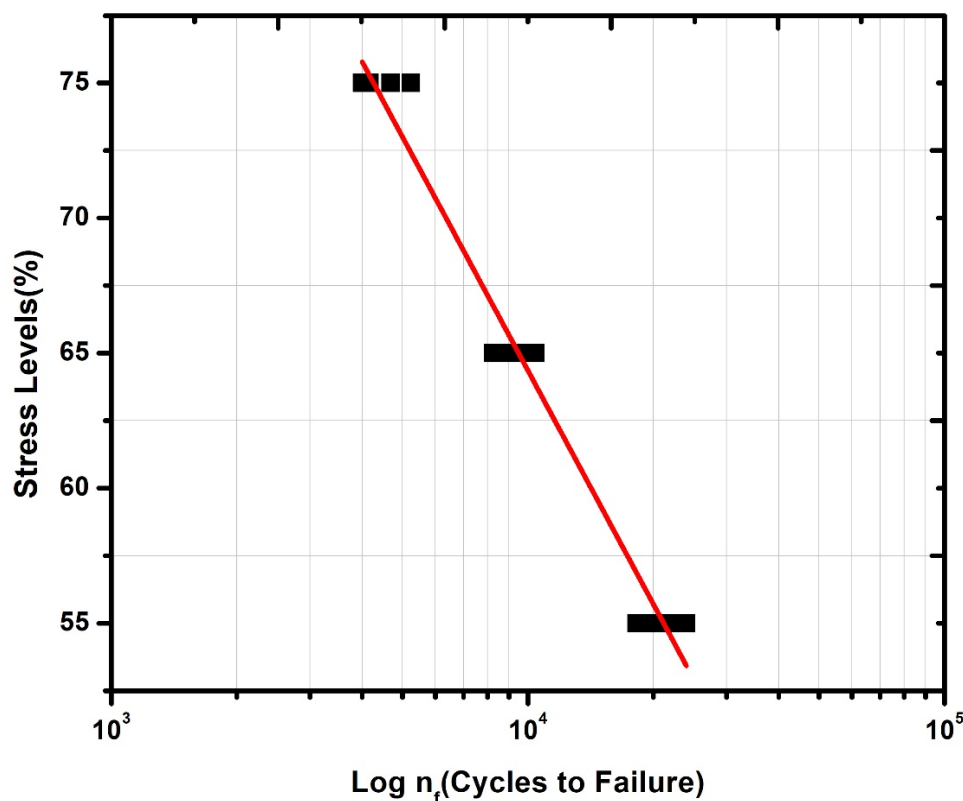


Figure 3: Experimental Stress-Life Curve.

The use of less stiff glass fiber allows large deformation in the matrix giving rise to early fatigue failure of GFRP composites. This is one of the reasons for the usage of GFRP composites in secondary structures such as luggage racks, floors, bulkheads and fairings. Fatigue life of GFRP composites can be assessed by the regression equation as given in Eq. (18),

$$\log n_f = S \log X_{\max} + T \tag{18}$$

Where,

n_f - Number of cycles to failure,

X_{\max} - Stress applied.

S, T - Constants

4.3 Residual Strength

Residual strength plays a major role in designing the service life of the components or structures. Hence, in this study, the GFRP composite specimens were subjected to a certain number of loading and unloading cycles for three different stress levels in MTS servo-hydraulic fatigue testing machine. Further, those specimens were tested in computerized Universal Testing Machine (UTM) to find its residual tensile strength. The experimental residual strength values were compared with the proposed residual strength model indicated in Eq.10 and also with the models seen in literature (Broutman and Sahu, 1972; Kassapoglou, 2012). Table 4, Table 5 and Table 6 provide the residual strength values obtained from the experiments conducted at 55%, 65% and 75% of ultimate tensile strength respectively along with the values calculated from the model (Eq.10) and with the models seen in literature.

Table 4: Residual Strength at 55% of Ultimate Tensile Strength.

Number of loading cycles (n)	Normalised cycles (n/n_f)	Residual Strength(N/mm ²)			
		Experimental	Present Model	Kassapoglou (2012)	Broutman and Sahu (1972)
2000	0.1012	278.21±13.83	288.76	310.00	314.96
4000	0.2025	259.39±9.59	268.93	293.15	299.92
8000	0.4051	231.02±11.28	239.70	258.00	269.84
16000	0.8102	192.6±10.99	196.59	203.25	209.68
19748	1	-	180.90	181.50	181.50

Table 5: Residual Strength at 65% of Ultimate Tensile Strength.

Number of loading cycles (n)	Normalised cycles (n/n_f)	Residual Strength(N/mm ²)			
		Experimental	Present Model	Kassapoglou (2012)	Broutman and Sahu (1972)
2000	0.2350	276.2±10.23	281.26	298.22	302.85
4000	0.4701	252.93±9.76	257.83	264.49	276.70
8000	0.9402	219.03±10.19	223.28	220.09	221.41
8509	1	-	214.50	214.50	214.50

Table 6: Residual Strength of at 75% of Ultimate Tensile Strength.

Number of loading cycles (n)	Normalised cycles (n/n_f)	Residual Strength(N/mm ²)			
		Experimental	Present Model	Kassapoglou (2012)	Broutman and Sahu (1972)
1000	0.2352	281.55±12.6	292.67	308.34	310.59
2000	0.4703	268.26±8.29	274.42	288.23	291.19
4000	0.9407	251.79±8.95	255.42	251.66	252.39
4252	1	-	247.50	247.50	247.5

Figure 4, Figure 5 and Figure 6 show a comparison of the residual strength versus normalized cycles for GFRP at 55%, 65% and 75% of ultimate tensile strength respectively. The curve shows some loss of strength at the beginning followed by a slow degradation in the middle stage and rapid loss in the final stage. From the figures (4, 5 and 6), it is clear that the behavior noted for the proposed model correlates well with the experimental results, also supported by high values of R^2 . The present residual strength model shows better prediction than the models seen in literature. This fact combined with the inclusion of stress ratio and applied stress makes it a useful and a strong rival for design purposes.

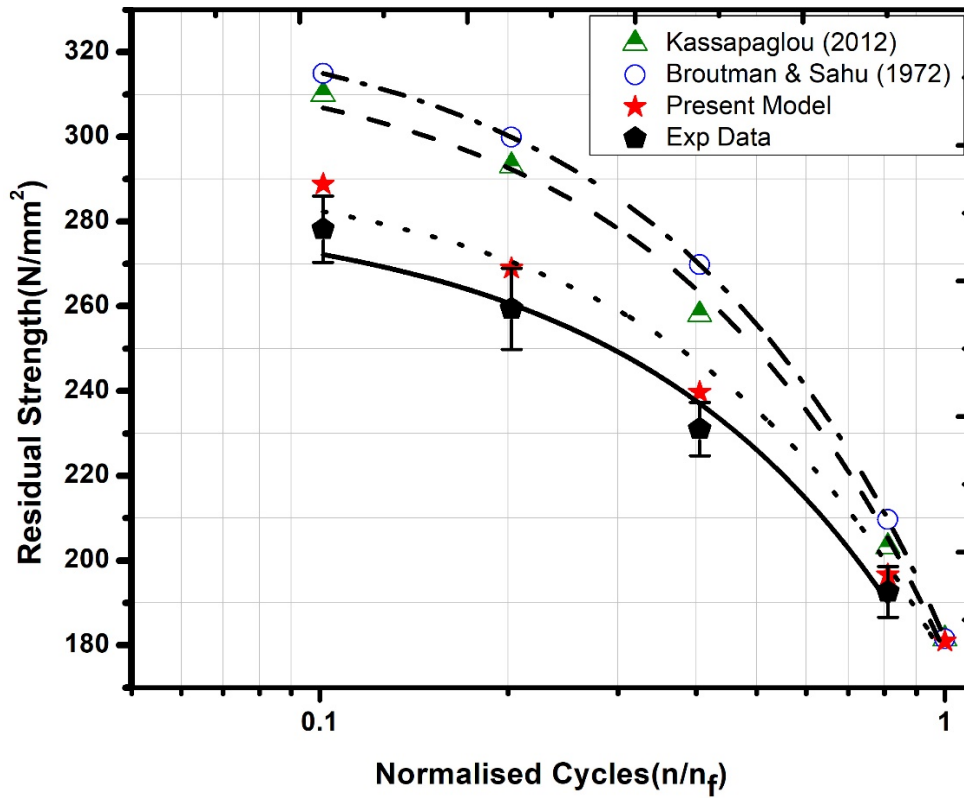


Figure 4: Residual Strength at 55% of Ultimate Tensile Strength.

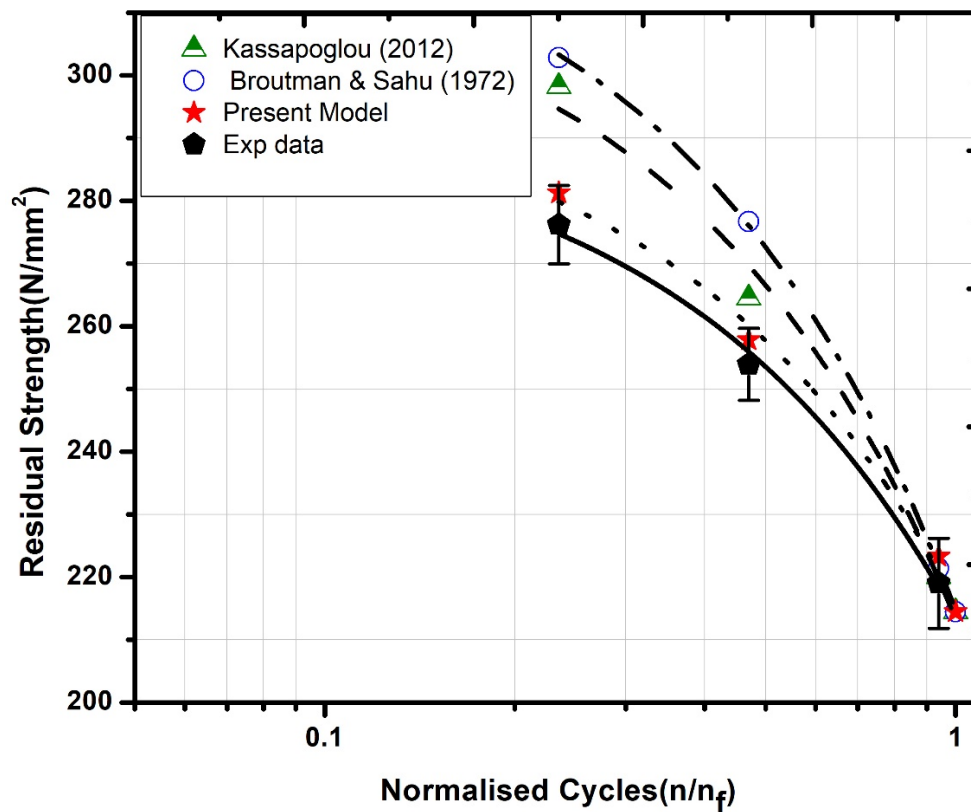


Figure 5: Residual Strength at 65% of Ultimate Tensile Strength.

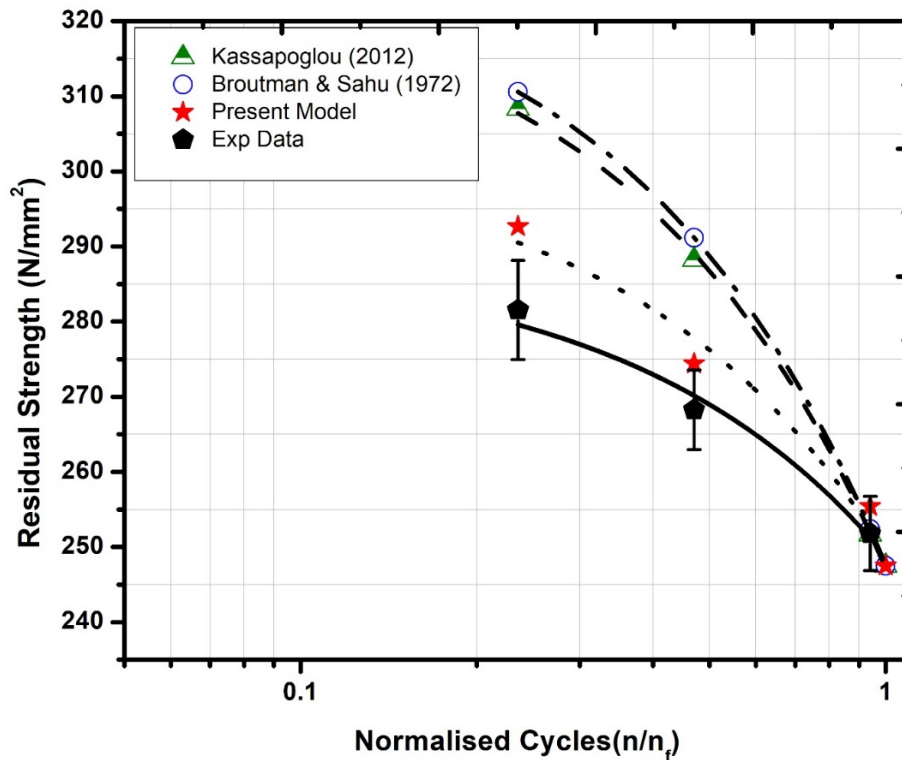


Figure 6: Residual Strength at 75% of Ultimate Tensile Strength.

Residual strength of GFRP composites can be assessed by the regression equation as given in Eq. (19),

$$X_r = c + d \left(\frac{n}{n_f} \right) \tag{19}$$

where

X_r - Residual Strength after ‘n’ number of cycles

n_f - Number of cycles to failure

c, d - Constants

The value of constants c, d and R^2 at different stress levels for both experimental and present model is given in Table 7.

Table 7: Regression equation constants for Experimental and Present model Residual Strength Results.

Stress levels	Material	$X_r = c + d \left(\frac{n}{n_f} \right)$		
		c	d	R^2
55%	GFRP (Experimental)	281.94	-106	0.969
	GFRP (Present model)	294.27	-117.7	0.987
65%	GFRP (Experimental)	293.36	-79.31	0.995
	GFRP (Present model)	299.43	-83.5	0.992
75%	GFRP (Experimental)	290.61	-42.08	0.9838
	GFRP (Present model)	303.24	-54.02	0.975

4.4 Fatigue life

Fatigue life of GFRP specimens at different stress levels predicted by experiments and by the proposed models (Eq. 10 and Eq.17) are listed in Table 8.

Table 8: Comparison of Fatigue Life.

Specimen	Stress Level (%)	No. of Cycles to Failure (Experimental)	No. of Cycles to Failure (Predicted)	
			Eq.13	Eq.17
1	55	19748	17351	17358
2	65	8509	7619	11542
3	75	4252	6063	7145

Figure 7 shows the experimental and predicted (Eq.13 and Eq.17) fatigue life curve for GFRP specimens. It is evident that fatigue life values predicted from Eq.13 show good agreement with experimental results. The reason for deviation in the results from the models Eq. (13) and Eq. (17) is that the former depends on the experimental stress-life curve whereas the latter depends only on the fatigue limit of the GFRP composite. The main advantage of the model (Eq.17) is the absence of dependence on any experimental data which save a lot of time and number of experiments to be tested, as the time limit and a large number of specimens to be tested are known as the main difficulty in conducting fatigue experiments.

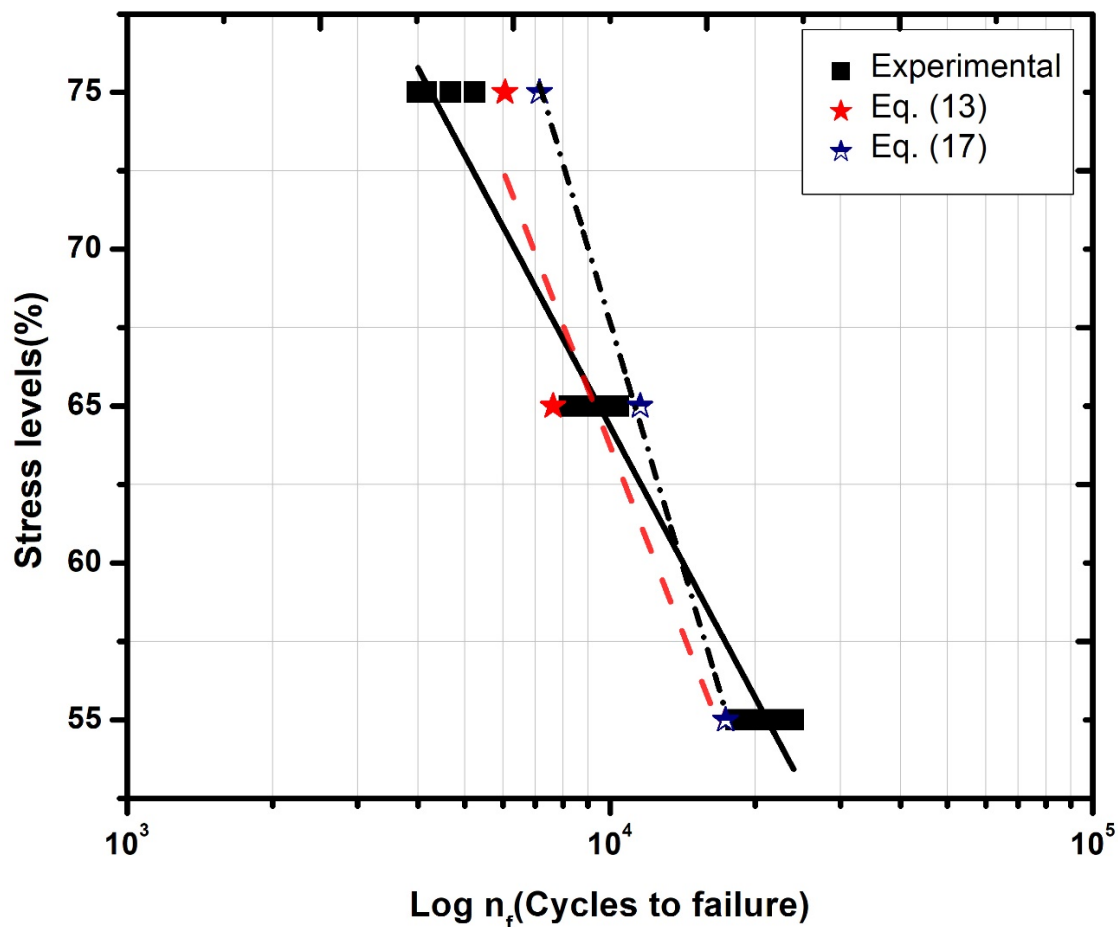


Figure 7: Stress-Life Curve for GFRP-Experimental and Predicted

5. CONCLUSIONS

Fatigue life and residual strength of GFRP composites have been computed experimentally and analytically in this paper. Tensile tests were conducted under different quasistatic strain rates and increase in tensile strength and strain with increasing strain rate was observed. Fatigue life and residual strength models have been proposed which include the stress ratio. Also, a simplified model is proposed for predicting the fatigue life of GFRP composites based on the fatigue endurance limit. The validity of the models proposed were compared with experimental results and models seen in literature were also referred. The proposed fatigue life model (Eq. 17) based on the endurance limit can predict the fatigue failure cycles without any experimental testing. The difference between the proposed fatigue life models (Eq. 13 and Eq. 17) is that the former (Eq. 13) is deduced from the residual strength model [Eq.(10)] which depends on experimental parameters whereas, the latter (Eq. 17) is based on the endurance limit of the material.

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