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Irregular vibrations in multi-mass discrete-continuous systems torsionally deformed

Abstract

In the paper irregular vibrations of discrete-continuous systems consisting of an arbitrary number rigid bodies connected by shafts torsionally deformed are studied. In the systems a local nonlinearity described by the polynomial of the third degree is introduced. It is assumed that the characteristic of the local nonlinearity is of a hard type. Governing equations are solved using the wave approach leading to equations with a retarded argument. Exemplary numerical calculations are done for the three-mass system. The possibility of occurring of irregular vibrations is discussed on the basis of the Poincaré maps, bifurcations diagrams and the exponents of Lyapunov.

Keywords

nonlinear dynamics, irregular vibrations, discrete-continuous systems, torsional systems, wave approach

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1 INTRODUCTION

The paper deals with nonlinear vibrations of discrete-continuous mechanical systems torsionally deformed with a local nonlinearity having the characteristic of a hard type. The systems consist of shafts with circular cross-sections connected by rigid bodies. Local nonlinearities, justified by many engineering solutions, are described by the polynomial of the third degree.

Regular vibrations in nonlinear multi-mass discrete-continuous torsional systems for the hard characteristic case are discussed in [5]. Irregular nonlinear vibrations including chaos are studied mainly in discrete systems, [1,2,4,12-15]. In the present paper an attempt to study irregular nonlinear vibrations in a discrete-continuous system is undertaken by the generalization of the approach used in [12-14] for discrete systems.

Governing equations for multi-mass discrete-continuous systems torsionally deformed are derived in [5,6] including the local nonlinearities having the characteristic of a hard as well as of a soft type. In the studies a wave approach leading to solving equations with retarded argument is used, [5-7].

In [8,9] irregular nonlinear vibrations in discrete-continuous systems torsionally deformed with local nonlinearities having a soft type characteristic are discussed. Here similar considerations are presented for systems having the hardening characteristics of the local nonlinearities. Numerical results are presented for the three-mass system. The possibility of occurring of irregular vibrations is discussed on the basis of bifurcation diagrams and the Poincaré maps. Exemplary diagrams of the exponent of Lyapunov are also given.

2 ASSUMPTIONS, GOVERNING EQUATIONS

The discrete-continuous system discussed in the paper is shown in Fig. 1.The i-th shaft in a multi-mass system, $i = 1,2,...,N$, is characterized by length l_i, density ρ, shear modulus G and polar moment of inertia I_{0i}, [5,6]. The mass moment of inertia of rigid bodies, $i = 1,2,...N+1$, are J_i. The first rigid body J_1 is loaded by the harmonic moment $M(t) = M_0 \sin pt$, where M_0 and p are the amplitude and frequency of the external moment, correspondingly. A local nonlinear discrete element, described by the polynomial of the third degree, with a hardening characteristic is located in the cross-section $x = 0$. Equivalent external and internal damping, having coefficients d_i and D_i, are taking into account in appropriate cross-sections. It is assumed that displacements and velocities of the shaft cross-sections are equal to zero at time instant $t = 0$.

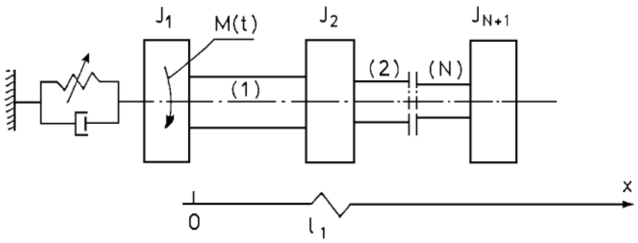


Figure 1 Multi-mass system torsionally deformed.

The determination of angular displacements θ_i of shaft cross-sections, in appropriate nondimensionless quantities given [5,6], is reduced to solving N equations

$$\theta_{i,tt} - \theta_{i,xx} = 0, \qquad i = 1,2,...,N \tag{1}$$

with the following nonlinear boundary conditions

$$\begin{gathered}
M_0 \sin pt - \theta_{1,tt} + K_r(D_1\theta_{1,xt} + \theta_{1,x}) - d_1\theta_{1,t} - k_1\theta_1 - k_3\theta_1^3 = 0 \quad \text{for} \quad x = 0, \\
\theta_i(x,t) = \theta_{i+1}(x,t) \quad \text{for} \quad x = \sum_{k=1}^{i} l_k, \quad i = 1,2,...,N-1, \\
-\theta_{i,tt} - K_r B_i E_{i+1}(D_i\theta_{i,xt} + \theta_{i,x}) + K_r B_{i+1} E_{i+1}(D_{i+1}\theta_{i+1,xt} + \theta_{i+1,x}) - E_{i+1} d_{i+1}\theta_{i,t} = 0 \\
\text{for} \quad x = \sum_{k=1}^{i} l_k, \quad i = 1,2,...,N-1, \\
-\theta_{N,tt} - K_r B_N E_{N+1}(D_N\theta_{N,xt} + \theta_{N,x}) - E_{N+1} d_{N+1}\theta_{N,t} = 0 \quad \text{for} \quad x = \sum_{k=1}^{N} l_k,
\end{gathered} \tag{2}$$

and with zero initial conditions. Comma denotes partial differentiation.

The solutions of equations (1) are sought in the form

$$\theta_i(x,t) = f_i(t-x) + g_i(t + x - 2\sum_{k=1}^{i-1} l_k), \quad i = 1,2,...,N \tag{3}$$

Substituting (3) into the boundary conditions (2) we obtain the following set of ordinary non-linear differential equations with a retarded argument for unknown functions f_i and g_i

$$\begin{gathered} r_{N+1,1}g_N''(z) + r_{N+1,2}g_N'(z) = r_{N+1,3}f_N''(z-2l_N) + r_{N+1,4}f_N'(z-2l_N), \\ g_i(z) = f_{i+1}(z-2l_i) + g_{i+1}(z-2l_i) - f_i(z-2l_i), \quad i = 1,2,....,N-1, \\ r_{11}f_1''(z) = M(z) + r_{12}g_1''(z) + r_{13}f_1'(z) + r_{14}g_1'(z) - M_{sp}(z) \\ r_{i1}f_i''(z) + r_{i2}f_i'(z) = r_{i3}g_i''(z) + r_{i4}g_i'(z) + r_{i5}f_{i-1}''(z) + r_{i6}f_{i-1}'(z), \quad i = 2,3,...,N, \end{gathered} \tag{4}$$

where

$$\begin{gathered} r_{11} = K_r D_1 + 1, \qquad r_{12} = K_r D_1 - 1, \\ r_{13} = -K_r - d_1, \qquad r_{14} = K_r - d_1, \\ r_{i1} = K_r E_i (B_i D_i + B_{i-1} D_{i-1}) + 1, \quad r_{i2} = E_i [K_r (B_i + B_{i-1}) + d_i], \\ r_{i3} = K_r E_i (B_i D_i - B_{i-1} D_{i-1}) - 1, \quad r_{i4} = E_i [K_r (B_i - B_{i-1}) - d_i], \\ r_{i5} = 2K_r B_{i-1} E_i D_{i-1}, \qquad r_{i6} = 2K_r B_{i-1} E_i, \qquad i = 2,3,...,N, \\ r_{N+1,1} = K_r B_N E_{N+1} D_N + 1, \qquad r_{N+1,2} = E_{N+1}(K_r B_N + d_{N+1}), \\ r_{N+1,3} = K_r B_N E_{N+1} D_N - 1, \qquad r_{N+1,4} = E_{N+1}(K_r B_N - d_{N+1}). \end{gathered} \tag{5}$$

Nonlinear equations (4) are solved numerically by means of the Runge-Kutta method. In the case of the local nonlinearities having the hard characteristic, $k_3 > 0$, such equations can be solved numerically with zero or nonzero initial conditions. It should be pointed out that ordinary differential equations with shifts in the arguments of unknown functions have an attention in the literature, eg., in [3].

3 NUMERICAL ANALYSIS

The aim of the numerical analysis is to study the possibility of occurrence of irregular vibrations in discrete-continuous systems considered. This is done on the basis of the bifurcation diagrams and the Poincaré maps for the three-mass torsional system, characterized by the following basic parameters: $N = 2$, $l_1 = l_2 = 1$, $B_1 = B_2 = 1$, $E_2 = E_3 = 0.8$, $K_r = 0.05$, $k_1 = 0.05$,

$k_3 = 0.005$, [5,11]. The three first natural frequency for linear the system are $\omega_1 = 0.089$, $\omega_2 = 0.261$ and $\omega_3 = 0.376$.

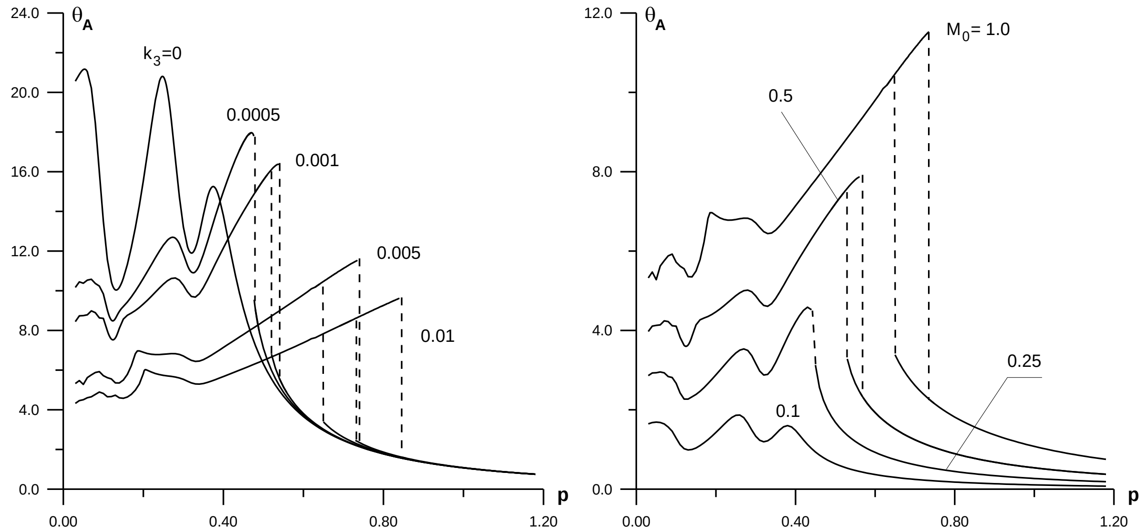


Figure 2 The effect of parameters k_3 and M_0 on displacement amplitudes.

Cases when solutions are harmonic vibrations with the period equal to the period of the external loading are presented in Fig. 2. They are given for large damping having coefficients equal $d_0 = d_i = D_i = 0.1$ and show the effect of the parameter k_3 with $M_0 = 1$ and the effect the amplitude of the external moment with $k_3 = 0.005$. Nonlinear effects are observed in the first three resonant regions, similarly to other results given in [5]. Especially, it is seen that in the third resonant region nonlinear effects have the form of amplitude jumps. Two amplitude jumps are observed. They correspond to zero and nonzero initial conditions, respectively. From diagrams in Fig. 2 it follows that distances between jumps increase with the increase of the parameter k_3 representing the local nonlinearity and with the increase of M_0. It appears that distances of jumps increase also with the decrease of damping, [5].

Further numerical results are presented in Figs. 3 - 7. They concern the amplitude of the external moment equal $M_0 = 1$ and $k_3 = 0.05$, however small damping, i.e., all damping coefficient are equal $d_0 = 0.001$. In Fig. 3 bifurcation diagrams are shown for the angular displacement as well as for the angular velocity in the cross-section $x = 0$. From these diagrams it follows that irregular vibrations can be expected for the frequency of the external moment $p < 1.2$. In bifurcations diagrams 100 periods of the solutions are taken into account.

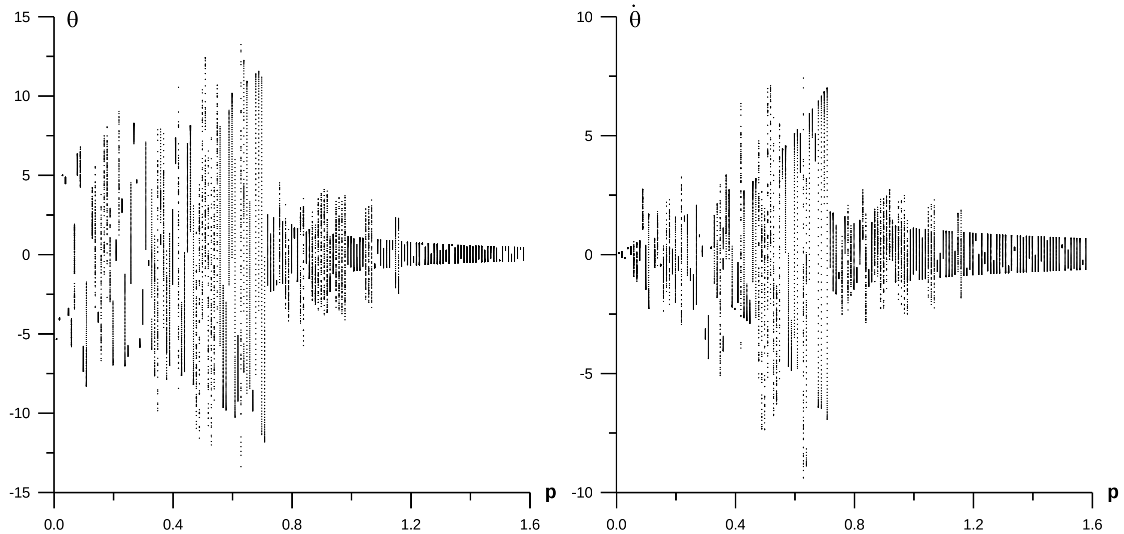
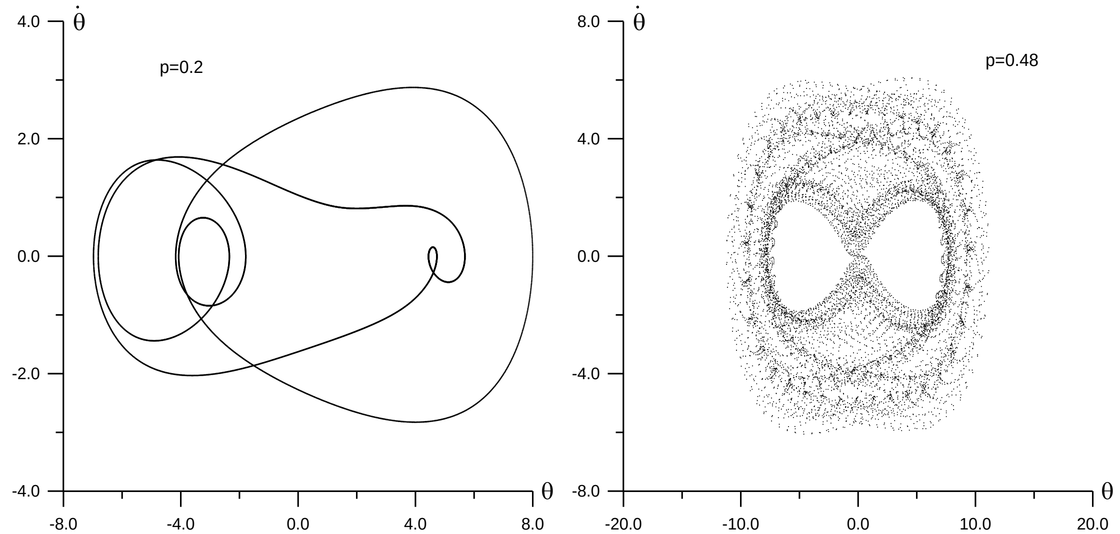


Figure 3 Bifurcation diagrams for angular displacements and velocities with $d_0 = 0.001$.

In Fig. 4 the Poincaré maps are presented for selected frequency p of the external moment, namely equal to $p = 0.2,\ 0.48,\ 0.78,\ 0.83$ with damping coefficients $d_0 = d_i = D_j = 0.001$ and the amplitude of the external moment $M_0 = 1$. One can see that the Poincaré maps have various shapes, in the dependence of the frequency p of the external moment $M(t)$.

In Fig. 4 strange atractors also are noticed. For this reason in the case of $p = 0.78$ and $p = 0.83$ maximal exponents of Lyapunov are checked. From Fig. 5 it follows that maximal exponents of Lyapunov are positive, so motions in these cases are chaotic. From the bifurcation diagrams as well as from detailed calculations with $\Delta p = 0.01$ it was found that irregular vibrations in the studied three-mass system can occur for frequency $p \le 1.15$.



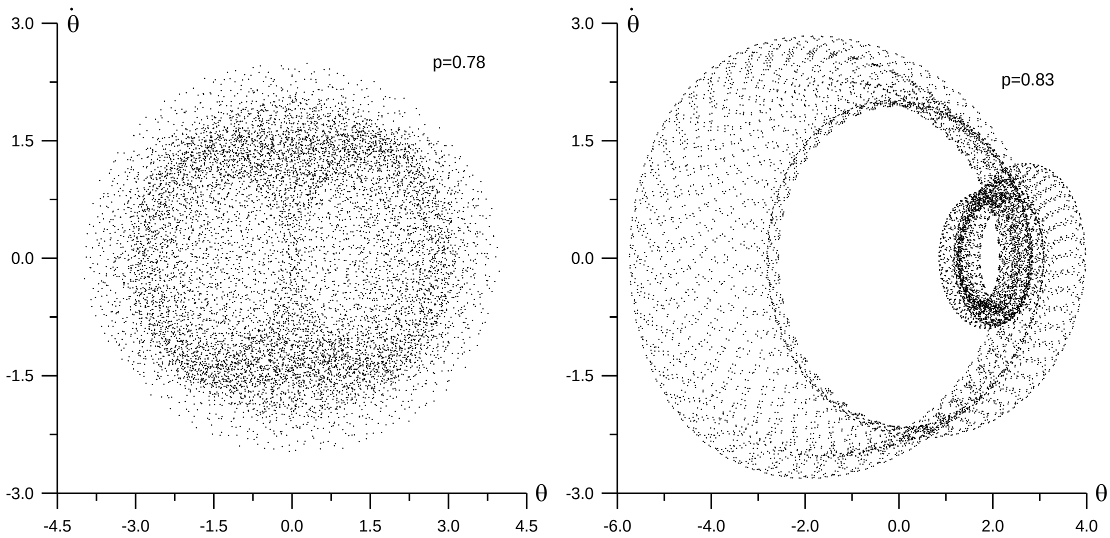


Figure 4 Poincaré maps for $p = 0.2,\ 0.48,\ 0.78,\ 0.83.$

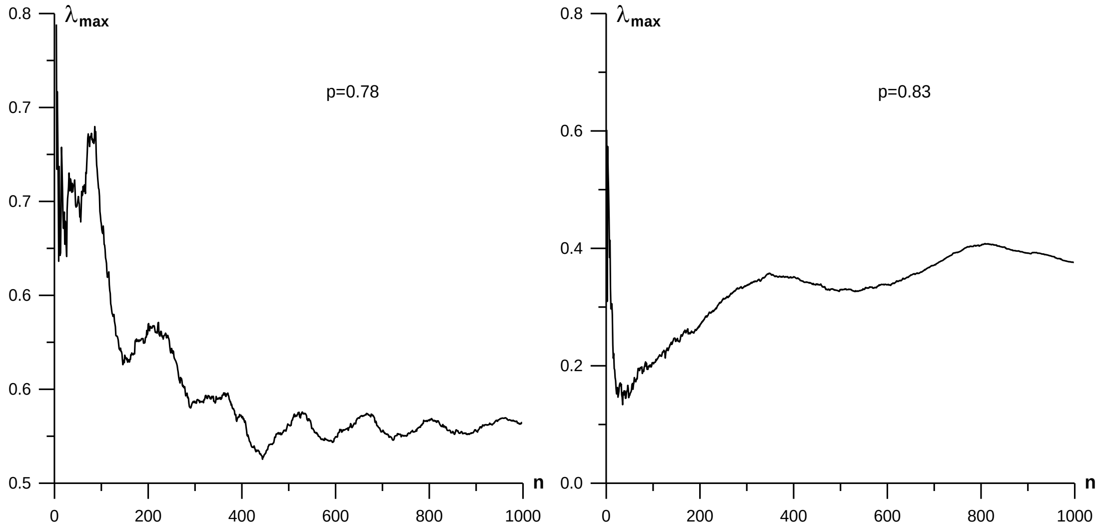


Figure 5 Maximal Lyapunov exponents for angular displacements with $p = 0.78,\ 0.83.$

The numerical results presented in Figs. 3 - 5 concern appropriate solutions in the cross-sections $x = 0$. Elastic elements in discrete-continuous systems have finite length. The wave approach applied in the papers allows to determine simultaneously solutions in required cross-sections of shafts.

In Fig. 6 the Poincaré maps for the frequency $p = 0.83$ and cross-sections $x = 0,\ 0.5,\ 1.0,\ 1.5,\ 2.0$ are presented with damping coefficients equal to $d_0 = 0.1$. From these diagrams it follows that the maximal angular velocities decrease with the increase of x.

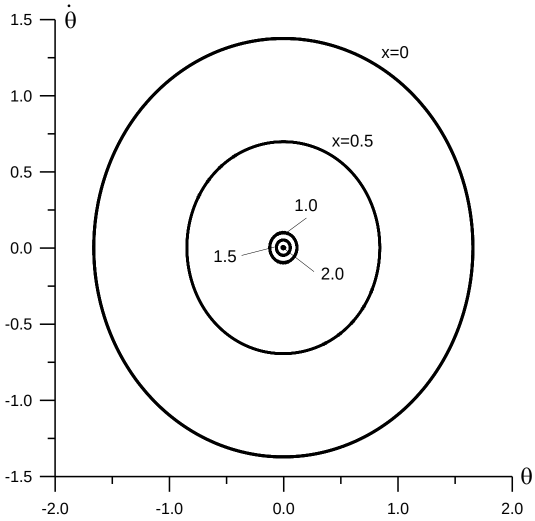


Figure 6 Poincaré maps for $p = 0.83$ in cross-sections $x = 0, 0.5, 1.0, 1.5, 2.0$ for $d_0 = 0.1$.

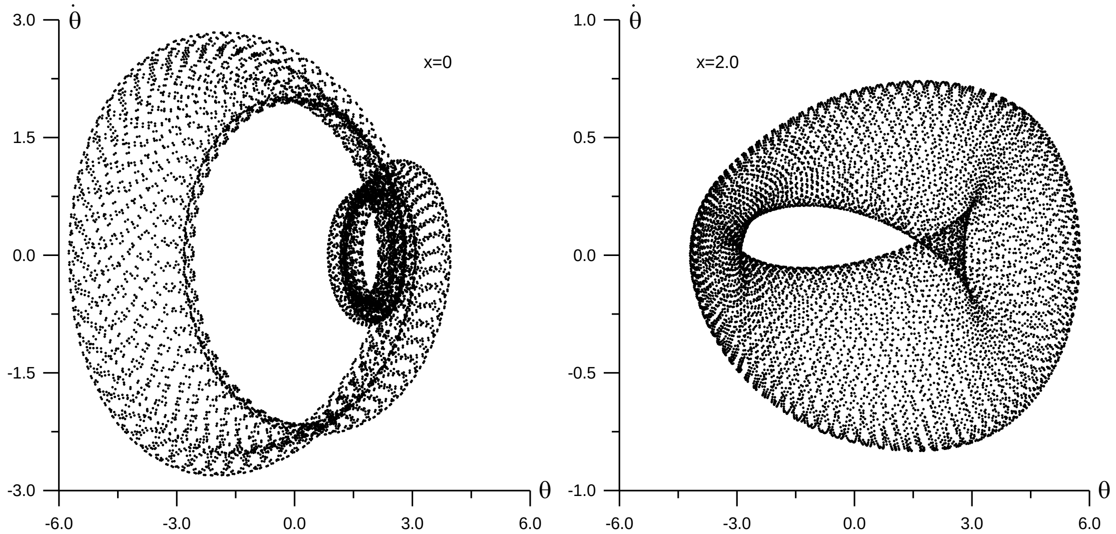


Figure 7 Poincaré maps for $p = 0.83$ in cross-sections $x = 0, 2.0$ for $d_0 = 0.001$.

The Poincaré maps for $p = 0.83$ and cross-sections $x = 0,\ 2.0$ with $d_0 = 0.001$, shown in Fig. 7, inform that diagrams have quite different shapes in the each considered cross-section.

The above numerical results concern the three-mass torsional system with the local nonlinearity having the characteristic of a hard type. Similar considerations were carried out in [10] in the case of a two-mass system. The possibility of occurrence of irregular vibrations were also done on the basis of the bifurcation diagrams and the Poincaré maps.

4 CONCLUSIONS

From the considerations in the paper it follows that in discrete-continuous systems torsionally deformed with a local nonlinearity having a hardening characteristic and loaded by the external moment harmonically changing in time, regular and irregular vibrations may appear. Different kinds of irregular vibrations including chaotic vibrations can be found in the limited range of the change of the parameters representing the system and the external moment. Presented numerical calculations concern the three-mass system, however governing equations allow us to widen considerations to other discrete-continuous systems. Exemplary diagrams show bifurcation diagrams and variety of Poincaré maps. Chaotic motions noticed for certain frequencies of the external moment are justified by the positive values of the maximal exponents of Lyapunov.

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