STATIONARY DYNAMIC RESPONSE OF A CIRCULAR RIGID FOUNDATION PARTIALLY SUPPORTED BY A FLEXIBLE PILE AND INTERACTING WITH A HALF-SPACE

Luis Filipe do Vale Lima[a]* (iD), **Josue Labaki[b]** (iD), **Euclides Mesquita[a]** (iD)

[a] Departamento de Mecânica Computacional, Faculdade de Engenharia Mecânica, Universidade Estadual de Campinas, Rua Mendeleyev 200; 13083-970, e Centro de Pesquisa em Engenharia e Ciências Computacionais CCES-Unicamp, Campinas, SP, Brasil. E-mail: lfvlima@fem.unicamp.br, euclides@fem.unicamp.br
[b] Departamento de Sistemas Integrados, Faculdade de Engenharia Mecânica, Universidade Estadual de Campinas, Rua Mendeleyev 200; 13083-970 Campinas, SP, Brasil. E-mail: labaki@fem.unicamp.br
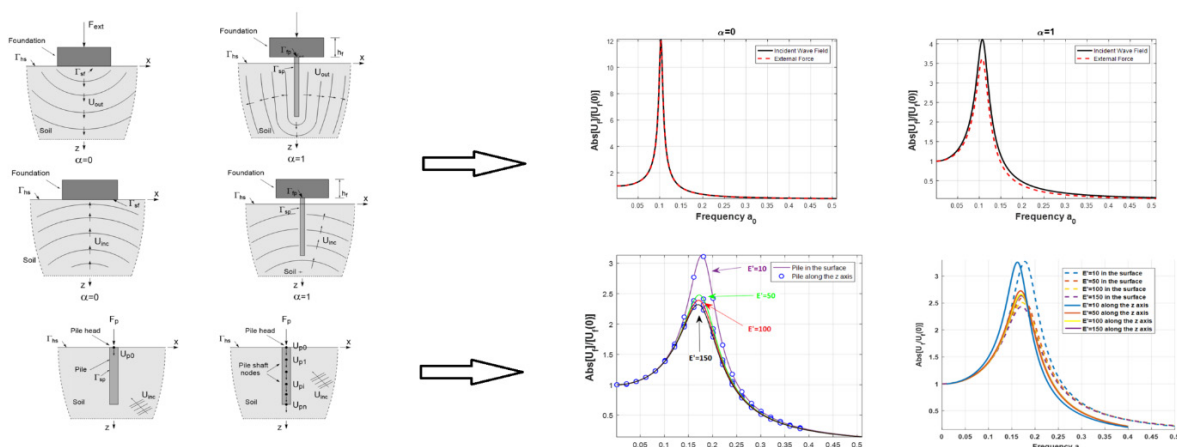
* Corresponding author

Abstract

This article describes an analysis of the dynamic response of coupled soil-pile-foundation systems. The soil and the pile models were developed in previous research works. This article describes the influence of three issues on the vertical dynamic foundation response. First, the influence of the foundation bearing mechanism is investigated. The foundation may be supported by the soil, by the pile or by a combination of both supporting mechanisms. A parameter is introduced to allow for a continuous change in the foundation supporting mechanism. Second, the influence of the excitation mechanism is investigated. The considered excitation mechanism are external forces applied directly at the foundation and an incoming wave field impinging the soil-pile-foundation system. Third, the article investigates if, for both excitation mechanism, the response at the pile head only is able to describe properly the soil-pile response. The analysis presented in the article contributes to an in-depth understanding of the dynamic response of coupled soil-pile-foundation systems.

Keywords

Dynamic Soil-Structure Interaction, Foundation Dynamics, Pile Dynamics, Soil-pile-foundation response.

Graphical Abstract

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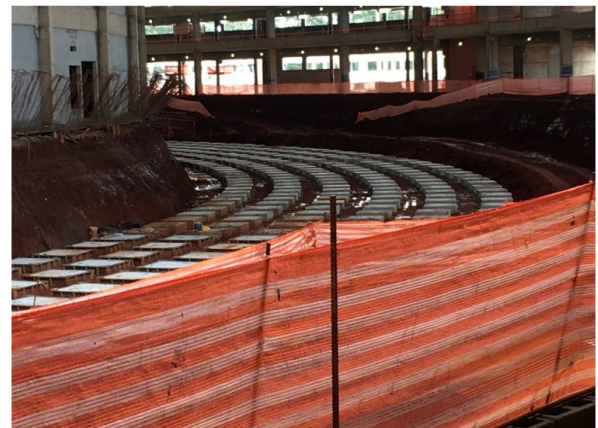
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1 INTRODUCTION

This article reports a study on the stationary dynamic behavior of soil-pile-foundation systems. The motivation of the present article is explained in Figures 1 and 2. Figures 1a) and 1b) show two partial views of the pile and pile caps for the actual foundations of the New Brazilian Synchrothron Light Source Laboratory, Sirius (Silva, 2018). A simplified scheme of the foundation system is shown in Figure 2. The foundation slab is resting on a soil-cement layer, which in turn is supported by more than 1300 piles. Considering that the soil-cement layer is not completely rigid, part of the system bearing response comes from the pile or pile caps, but there may be some soil reaction forces between pile caps. It is not simple to model and quantify how much bearing capacity is due to direct soil response. So, the idea is to develop a numerical model that allows to analyze the vertical dynamic response of the foundation slab partially supported by piles and partially supported by direct contact with the soil surface among the piles. The model that will be described in session 2 allows the soil-pile bearing response to be continuously changed from the pile-only response to the soil-only reaction. The influence of the change in this foundation bearing mechanism on the stationary response of the foundation slab is the main objective of this work.

Furthermore, the investigation addresses the role of external and internal excitation sources on the foundation response. An excitation is considered to be external when it is applied directly to the foundation slab. These loads are usually created by the equipment installed on the slab. An internal excitation is created by incoming waves that travel through the soil and impinge the soil-pile-foundation system. The article investigates whether the dynamic behavior due to external forces present the same rationale for the case of vertical incident wave fields. The article also investigates if the pile response may be considered only at the center of the pile cap or if the response pile response must consider the modelling of the completely embedded pile within the soil.

a) Piles and pile caps   b) Pile caps

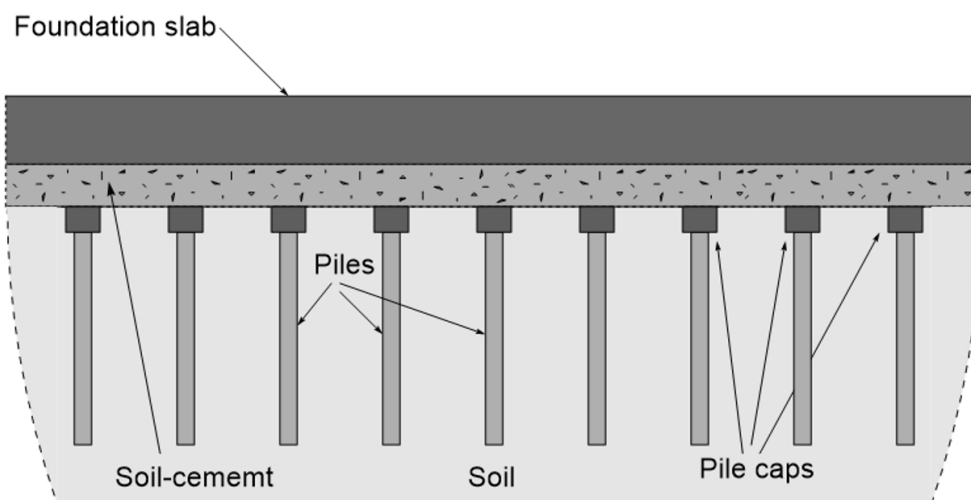**Figure 1**: Partial view of the foundations of the Sirius synchrothron light lab

Figure 2: Simplified foundation scheme for Syrius lab

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It is well established that the soil has an important effect on the dynamic response for foundations and foundation arrangements that may include piles. The solution of coupled soil-structure problems cannot be accomplished through analytical techniques and either a mix of analytical-numerical or purely numerical methods are required (Hall and Oliveto, 2003). Time and frequency domain dynamic response of foundations interacting with distinct soil profiles have been analyzed by many authors, including (Idriss et al., 1979; Wong and Luco, 1976; Gazetas, 1983; Wolf, 1985; Spyrakos and Beskos, 1986). One important topic in the Dynamic Soil Structure Interaction (DSSI) research is the study of coupled soil-pile-foundation systems. Fowler and Sinclair (1978) and Rajapakse and Shah (1987) presented the frequency domain response of a circular bar embedded in an elastic half-space. Barros (2006) and Lima et al. (2016) showed that the presence of a pile has a significant impact in reducing vibration amplitude of a foundation system subject to external excitation. Taherzadeh et al. (2009) presented a dynamic analysis of pile groups in which approximate formulas to describe the piles cross interaction were given. The dynamic response of a pile embedded in a poroelastic half-space has been presented by Zeng and Rajapakse (1999). The frequency domain vertical response of a single pile embedded in a poroelastic layered media has been presented by Lu et al. (2009). An extension to include the dynamic analysis of pile groups in poroelastic layered half-spaces has been authored by Xu et al. (2011). Labaki, et al. (2019) studied the horizontal and rocking vibratory response of a piled raft foundation using boundary element method (BEM) to analyze the effects of foundation and soil parameters under external and seismic excitations.

In the last years, studies on piled foundations have discussed issues as monopile foundations supports, pile groups, poroelastic and liquefiable soil models, offshore and wind turbines foundations. A numerical study on the static behavior of four different foundation schemes for wind turbines was provided by Pam et al. (2018). The study of pile groups in offshore applications for undrained soil conditions and static loading was presented by Mali and Singh (2018). The dynamic response of monopile including soil-structure interaction effects have been performed by Lombardi et al. (2013) and Gávin et al. (2017). The dynamic response of offshore wind turbines under the action of wind, waves and earthquake and supported by monopile embedded in clay has been discussed by Wang et al. (2018). A dynamic analysis of pile and raft responses to seismic loads in which the piles are not rigidly connected to the raft was presented by Saadatinezhad et al. (2019). More recently Kwon and Yoo (2020) presented a study on the dynamic response of piles interacting with liquefiable soils, showing that this is very actual research topic.

After this introduction, session 2 describes the statement of the problem, raising the questions the article tries to answer. Session 3 describes the models used in the article. The basic elements such as the half-space and the pile models leading to the respective dynamic flexibility matrices are addressed. The coupling of the three main components, the soil, the pile and the foundation is also described. Formulations for external forces, and internal excitations, incoming wave fields, are given. Session 4 brings the numerical results. Initially a dynamic and a static validation for the soil-foundation and pile-foundation are presented. After validation, a major study on the influence of the foundation bearing mechanism under both excitation sources is presented. Parametric studies investigating the role of the mass ratio between pile and foundation, the relative stiffness and density of soil and pile are discussed. The role of the scatter mechanism at the pile head or along the whole pile axis on the foundation response is also given. Session 5 furnishes the mains conclusions of the numerical study.

2 STATEMENT OF THE PROBLEM

The model conceived to analyze the dynamic response of the soil-pile-foundation system discussed in the previous session is shown in Figure 3. There are three main components in the model, namely, the soil, the pile and the rigid foundation. In this model the index 'hs' is related to the soil, assumed to be a three-dimensional homogeneous, isotropic half-space, with shear modulus G_{hs}, Poisson's ratio v_{hs}, mass density ρ_{hs} and material damping coefficient η_{hs}. The index 'p' is related to the pile, with Young's modulus E_p, radius a_p and length h_p, mass density ρ_p and total mass M_p. The index 'f' is related to the rigid circular foundation with mass M_f, radius a_f and height h_f. The half-space free surface is designated by Γ_{hs}. The soil-rigid foundation interface is Γ_{sf} and the pile-foundation interface or point of contact is called Γ_{fp}. The soil-pile interface is named Γ_{sp}. The coordinate system is placed so that the x-y plane is aligned with the surface of the soil, and the pile is aligned along the z-axis. The center of the coordinate system coincides with the geometric center of the circular foundation at the same level of the half-space surface. The center of the soil-foundation interface is also the connecting point between pile and foundation. The vertical displacement of the rigid foundation is U_f. The vertical displacement of the pile head is designated by U_{p0}. The soil vertical displacement at the origin of the coordinate system is U_{hs0}.

The system may be subjected to external excitation F_{ext} or an incident vertical wave field U_{inc}. The forces between pile and foundation are designated by F_{p0} and act at the surface (point) Γ_{fp}. The forces between the soil and the

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foundation area directly interfacing the soil, Γ_{sf}, is designated by F_{hs0}. The total soil and foundation reaction applied to the soil- and pile-foundation interface ($\Gamma_{sf} + \Gamma_{fp}$) and designated by F_{rea} is, consequently, composed of the two interface forces F_{p0} and F_{sh0}.

2.1 Influence of bearing mechanism

Regarding the total interaction force at the soil- and pile-foundation F_{rea}, three distinct possibilities are investigated in this article. In the first case, the foundation is completely supported by the soil reaction, so that $F_{hs0}=F_{rea}$, and $F_{p0}=0$. In the second case, the foundation is solely supported by the pile, leading to $F_{hs0}=0$ and $F_p=F_{rea}$. The third possibility is the case in which the total reaction on the foundation, F_{rea}, is composed by the soil and pile reaction forces. The model will allow a continuous change in the composition of F_{rea}. To model this effect, a dimensionless factor α is introduced (Equation 1) in order to allow the bearing mechanism, the reaction forces F_{rea}, to shift continuously from the first condition, i.e., all reactions act at the soil-foundation interface Γ_{sf} ($\alpha=0$), to the second extreme, all soil-pile reactions are concentrated at the pile head ($\alpha=1$). The influence of the mixed bearing mechanism can be captured in the model by varying the value of α. These conditions are schematically shown in Figure 4.
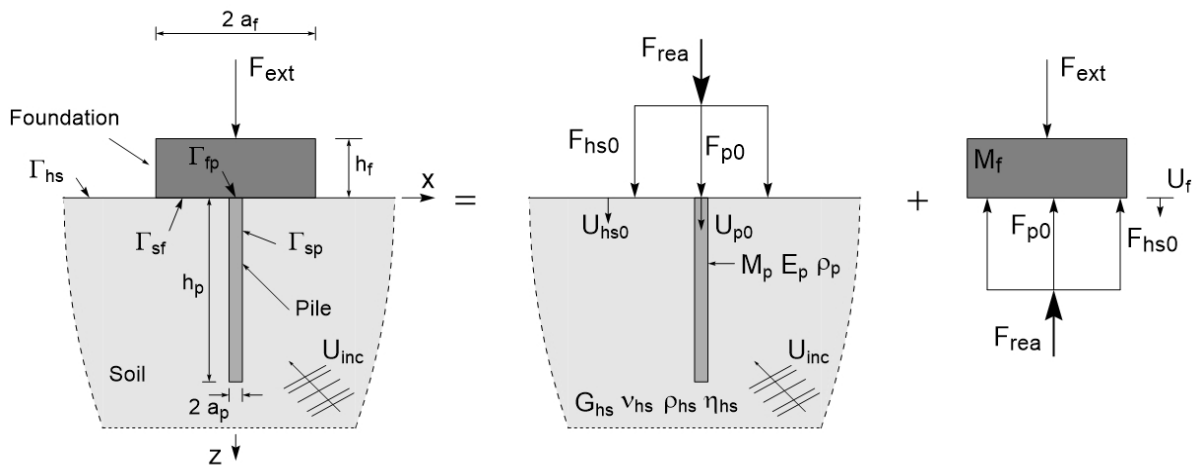
$$F_{rea} = (1-\alpha) F_{hs0} + \alpha F_{p0}$$

(1)
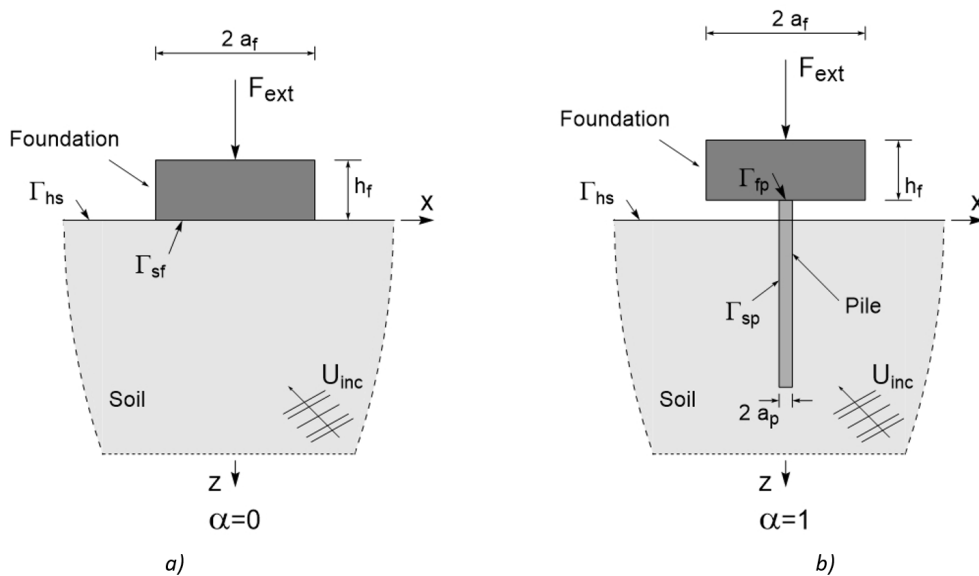

Figure 3: Pile-soil-foundation model – definitions

Figure 4: Bearing mechanisms – the limiting cases.

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2.2 Influence of excitation sources

A second question is investigated in this article, namely, the influence of the pile presence on the foundation response when distinct excitation mechanisms are considered an external force F_{ext} or an incoming wave field U_{inc} impinging upon the soil-pile-foundation system.

The rationale of the question being investigated is illustrated in Figures 5 and 6. Consider two systems. The first one is the soil-foundation (Figure 4a) and the second one is the soil-pile foundation system (Figure 4b). Consider the case of a dynamic external excitation force F_{ext} acting directly upon the foundation. In both systems, there will be reacting forces created either at the soil-foundation interface Γ_{sf} (Figure 4a) or at the soil-pile interface Γ_{sp} (Figure 4b). Dynamic loads acting upon the soil will create a soil reaction but also induce waves propagating from the loading surfaces Γ_{sf} and Γ_{sp} into the soil interior. If the soil is unbounded in one of its dimensions, these propagating and non-reflected waves will carry energy away from the source and act as a damping mechanism, known as geometric damping.

Consider the soil interface excited in each case. If the pile is not very short, the pile-soil interface Γ_{sp} (Figure 5b) may be larger than the soil-foundation interface Γ_{sf} (Figure 5a). Larger reaction surfaces should induce a larger system of emanating waves which should carry more energy away from the system and present a larger geometric damping effect. So, in principle, for the same external excitation force, F_{ext}, foundations supported by piles (Γ_{sp}) should have smaller vibration amplitudes that its counterpart, supported only at the soil-foundation interface (Γ_{sf})

Now analyze the same two systems but subjected to an incident vertical wave field, U_{inc}, as shown in Figures 6a and 6b. Again, if the pile is not too short, it has a larger soil interface Γ_{sp} and will be impinged by a larger amount of the incident vertical wave field. This larger surface (Figure 6b) being impinged by the incoming wave field should lead to a larger vibratory response when compared to the soil-foundation system, Figure 6a, which has a smaller interface Γ_{sf}. So, from this reasoning, the response of soil-foundation and soil-pile-foundation systems should have opposite amplitude responses according to the type of excitation, external force or incoming wave field. This article addresses the veracity of this reasoning. Note that the wave propagation frontlines shown in Figures 5 and 6 are merely illustrative.

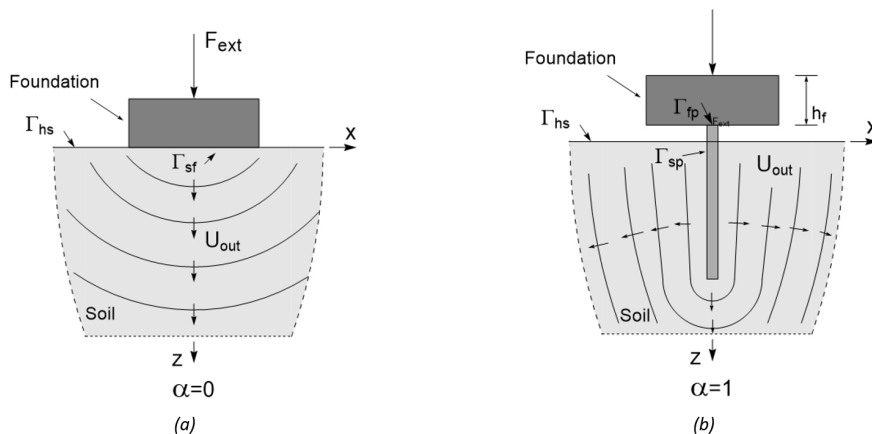
Figure 5: Excited interfaces for each supporting mechanism when the foundation is under an external force.
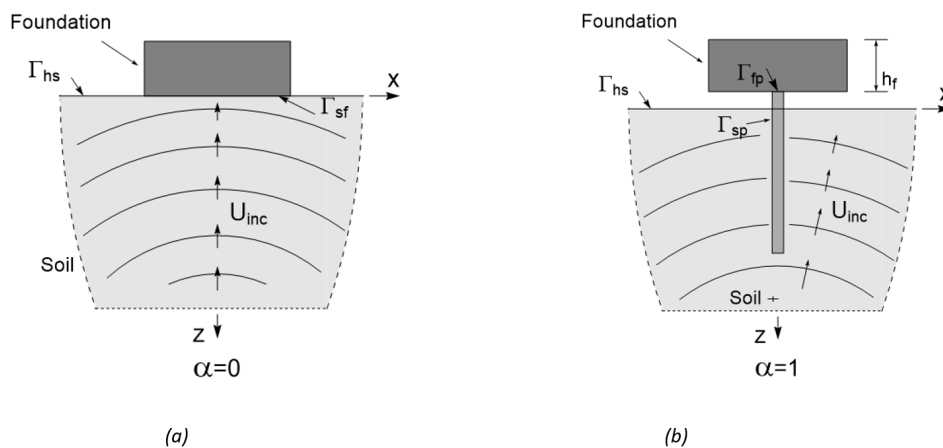

Figure 6: Excited interfaces for each supporting mechanism when the foundation is subjected to an impinging vertical wave field.

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2.3 Influence of pile internal nodes

The third point analyzed in this article is concerned with the type of pile response considered in the analysis. The pile is fully embedded in the half-space, as mentioned in the problem statement. Two distinct pile responses will be considered in the present article. The response of the pile interacting with the soil can be obtained only at the pile head U_{p0}, see Figure 7a, or additionally along a set of n discrete points along the pile length U_{pi} (i=0,n), as shown in Figure 7b. The article addresses the question whether these two soil-pile response modes have an influence on the foundation dynamic response due to external forces and incoming wave fields. In principle, for the external force acting upon the foundation, considering only the pile response at the pile head U_{p0} or the pile response in many other points along its shaft U_{pi} should be immaterial. On the other hand, for the case of an incoming wave field, the response of a pile being considered only as a point at the pile head or as a series of discrete nodes at the pile length should lead to distinct responses because of the wave scattering phenomena occurring along the pile shaft. This effect is investigated.
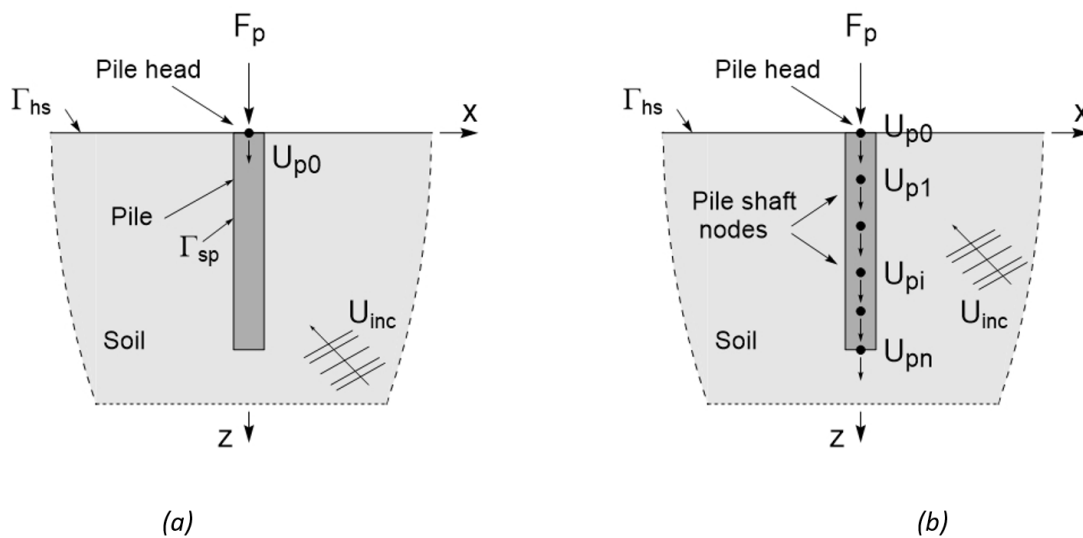

(a)   *(b)*

Figure 7: Two models of the pile response: (a) response concentrated at pile head and (b) response distributed along the pile length.

3 FORMULATION

In this session the models used in this study are described. First the soil model and response is addressed. Next the pile model is reported. In the sequence the soil-foundation and the soil-pile coupling equations are formulated. The soil-pile response is formulated in two distinct ways: coupling at the pile head only and coupling along the entire pile shaft. The last two sessions present the formulation for the complete system, soil-pile-foundation, subjected to external forces and incident wave fields.

3.1 Soil and Foundation Models

3.1.1 Soil Model

In this work, the soil is modeled as a three-dimensional, homogeneous, isotropic, half-space under stationary loads of circular frequency ω. Rajapakse and Wang (1993) have derived the Green's function describing the dynamic behavior of such medium in terms of Hankel transforms and series expansion. Based on the Green's functions synthesized by Rajapakse and Wang (1993), Labaki, Mesquita and Rajapakse (2014) were able to obtain a dynamic flexibility matrix for distinct soil profiles $S_{hs}(i,j)$, connecting the loads applied at the j-th point within the soil, $F_{hs}(j)$, with the soil displacement response at the i-th point, $U_{hs}(i)$ for a given circular frequency ω. In mathematical terms, the stationary response of the considered three-dimensional transversely isotropic half space may be written as:

$$\{U_{hs}(i)\} = \left[S_{hs}(i,j,\omega)\right]\{F_{hs}(j)\} \tag{2}$$

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3.1.2 The pile model

The Finite Element Method (FEM) is used to model the pile with bar elements of length l_e, with two nodes and a linear interpolation within the element. The pile is divided into n_p elements, which results in $n_n = n_p+1$ number of the pile nodes in the model. The standard finite element stiffness $[K_e]$ and mass $[M_e]$ matrices of a pile element are given by:

$$[K_e] = \frac{\pi a_p^2 E_p}{l_e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

(3)

$$[M_e] = \frac{\pi a_p^2 \rho_p l_e}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

(4)

The global stiffness $[K]$ and mass $[M]$ matrices of pile are achieved from the matrices of elements by the standard FEM assembly procedure. Assuming time harmonic behavior, a pile flexibility matrix relating the $[S_p(i,j)]$ relating the forces applied at the pile node j, $F_p(j)$, with the pile displacements at node I, $U_p(i)$ can be written:

$$\{U_p(i)\} = [S_p(i,j,\omega)]\{F_p(j)\}$$

(5)

With

$$[S_p(i,j,\omega)] = \{[K(i,j)] - \omega^2 [M(i,j)]\}^{-1}$$

(6)

3.2 The Coupled Models

3.2.1 Response of Rigid and Massless Foundation on the Half-space surface

The response of the rigid and massless foundation interacting with the surface of the homogeneous half-space was given by Labaki et al. (2014). It uses a standard procedure superposing the solution of concentric annular surface disks with constant spatial loads amplitude and imposing rigid body kinematic rigid body compatibility and equilibrium conditions at the soil-foundation interface Γ_{hs} (Wong and Luco, 1976). This approach leads to a flexibility function S_{hs0} relating the half-space soil reaction F_{hs0} to the response at the half-space surface U_{hs0}:

$$U_{hs0} = S_{hs0}\, F_{hs0}$$

(7)

3.2.2 First soil-pile model: Response only at the pile head

The first soil-pile model used in this analysis was presented by Rajapakse and Shah (1987) and simplified more recently by Labaki, Mesquita and Rajapakse (2015). The embedded pile is modeled by a one-dimensional elastic bar of Young's modulus $E^* = E_p - E_{hs}$ and mass density $\rho^* = \rho_p - \rho_{hs}$. The soil is considered to be subjected to a series constant stress distributions in the form of shell-like elements. The strain and kinetic energy of the bar, together with the strain energy of the soil and the potential energy of the external load can be written in terms of trial functions, containing unknown generalized parameters. The subsequent minimization of the Lagrange's equation of the bar-soil system according to Hamilton's principle yields (Rajapakse and Shah, 1987) the vertical displacement of the pile head U_{p0}. This displacement at the pile head can also be related to exciting force at the pile head, F_{p0}, through a dynamic flexibility function S_{P0}

$$U_{p0} = S_{p0}\, F_{p0}$$

(8)

3.2.3 Second coupled soil-pile model: Response along the pile length.

In session 3.2.2, the response of the coupled pile-soil system at the pile head was described. In this session the complete coupling of the pile along its length with the soil is described. The methodology in this case is a standard direct

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coupling procedure, in which displacement compatibility and equilibrium conditions among the pile and half-space nodes are imposed at Γ_{sp}. Considering Equations (2) and (5), kinematic compatibility requires that:

$$\left\{ U_p\left(i\right)\right\} = \left\{ U_{hs}\left(i\right)\right\} \tag{9}$$

And force equilibrium conditions at interface are:

$$\left\{ F_p\left(i\right)\right\} + \left\{ F_{hs}\left(i\right)\right\} = \left\{ F_{p-ext}\left(i\right)\right\} \tag{10}$$

In Equation (10) a vector of external loads applied to the pile F_{p-ext} has been introduced. In practice the external loads act only at the pile head so that $\{F_{p-ext}\} = \{F_{po}, 0, 0, ...,0\}^T$. With these definitions the coupled soil-pile response, with an excitation at the pile head may be written as:

$$\left\{ U_p\left(i\right)\right\} = \left\{ \left[S_p\right]^{-1} + \left[S_{hs}\right]^{-1}\right\}^{-1}\left\{ F_{p-ext}\right\} \tag{11}$$

3.3 Response of the Coupled Models to External and Internal Excitations

3.3.1 Formulation of the foundation-pile-soil system subjected to an external force

In this section, the complete soil-pile-foundation system subjected to external excitations will be formulated. The rigid foundation interacts with the soil and the pile, as shown in Figure 1a. Observing Figure 1b, which shows the forces acting on the structure, the equilibrium equation of the structure in terms of its displacement U_f may be written as:

$$F_{ext} - F_{rea} = -\omega^2 M_f U_f \tag{12}$$

By inserting Equation (1) into Equation (12), an equation that relates the displacement of the foundation U_f with the forces acting on the pile and the soil can be founded:

$$-\omega^2 M_f U_f + F_{ext} = \left(1-\alpha\right) F_{hs0} + \alpha\, F_{p0} \tag{13}$$

The pile and soil forces acting at the soil-pile-foundation interface (z=0) may be obtained from Equations (7) and (8) and substituted into Equation (13) to obtain:

$$-\omega^2 M_f U_f + F_{ext} = \left(1-\alpha\right)\left(U_{hs0} S_{hs0}^{-1}\right) + \alpha\left(U_{p0} S_{p0}^{-1}\right) \tag{14}$$

This system must satisfy the criteria of kinematic compatibility, which stipulates that the displacement of the foundation U_f must be equal to the displacement of the pile head U_{p0} and must be equal to the displacement to the soil-foundation interface U_{hs0}:

$$U_f = U_{hs0} = U_{p0} \tag{15}$$

Applying Equation (15) into Equation (14) and isolating U_f:

$$U_f = \frac{F_{ext}}{\left[\left(1-\alpha\right)\left(S_{hs0}^{-1}\right) + \alpha\left(S_{p0}^{-1}\right) + \omega^2 M_f\right]} \tag{16}$$

Equation (16) gives the foundation response U_f in which the foundation is supported by the pile and the soil and excited by the external force F_{ext}.

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3.3.2 Formulation of the foundation-pile-soil system subjected to an incident vertical wave field

In this section the formulation for the foundation response of a soil-pile-foundation system due to an impinging incident vertical wave field U_{inc} is presented. No external forces F_{ext} are considered, in this case $F_{ext}=0$. The equilibrium equation of the foundation in this system is:

$$F_{rea} = -\omega^2 M_f U_f \tag{17}$$

The total soil displacement field U_t is composed of two parts, the incident wave field U_{inc} and the scattered part U_ζ The total displacement field of the surface of the soil (z=0) is

$$U_t = U_{inc} + U_\zeta \tag{18}$$

The corresponding forces at the free surface due to U_{inc} are:

$$F_t = F_{inc} + F_\zeta \tag{19}$$

In the absence of the foundation, the force at the free surface due to an impinging wave field is zero, $F_{inc}=0$. Hence, the total reaction force F_{rea} at the foundation-pile-soil interface equals the scatter force F_ζ,

$$F_{rea} = F_t = F_\zeta \tag{20}$$

The scattered wave field U_ζ and the scatter force F_{rea} are related through the soil flexibility S_{hs0} and pile flexibility S_{p0} balanced by the dimensionless parameter α:

$$U_\zeta = (1-\alpha)(S_{hs0} \, F_{hs0}) + \alpha \left(S_{p0} \, F_{p0}\right) \tag{21}$$

In view of Equations (17) to (21), the dynamic response of the foundation U_f to an incident wave field U_{inc} can be written as:

$$U_f = \frac{\left(1 - \dfrac{(1-\alpha)}{S_{p0}} - \dfrac{\alpha}{S_{hs0}}\right)\left(\dfrac{1}{S_{p0}} + \dfrac{1}{S_{hs0}}\right)^{-1}}{1 + \left(\omega^2 M_f - \dfrac{(1-\alpha)}{S_{p0}} - \dfrac{\alpha}{S_{hs0}}\right)\left(\dfrac{1}{S_{p0}} + \dfrac{1}{S_{hs0}}\right)^{-1}} \, U_{inc} \tag{22}$$

The Equation (22) gives the foundation displacement of the coupled system U_f for a foundation excited by an incident wave field, U_{inc}. When α = 0 the foundation is entirely supported by the soil (Figure 4a). When α= 1 the foundation is only supported by the pile (Figure 4b). The formulation described in sections 3.3.1 and 3.3.2 is used to analyze both the pile considered on the pile head or coupled along the entire z axis choosing the appropriate dynamic flexibility of the soil S_{hs0} or $[S_{hs}]$ and dynamic flexibility of the pile S_{p0} or $[S_p]$.

4 VALIDATION AND NUMERICAL RESULTS

In this section, the validations and the numerical results will be presented. Validations will be made in relation to the displacement of a foundation supported by the soil (α =0) and displacement of the foundation supported by a pile (α =1) as well as the static stiffness of a foundation supported by the soil. After the validations, new results will be presented. For the purpose of presentation of numerical results, the following normalizations are defined: ratio of modulus of elasticity $E'=E_p/E_{hs}$, ratio of density $\rho'= \rho_p/ \rho_{hs}$ and mass ratio $B=M_f/M_{soil}$, in which M_f is the mass of the foundation and M_{soil} is the mass of the soil. The soil mass, M_{soil}, is defined as the mass

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comprised by a volume formed by the area of the soil-foundation interface possessing a unit depth, $M_{soil} = \pi a_f^2 1 \rho_s$. In DSSI problems it is usual to use the dimensionless frequency parameter $a_0 = \omega a_f / c_s$, in which the shear wave velocity of the soil is $c_s = \sqrt{G_{hs} / \rho_{hs}}$. The normalization of the soil response in relation to its frequency parameter a_0 and the amplitude response U_f under the influence of the mass ratio B will be shown. The influence of α, E' and ρ' in the displacement of the foundation subjected to an external force F_{ext} or an incident wave field U_{inc} considering only the flexibility at the pile head will be presented. Finally, a comparison of the pile response will be shown considering only the pile head or the pile coupled along the half-space.

4.1 Validations

4.1.1 Validation of dynamic response of foundation on the soil and on the pile

In this session we will validate the results for the limiting cases (α=0) and (α=1). For the case of the foundation supported by the half-space, the normalized absolute value of the half-space flexibility function is compared to similar results reported by Barros et al. (2019) and shown in Figure 8a. The results are normalized with respect to the static value of the flexibility function $S_{hs0}(\omega=0)$. The parameters used to obtain this solution are: $M_f=0$, $a_f=1.0m$, $E_{hs}=2.5Pa$, $\eta_{hs}=0.01$, $\nu_{hs}=0.25$, $\rho_{hs}=1kg/m^3$. The results show a very good agreement.

For the foundation supported by only by the pile (α=1), the absolute value of frequency dependent pile head flexibility is also compared to the values obtained by Barros et al. (2019) and shown in Figure 8b. The soil parameters remain the same of the previous example. The pile parameters are: $h_p=10m$, $a_p=1m$, $\rho_p=1kg/m^3$ and $E_p=2500Pa$. The foundation mass is $M_f=0$. The results are normalized with respect to the static value of the flexibility function $S_{p0}(\omega=0)$. There is a good agreement between both calculations, specially in the lower frequency range. It should be noted that there are some methodology differences between the present approach and the one reported by Barros et al. (2019). In the present study the pile is incorporated in the half-space without an excavation. The presence of the additional soil mass in the position of the pile is accounted for by introducing a modified Young's modulus E* and density ρ*, defined as: $E^* = E_p - E_{hs}$ and $\rho^* = \rho_p - \rho_{hs}$. For larger frequency responses both methodologies should lead to slightly distinct results, what is consistent with the results shown in Figure 8.
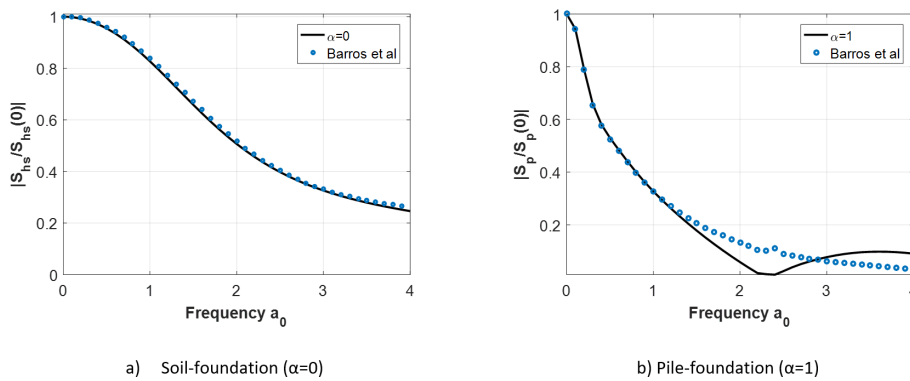

a)   Soil-foundation (α=0)

b) Pile-foundation (α=1)

Figure 8: Validation foundation response for the extreme cases

4.1.2 Static stiffness of soil foundation systems

As the results presented above are normalized by their static values, a comparison of the static stiffness value of vertical static stiffness of a rigid circular massless foundation resting on the surface of the soil is now presented. According to Gazetas (1983), the value of vertical static stiffness of a rigid circular massless foundation resting on the surface of the soil is given by the Equation (23):

$$K_{static} = \frac{4 G_{hs} a_f}{1 - \nu_{hs}}$$

(23)

Where G_{hs} is the modulus of elasticity of the soil, $a_f=1.0m$ is the radius of the foundation and ν_{hs} is the Poisson's ratio of the soil. In this validation, four properties are considered. The soil data used in this numerical example are shown in Table 1.

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Table 1 Properties of the soils used in the study.

	E(Pa)	v	G (Pa)	ρ (kg/m³)	c_s(m/s)
Soil 1	2.5	0.25	1	1	1
Soil 2	250	0.25	100	4	5
Soil 3	1200	0.25	480	4.8	10
Soil 4	200e6	0.25	80e6	8000	100

In this study, α=0. In the other words, pile is not presence in the system. Table 2 shows the values of the soil vertical static stiffness obtained by the present formulation, the values of the vertical static stiffness found according to Equation (23) and the relative errors between them.

Table 2 Comparison between values of the vertical static stiffness.

	K_{static} Present (N/m)	K_{static} Gazetas (N/m)	Relative error
Soil 1	5.2767	5.333	1.06%
Soil 2	527.6767	533.333	1.06%
Soil 3	2532.9	2560	1.06%
Soil 4	4.2215e08	4.2667e8	1.06%

Table 2 shows a good agreement of the present formulation with their alternative equivalents for different sets of data.

4.2 Results for the present study

In this session a series of studies is presented, including the effect of the mass ratio B, the influence of the bearing mechanism (α), the relative stiffness and density of pile and soil and finally the influence of the internal pile nodes in the scattering process.

4.2.1 Influence of the mass ratio B on the response of the soil-foundation system (α=0) - External excitation

This session describes the influence of the mass ratio $B=M_f/M_{soil}$ on the frequency response of a rigid foundation interacting with the soil without any pile (α=0) and subjected to an external excitation F_{ext}. Figures 9a to 9c show the normalized Real, Imaginary and Absolute value of the foundation displacement U_f for distinct values of the mass ratio parameter. Soil 1 was used (see Table 1) and a_f=1m. Figures 9 show that as the mass ratio B increases a resonance-like peak appears at the foundation response. As expected, the larger the mass, the lower the frequency of the displacement peak.
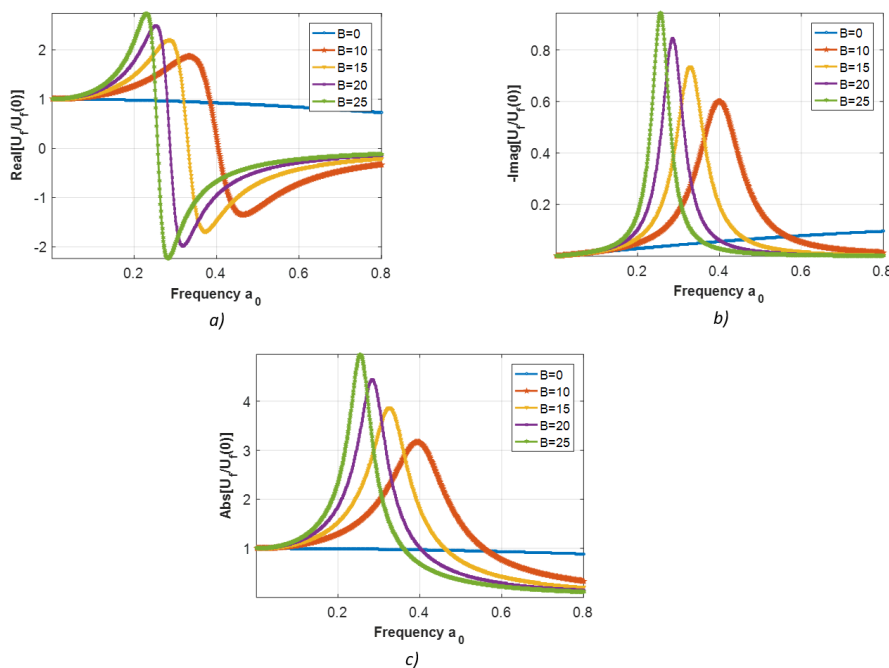

Figure 9: Foundation response for distinct mass rations B.

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4.2.2 Influence of the dimensionless parameter α on the foundation response

This session will provide the response of the foundation with distinct bearing mechanism, in which the parameter α varies in the range (0< α <1). External and internal excitations are considered. The influence of distinct values for the Elasticity modulus ratio pile $E'=E_p/E_{hs}$ is also included in the analysis. The following parameters are adopted: E_{hs}=2.5Pa, η_{hs}=0.01, ν_{hs}=0.25, ρ_{hs}=1kg/m^3; a_f=1m, a_p=1m, h_p/a_p=35, ρ_p=1kg/m^3. Different values of E' (E'=10, 50, 100 and 150) are considered.

External excitation. Figures 10a to 10d show absolute value of the foundation displacement U_f, determined by Equation (16) and normalized by the static value $U_f(\omega=0)$ for distinct values of E' and α. In all cases of the E' parameter, as α increases the peak displacement amplitude decreases. This means that the largest amplitudes occur when the foundation rests only on the soil. As the load on the pile increases, the peak displacement amplitude decreases. The smallest displacement amplitudes occur for the foundation supported only by the pile (α=1), regardless of the relative rigidity measure E'. These results are, so far, in agreement with the reasoning that the pile would generate a larger amount of outgoing waves resulting in a larger radiation damping in the system.
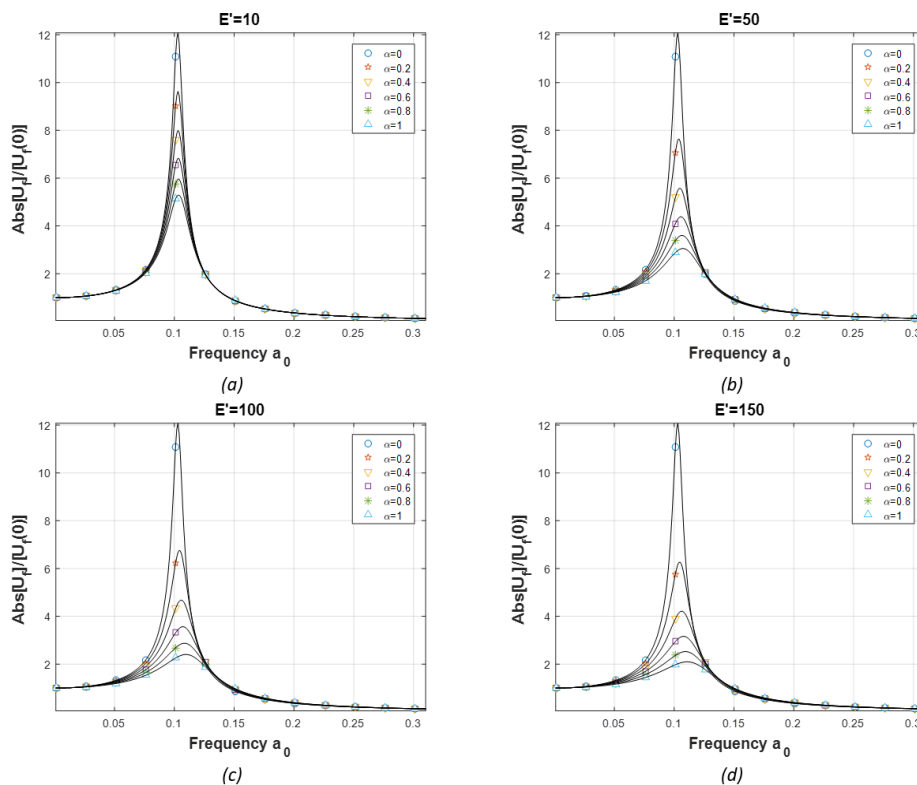

Figure 10: Influence of the parameters α and E' on the foundation displacement for external excitations.

Internal excitation. The internal excitation in all the examples of this article is an incoming wave field $U_{inc}=U_0\cos((\omega z_p)/c_p)$, where U_0 is the wave amplitude, z_p is the p-nodal coordinate at the pile in the z direction and c_p is pressure wave propagation velocity in vertical direction, in which $c_p = \sqrt{E_{hs}/\rho_{hs}}$. Figures 11a to 11d show the absolute value of the normalized foundation displacement due to a vertical incoming wave field, determined according to Equation (22). The same parameters of the previous example are used.

An analysis of Figures 10 and Figure 11 show a well-defined resonance-like region for all cases considered. It also shows that when α=0, the displacements of the foundation U_f is always the same independently of the value of E'. That is physically consistent because when α=0, the foundation is fully supported by the soil, and the soil has the same properties in all considered cases. Besides that, it is observed that an increase in α corresponds to a decrease in the displacement of the foundation at the peak response. In the previous results, the elastic modulus ratios were fixed (E'=10, 50, 100 or 150) and the value of the parameter α was changed. Now the opposite is done, the value of α is fixed (α = 0.5) and the modulus of elasticity ratio E' is varied, as can be seen in Figures 12. For the results presented in Figure 12a, the foundation is subjected to an external force. For Figure 12b, the foundation is subjected to an incident vertical wave field.
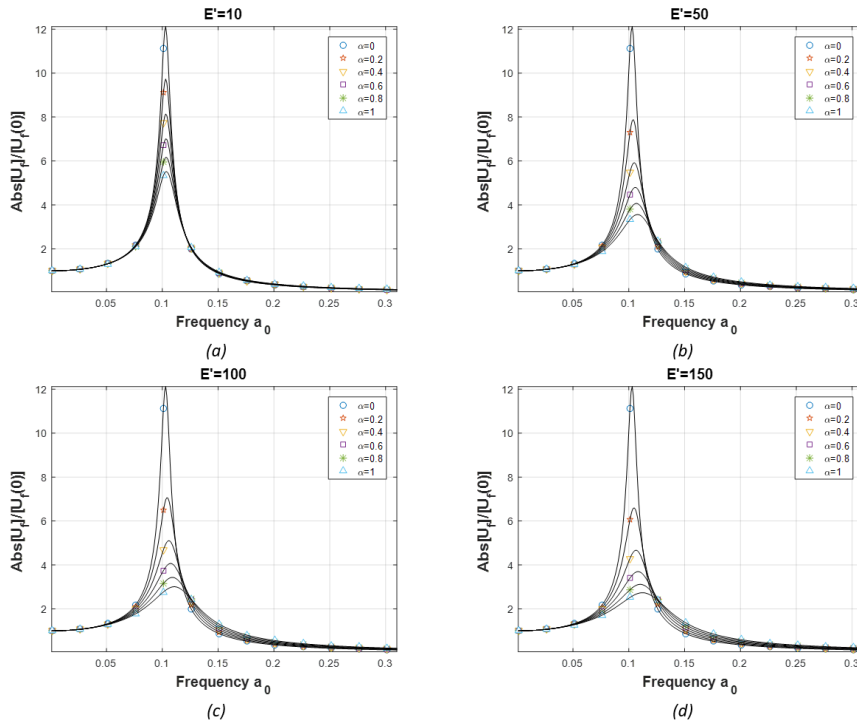
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Figure 11: Influence of the parameters α and E' on the foundation displacement for internal excitation.

Analyzing Figure 12, for both forms of excitation, as the modulus of elasticity of the pile increases the foundation vibration amplitude peak decreases. It is also noted that as the modulus of elasticity of the pile increases, there is a slight increase in the frequency of the peak response. Figure 13 shows a comparison of the displacement of the foundation when it is subjected to an external force or an incident wave field. The following parameters are considered: for the half-space, E_{hs}=2.5Pa, η_{hs}=0.01, ν_{hs}=0.25, ρ_{hs}=1kg/m^3; for the foundation, a_f=1m, and for the pile: a_p=1m, h_p/a_p=35, ρ_p=1kg/m^3 and B=30. As can be seen in Figure 13, the behavior of the displacement of the foundation subjected to an external force or to an incident wave field is very similar. However, the normalized amplitude of the foundation displacement is slightly greater when subjected to an incident vertical wave field. Nevertheless, as expected, the peak frequency is the same for both exciting mechanisms, external force or incident wave field.

As can be observed in Figures 10 to Figure 13, the presence of a pile has a significant impact in reducing vibration amplitude of the system, regardless of the type of excitation (external wave or incident wave field). Despite the increase of contact area by the introduction of a pile, the vibration levels of the coupled system are lower when the pile is present in the system. This shows that the initial hypothesis formulated in this article, which considered that the increase of contact area by the introduction of a pile would increase the peak amplitude of the foundation displacement at resonance when subjected to an incident wave field is wrong. The introduction of a pile reduces the foundation vibration amplitude at resonance for both exciting mechanisms, external excitation or incoming wave field. This is a core result from the present analysis.
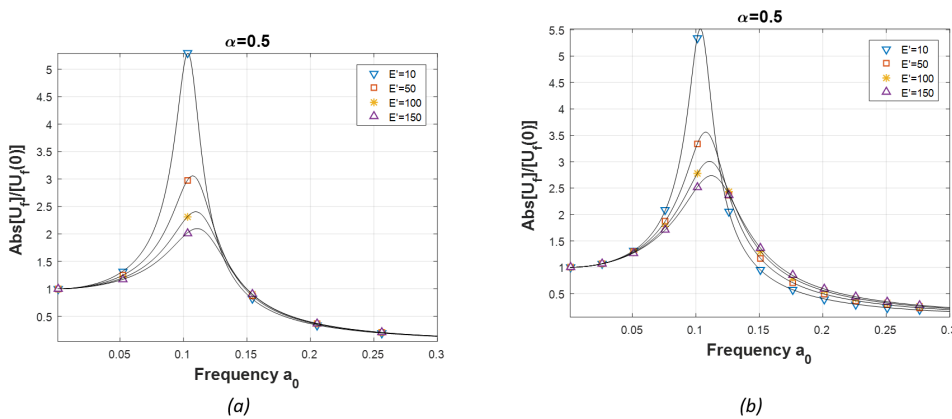

Figure 12: Influence of the E' for a fixed dimensionless parameter α: (a) external force (b) vertical incident wave field.

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4.2.3 Influence of the relative density of the pile and soil in the foundation response

Figure 14 shows the influence of the relative density of the pile and soil $\rho'=\rho_p/\rho_{hs}$ in the foundation response of the coupled system with a given mass ratio B=30. Two cases are shown. Figure 14a shows the case that the foundation is submitted to an external force and the Figure 14b shows the case when the foundation is submitted to an incident wave vertical field. For these results, the following parameters are considered: for the half-space, $E_{hs}=2.5$Pa, $\eta_{hs}=0.01$, $\upsilon_{hs}=0.25$, $\rho_{hs}=1$kg/m³; for the foundation, $a_f=1$m, and for the pile $a_p=1$m, $h_p/a_p=35$, $\rho_p=100$kg/m³. In the both cases E'=10 and $\rho'=100$.

In Figure 14, it is to be noticed that for this case, in which the pile is stiffer and more dense that the supporting soil, many amplitude peaks appear at the response amplitude. For the tested case, the pile is more sensitivity to the incident wave field than to an external excitation. These peak amplitudes in Figures 14 are related to the dynamics of the embedded pile response as can be seen by the vertical dynamic flexibility of the pile-soil system at the pile head shown in Figure 15, for the same parameters of the previous example.
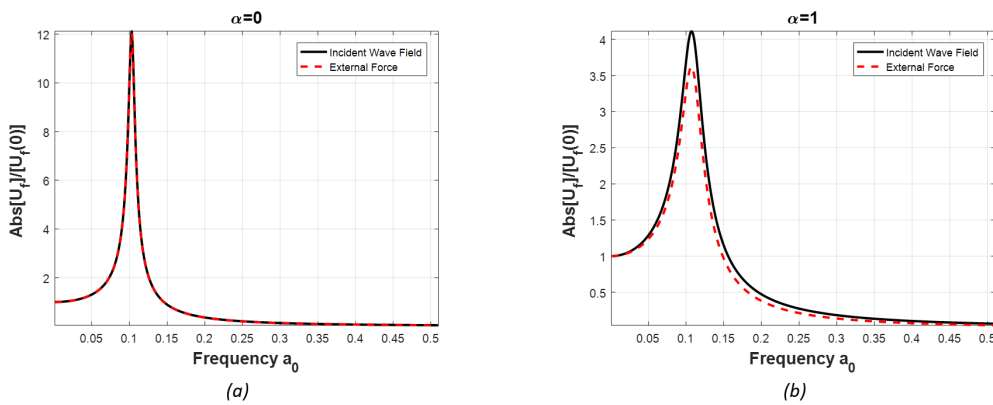

Figure 13: Comparison of the displacement of the foundation for extreme values of α: (a) subject to an external force and (b) subjected to an incident wave field.
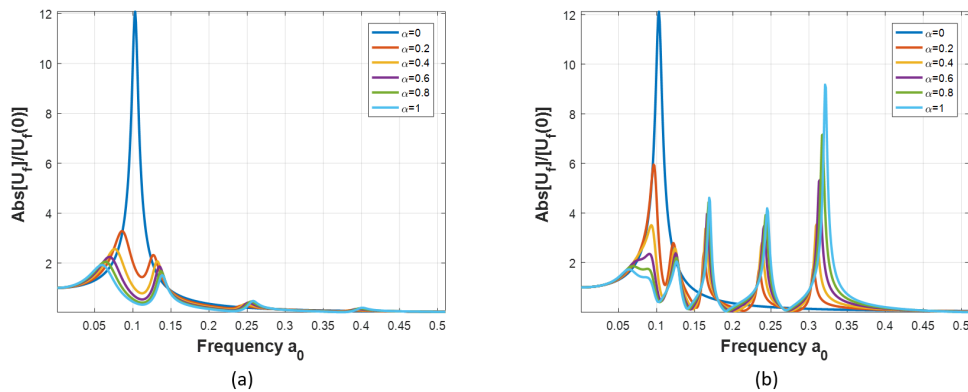

Figure 14: Influence of the density of the pile in the displacement of the foundation: (a) subject to an external force and (b) subjected to an incident wave field.
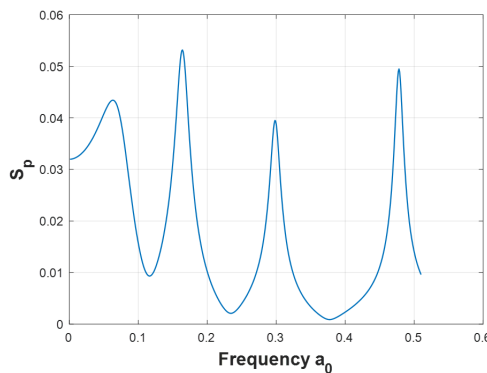

Figure 15: Vertical dynamic flexibility of the pile-soil system at the pile head.

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4.2.4 Influence of Pile response: degrees of freedom along the shaft and at the pile head.

In the second pile model, the pile is discretized by FE elements and the pile nodes are fully coupled to the corresponding nodes at the surface and interior of the half-space. Regardless of being fully coupled with the half-space along its shaft, the pile response used to calculate the previous examples was determined only at the pile-head U_{p0}. The displacements of the internal pile nodes were not considered. For an external excitation F_{ext}, in which the foundation force was only transmitted to the pile at the pile head, at the surface Γ_{fp}, the inclusion of the displacement of the internal pile nodes has no effect on the foundation response. But, for the case when the excitation is an incident wave field U_{inc} that will hit and be scattered by the pile along all its length, the response at the pile head, connected to the foundation, may be influenced the scatter caused by the pile internal nodes. In this analysis B=50, h_p/a_p=5, α=1 and the remain parameters are the same of the previous example. Different values of E' used in the model, as indicated in Figure 16.

Figure 16 shows the comparison of the foundation displacement considering only the response at the pile head and when the pile all internal nodes along the z axis are considered. Foundation is submitted to an external force (Figure 16a) and subjected to an incident vertical wave field (Figure 16b).

From the Figure 16a it can be seen that when the foundation is subjected to external excitation, the vibratory response of the foundation does not change regardless of whether the pile response considers only the pile head or the internal nodes. This is the expected behavior. However, if the excitation is an incident vertical wave field (Figure 16b), it is clear that the vibratory response of the foundation depends on whether the incoming wave field is only scattered at the pile hear or if it is scattered along the whole pile length. But the results also show that this effect is not very significant, so that, for an initial study, it is possible to consider only the response at the pile head to calculate the displacement of the coupled system for both excitation mechanisms.
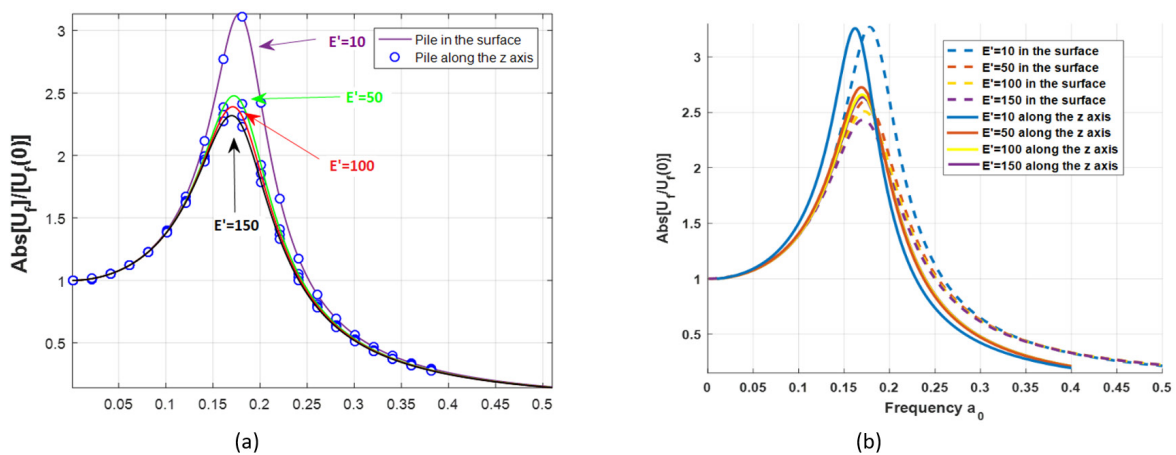

Figure 16: Foundation response for distinct modelling of the pile. (a) external force and (b) incident wave field.

5 CONCLUSIONS

The article presented a formulation to analyze the dynamic response of a soil-pile-foundation systems. The soil and the pile models were taken from previous works reported in the literature. This article addressed the important issue of the foundation bearing mechanism, which can be the soil-foundation interface, the pile foundation interface or still a distribution of the foundation loads between the soil and the pile. A dimensionless parameter was introduced to allow for the continuous change in the foundation bearing mechanism, from soil-only support to pile-only support. The question raised at the beginning about the role of the bearing mechanisms under the two distinct excitation sources was unequivocally answered. Vertical vibration amplitudes of the foundation are smaller for the pile-only supported foundation, regardless of the excitation source, external forces or incoming wave fields. The reduction of the vibration amplitude at the response peaks for the pile supported system are significant. Furthermore, depending on the relative properties of soil and pile, multiple resonance-like vibration amplitude peaks may be present at the pile response. These multiple amplitude peaks at the vibration spectrum are mainly due to the dynamic behavior of the pile embedded at the soil. An investigation about the influence of the pile modelling, taking or not into the formulation the internal pile nodes within the soil, has shown that for external force excitation only the response of the pile head need to be considered in the analysis of the coupled system. For a vertical incoming wave field, the inclusion of the internal pile nodes in the scattered response does present only a minor change in the foundation vibration amplitude. This suggests that for a

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preliminary analysis of the soil-pile-foundation vertical scatter phenomena, it suffices to consider the response of the pile head only.

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