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High precise analysis of lateral vibration of quintic nonlinear beam

Abstract

This article intends to achieve a new formulation of beam vibration with quintic nonlinearity, including exact expressions for the beam curvature. To attain a proper design of the beam structures, it is essential to realize how the beam vibrates in its transverse mode which in turn yields the natural frequency of the system. In this direction, new powerful analytical method called Parameter Expansion Method (PEM) is employed to obtain the exact solution of frequency-amplitude relationship. Afterwards, it is clearly shown that the first term in series expansions is sufficient to produce a highly accurate approximation of mentioned system. Finally, preciseness of the present analytic procedures is evaluated in contrast with numerical calculations methods, giving excellent results.

Keywords

Quintic nonlinearity, Parameter Expansion Method, Nonlinear vibration of beam, Exact formulation of beam curvature.

Hamid M. Sedighi* and Arash Reza

Department of Mechanical Engineering, Ahvaz Branch, Islamic Azad University, Ahvaz, Iran

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* Author email: hmsedighi@gmail.com

1 INTRODUCTION

Nonlinear vibration of beams is of substantial interest for engineers and has been studied, considerably. From the engineering outlook, structures like helicopter rotor blades, space craft antennae, flexible satellites, airplane wings, robot arms, high-rise buildings, long-span bridges, drill strings and vibratory drilling can be modelled as a beam-like member. The problem of the transversely vibrating beam was recently formulated in terms of the partial differential equation of motion by many researchers [1-17] with different boundary conditions. Sedighi et al. [1] presented the advantages of recent modern analytical approaches applied on the governing equation of transversely vibrating cantilever beams. A comprehensive study on the analytical investigation of cubic nonlinear vibrating tapered beams has been conducted by Bayat et al. [2]. The analytical expression for geometrically non-linear vibration of clamped-clamped Euler-Bernoulli beams including cubic non-linear strain-displacement relationship has been obtained by Barari et al. [3]. The application of new analytical approaches on the dynamical behavior of beams vibration with different boundary conditions has been investigated by Sedighi et al. [5-8]. Closed form solution to free vibration of beams with mixed boundary conditions has been proposed by Motaghian et al. [9]. Nikkhah Bahrami et al. [10] used modified wave approach for calculation of natural frequencies and mode shapes of arbitrary non-uniform beams. Non-linear modal analysis of a rotating beams studied by [11, 12]. When the vibration amplitudes are moderate or large, the geometric nonlinearity must be included.

It is very important to provide an accurate analysis towards the understanding of the non-linear vibration characteristics of these structures. Most models dealing with nonlinear dynamics of flexible beams include cubic nonlinear terms in the equations of motion. New predictions and understanding require higher order nonlinear terms in the equation of motion. However, research on flexible beams has so far been restricted to cubic nonlinearity. Literature which considered high order of nonlinearities is very limited [5].

In recent times, substantial progresses had been made in analytical solutions for nonlinear equations without small parameters. There have been several classical approaches employed to solve the governing nonlinear differential equations to study the nonlinear vibrations including perturbation methods [18], He's Max-Min Approach (MMA) [2], He's Energy Balance Method [19], Combined Homotopy Variational Approach [20], Iteration perturbation method [21], Homotopy perturbation method (HPM) [22, 23], Multistage Adomian Decomposition Method [24], Variational iteration method [3], Multiple scales method [25], Monotone iteration schemes [26], ADM–Padé technique [27], Navier and Levy-type solution [28], Hamiltonian approach [29], Parameter Perturbation Method [30], Differential Transform method [31], Laplace Transform method [32] . The application of new equivalent function for deadzone and preload nonlinearities on the dynamical behavior of beam vibration using PEM has been investigated by [6-8]. Therefore, many different methods have recently introduced various ways to eliminate the small parameter. The PEM have been shown to solve a large class of nonlinear problems efficiently, accurately and easily, with approximations converging very rapidly to solution. Usually, few iterations lead to high accuracy of the solution. The parameter expansion method is proved to be a very effective and convenient way for handling the nonlinear problems, one iteration is sufficient to obtain a highly accurate solution.

To extend study and understanding on nonlinear frequency, this paper brings quintic nonlinearities into consideration. The significant aim of this paper is to achieve analytical expressions for geometrically nonlinear vibration of Euler–Bernoulli beam including exact expression for the curvature of beam with quintic nonlinearity using PEM. The nonlinear ordinary differential equation of beam vibration is extracted from partial differential equation with first mode approximation, based on a Galerkin theory. The results presented in this paper exhibit that the analytical methods are very effective and convenient for nonlinear beam vibration for which the highly nonlinear governing equations exist. The proposed analytical method demonstrates that one term in series expansions is sufficient to obtain a highly accurate solution of beam vibration.

2 EQUATION OF MOTION

Consider the simply supported beam of length l, a moment of inertia I, mass per unit length m and a modulus of elasticity E, which is axially compressed by a loading P as shown in Fig. 1. Denoting by w the transverse deflection, the differential equation governing the equilibrium in the deformed situation is derived as:

$$\frac{d^2}{dx^2}\left\{\frac{EIw''(x,t)}{\left[1+w'^2(x,t)\right]^{3/2}}\right\}+Pw''(x,t)\left[1+\frac{3}{2}w'^2\right]+m\ddot{w}(x,t)=0 \tag{1}$$

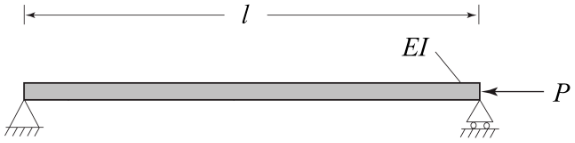


Figure 1 A uniform simply supported beam

where $w''(x,t)/\left(1+w'^2(x,t)\right)^{3/2}$ is the "exact" expression for the curvature, using the approximation

$$\frac{w''(x,t)}{\left[1+w'^2(x,t)\right]^{3/2}}\cong w''(x,t)\left[1-\frac{3}{2}w'^2(x,t)+\frac{15}{8}w'^4(x,t)\right] \tag{2}$$

which the nonlinear term $Pw''(x,t)\left[1+\frac{3}{2}w'^2\right]$ has been extracted from [5]. The governing quintic nonlinear equation (3) can be expressed as:

$$\begin{aligned}&EIw^{(4)}\left[1-\frac{3}{2}w'^2+\frac{15}{8}w'^4\right]-9EIw'''w'w''+\frac{45}{2}EIw'''w'^3w''-3EIw''^3\\&+\frac{45}{2}EIw'^2w''^3+Pw''\left[1+\frac{3}{2}w'^2\right]+m\ddot{w}=0\end{aligned} \tag{3}$$

which is subjected to the following boundary conditions

$$w(0,t)=\frac{\partial^2 w}{\partial x^2}(0,t)=0,\ w(l,t)=\frac{\partial^2 w}{\partial x^2}(l,t)=0 \tag{4}$$

Assuming $w(x,t)=q(t)\phi(x)$, where $\phi(x)$ is the first eigenmode of the simply supported beam and can be expressed as:

$$\phi(x)=\sin(\pi x/l) \tag{5}$$

Applying the Bubnov-Galerkin method yields:

$$\int_0^l \left(EIw^{(4)}\left[1-\frac{3}{2}w'^2+\frac{15}{8}w'^4\right]-9EIw'''w'w''+\frac{45}{2}EIw'''w'^3w''-3EIw''^3 \right.$$
$$\left. +\frac{45}{2}EIw'^2w''^3+Pw''\left[1+\frac{3}{2}w'^2\right]+m\ddot{w}\right)\phi\left(x\right)dx=0 \tag{6-a}$$

By introducing the following non-dimensional variables

$$\tau=\sqrt{\frac{EI}{ml^4}}t,\ \overline{q}=\frac{q}{l} \tag{6-b}$$

the non-dimensional nonlinear equation of motion about its first buckling mode can be written as

$$\frac{d^2\overline{q}\left(\tau\right)}{d\tau^2}+\gamma_1\overline{q}\left(\tau\right)+\gamma_2\left(\overline{q}\left(\tau\right)\right)^3+\gamma_3\left(\overline{q}\left(\tau\right)\right)^5=0 \tag{7}$$

where

$$\gamma_1=\pi^4-\frac{Pl^2\pi^2}{EI},\ \gamma_2=-\frac{3}{8}\pi^6-\frac{3}{8}\frac{Pl^2\pi^4}{EI},\ \gamma_3=\frac{15}{64}\pi^8 \tag{8}$$

3 OERVIEW OF PARAMETER EXPANSION METHOD

Consider the equation (7) for the vibration of a cantilever Euler-Bernoulli beam with the following general initial conditions

$$q\left(0\right)=A\ ,\ \ \dot{q}\left(0\right)=0 \tag{9}$$

Free oscillation of a system without damping is a periodic motion and can be expressed by the following base functions

$$\cos\left(m\omega\tau\right),\ m=1,2,3,... \tag{10}$$

We denote the angular frequency of oscillation by w and note that one of our major tasks is to determine $\omega\left(A\right)$,i.e., the functional behavior of w as a function of the initial amplitude A. In the PEM, an artificial perturbation equation is constructed by embedding an artificial parameter $p\in\left[0,1\right]$ which is used as an expanding parameter.

According to PEM the solution of equation (7) is expanded into a series of p in the form

$$q\left(\tau\right)=q_0\left(\tau\right)+pq_1\left(\tau\right)+p^2q_2\left(\tau\right)+... \tag{11}$$

The coefficients 1 and γ_1 in the equation (7) are expanded in a similar way

$$\begin{aligned} 1 &= 1 + pa_1 + p^2 a_2 + ... \\ \gamma_1 &= \omega^2 - pb_1 - p^2 b_2 + ... \\ 1 &= pc_1 + p^2 c_2 + ... \end{aligned} \tag{12}$$

where $a_i, b_i, c_i \left(i = 1,2,3,...\right)$ are to be determined. When $p = 0$, equation (7) becomes a linear differential equation for which an exact solution can be calculated for $p = 1$. Substituting equations (12) and (11) into equation (7)

$$\begin{aligned} &\left(1 + pa_1\right)\left(\ddot{q}_0 + p\ddot{q}_1\right) + \left(\omega^2 - pb_1\right)\left(q_0 + pq_1 + p^2 q_2\right) \\ &+\left(pc_1 + p^2 c_2\right)\left[\gamma_2\left(q_0 + pq_1\right)^3 + \gamma_3\left(q_0 + pq_1\right)^5\right] = 0 \end{aligned} \tag{13}$$

collecting the terms of the same power of p in equation (13), we obtain a series of linear equations which the first equation is

$$\ddot{q}_0\left(\tau\right) + \omega^2 q_0\left(\tau\right) = 0, \qquad q_0\left(0\right) = A, \quad \dot{q}_0\left(0\right) = 0 \tag{14}$$

with the solution

$$q_0\left(\tau\right) = A\cos\left(\omega\tau\right), \tag{15}$$

substitution of this result into the right-hand side of second equation gives

$$\begin{aligned} \ddot{q}_1\left(\tau\right) + \omega^2 q_1\left(\tau\right) = -\left(\frac{3}{4}c_1\gamma_2 A^3 + \frac{5}{8}c_1\gamma_3 A^5 - a_1 A\omega^2 - b_1 A\right)\cos\left(\omega\tau\right) \\ -\left(\frac{1}{4}c_1\gamma_2 A^3 + \frac{5}{16}c_1\gamma_3 A^5\right)\cos\left(3\omega\tau\right) - \frac{1}{16}c_1\gamma_3 A^5\cos\left(5\omega\tau\right), \end{aligned} \tag{16}$$

No secular terms in $q_1\left(\tau\right)$ requires eliminating contributions proportional to $\cos\left(\omega\tau\right)$ on the right-hand side of equation (16)

$$c\left(\omega\right) = \frac{3}{4}c_1\gamma_2 A^3 + \frac{5}{8}c_1\gamma_3 A^5 - a_1 A\omega^2 - b_1 A = 0 \tag{17}$$

But equation (12) for one term approximation of series respect to p and for $p = 1$ yields

$$a_1 = 0,\ b_1 = \omega^2 - \gamma_1,\ c_1 = 1 \tag{18}$$

From equations (18) and (17) we can easily find that the solution w is

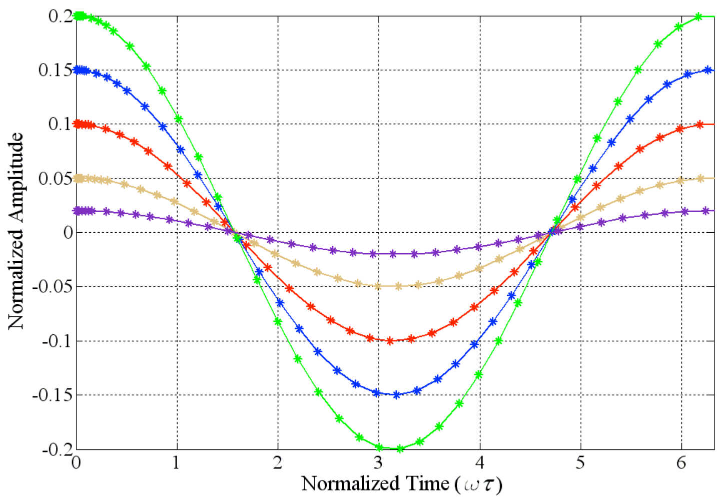
$$\omega(A) = \pm\sqrt{\gamma_1 + \frac{3}{4}\gamma_2 A^2 + \frac{5}{8}\gamma_3 A^4} \tag{19}$$

Replacing w from equation (19) into equation (11) yields:

$$q(\tau) \approx q_0(\tau) = A\cos\left(\sqrt{\gamma_1 + \frac{3}{4}\gamma_2 A^2 + \frac{5}{8}\gamma_3 A^4}\,\tau\right) \tag{20}$$

4 RESULTS AND DISCUSSION

In order to verify the integrity of the proposed solutions, the authors plot the analytical solutions at the side of corresponding numerical results in Fig. 2, where the first approximated amplitude-time curves of a uniform beam subjected to axial compression is presented for different initial conditions. It should be noted that, in order to verify the analytical solutions, Matlab built-in function ode45 has been used to solve the governing equation numerically. Ode45 uses simultaneously fourth and fifth order RK formulas to make error estimates and adjust the time step accordingly. This causes that ode45 function uses an automatically chosen variable time step to solve systems of ODEs. The built-in default accuracy of ode45 (10^{-3} relative error, 10^{-6} absolute error) is usually quite adequate for most purposes. Calculations are actually performed at internal time points that are chosen automatically by the routine to give the required accuracy [33].



Symbols: numerical solution, Solid line: analytical solutions

Figure 2 Comparison between analytical and Numerical results for $\gamma_1 = 80$

As can be seen, the first order approximation of $q(\tau)$ from analytical method is in excellent agreement with numerical results from fourth-order Runge-Kutta method. The exact analytical solutions reveal that the first term in series expansions is sufficient to result in a highly accurate solution of the problem. Furthermore, these equations provide excellent approximations to the exact period regardless of the oscillation amplitude. The material and geometric properties adopted here have been prepared in the Appendix.

For a vibrating Euler-Bernoulli beam, the Euler-Lagrange equation is as follows:

$$\frac{d^2}{dx^2}\left(EIw''(x,t)\right) + Pw''(x,t) + m\ddot{w}(x,t) = 0 \tag{21}$$

Applying the Bubnov-Galerkin method and using the first eigenmode of the simply supported beam yields:

$$\frac{d^2\overline{q}}{dt^2} + \gamma_1\overline{q}(\tau) = 0 \tag{22}$$

where

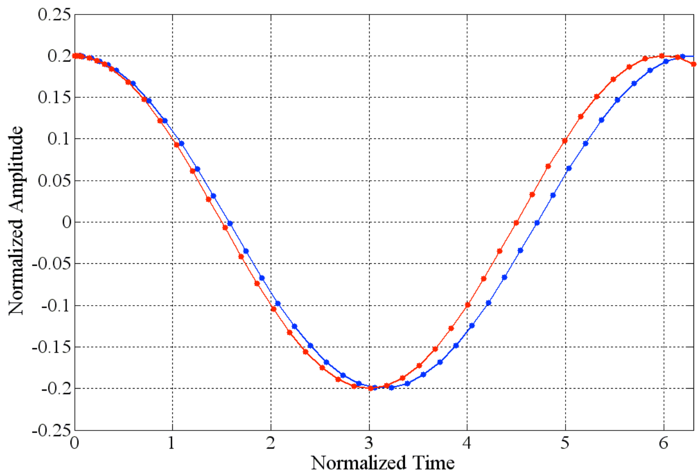
$$\gamma_1 = \pi^4 - \frac{Pl^2\pi^2}{EI} \tag{23}$$

Table 1 shows the governing equations of motion for transversely vibrating usual beam, cubic and quintic nonlinear beams.

Table 1 Equations of motion for different types of vibrating beams

Type of vibrating beam	Equation of motion
Usual beam	$\ddot{\overline{q}}(\tau) + \gamma_1\overline{q}(\tau) = 0$
Cubic nonlinear beam	$\ddot{\overline{q}}(\tau) + \gamma_1\overline{q}(\tau) + \gamma_2\left(\overline{q}(\tau)\right)^3 = 0$
Quintic nonlinear beam	$\ddot{\overline{q}}(\tau) + \gamma_1\overline{q}(\tau) + \gamma_2\left(\overline{q}(\tau)\right)^3 + \gamma_3\left(\overline{q}(\tau)\right)^5 = 0$

Figs. 3 and 4 display the effect of normalized amplitude on the nonlinear behavior of a vibrating beam. From Eq. (20), the nonlinear natural frequency is a function of amplitude that means when the oscillation amplitude becomes larger, the accuracy of approximated frequencies in usual beam theory ($\gamma_2 = \gamma_3 = 0$) and cubic nonlinear beam ($\gamma_3 = 0$) decreases. It confirms that the normalized amplitude has a significant effect on nonlinear behavior of the beams. From these Figs. it is observed that the results from usual beam theory are incompatible with quintic nonlinear beam when the initial condition becomes larger. In other words, the usual beam response is in more agreement with quintic nonlinear beam, when the amplitude of vibration approaches to zero.



usual beam theory quintic nonlinear beam

Figure 3 The impact of nonlinear terms on the beam dynamical behavior for $A = 0.2$ and $\gamma_1 = 80$

To extend study and understanding on beam nonlinear frequency, this paper brings quintic nonlinearities into consideration. Therefore, the influence of cubic and quintic terms on the natural frequency of beam as a function of amplitude has been investigated. As illustrated in Fig. 5, when the amplitude of vibration increases, the difference between natural frequencies of usual, cubic and quintic nonlinear beam gets larger. It is emphasizes that, in the case of large deformation, new predictions and understanding of nonlinear dynamical behavior of beams, requires higher order nonlinear terms in the equation of motion. Thus, a simply supported flexible beam represents rich nonlinear dynamics when amplitude of vibration increases. The percent of error in natural frequency approximation as a function of amplitude has been depicted in Fig. 6.

In order to investigate the effect of parameter γ_1 on the nonlinear behavior of quintic nonlinear beam, the natural frequency as a function of γ_1 has been illustrated in Fig. 7 for different values of amplitude. It is observed that the difference between nonlinear fundamental frequency and usual beam frequency increases with vibration amplitude. Also, the percent of error in approximating natural frequency as a function of γ_1 for different values of oscillation amplitude has been depicted in Fig. 8. The relative error of approximated simple beam theory frequency progressively increases at lower values of γ_1 for all values of vibration amplitudes.

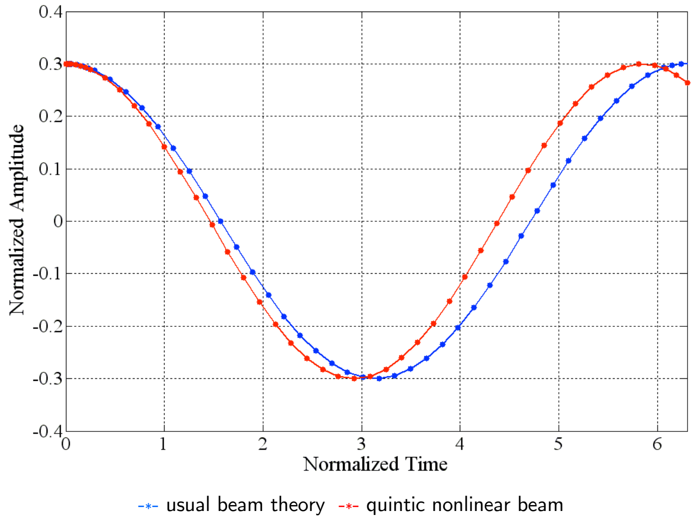


Figure 4 The impact of nonlinear terms on the beam dynamical behavior for $A = 0.3$ and $\gamma_1 = 80$

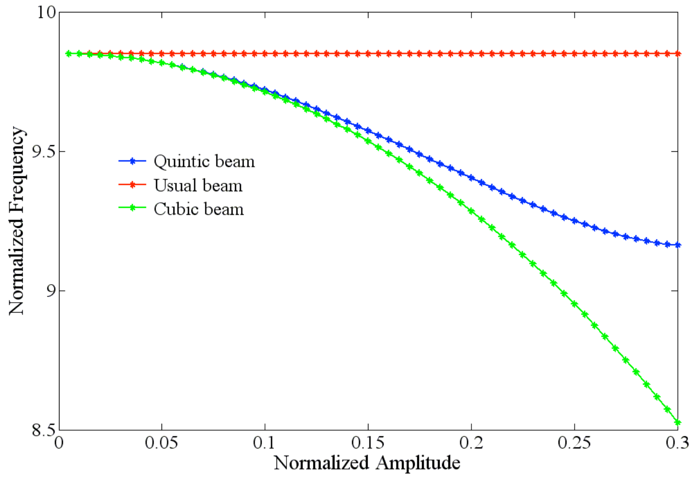


Figure 5 Comparison between of usual beam, cubic and quintic nonlinear beam for $\gamma_1 = 80$

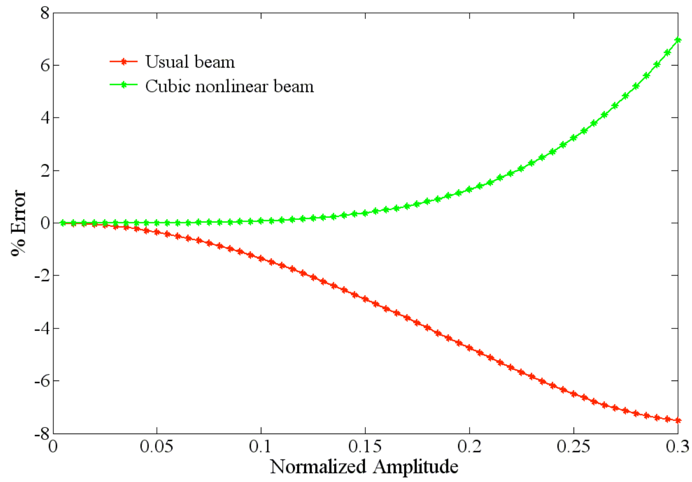


Figure 6 The percent of error in approximating natural frequency with respect to quintic nonlinear beam for $\gamma_1 = 80$

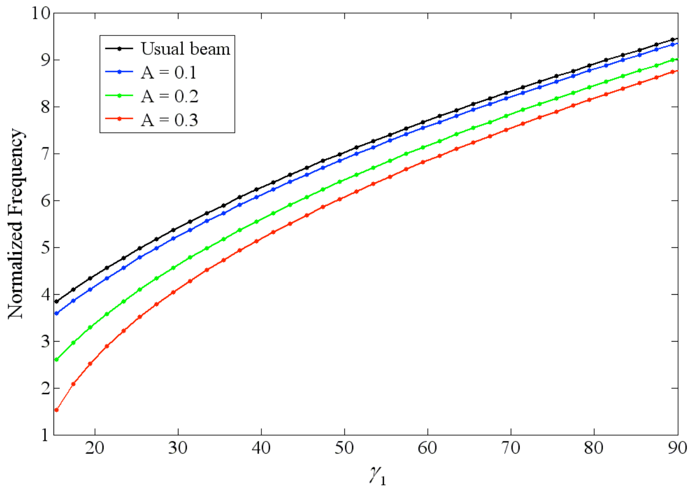


Figure 7 Comparison of usual beam and quintic nonlinear beam as a function of γ_1

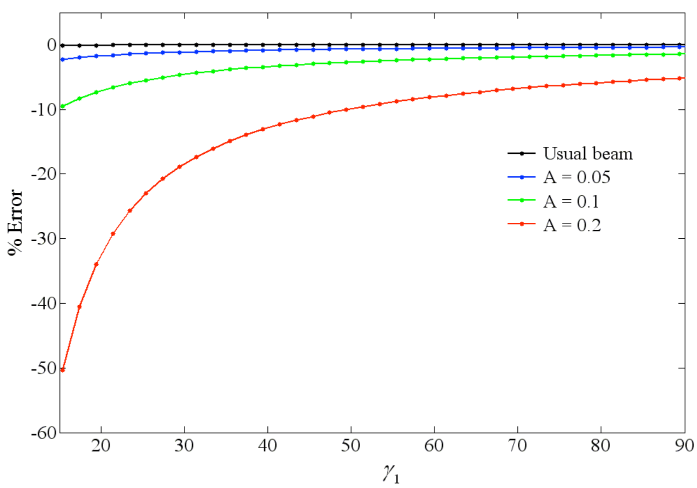


Figure 8 The percent of error in approximating natural frequency of usual beam as a function of γ_1

As mentioned before, it is very important to provide an accurate analysis towards understanding of non-linear vibration characteristics of beams. To demonstrate the necessity of quintic nonlinear terms, the percent of error in approximating cubic natural frequency as a function of γ_1 for different values of oscillation amplitude has been illustrated in Fig. 9. When the parameter γ_1 decreases, the relative error of approximated cubic beam frequency increases considerably, for all values of vibration amplitudes. In other words, in larger amplitudes this error cannot be ignored, as indicated in Fig. 9.

Note that the proposed method can be expanded to predict the beam response under different boundary conditions, easily. The mode shape which satisfies the boundary conditions at two ends of the beam should be placed in the Equation (6-a).

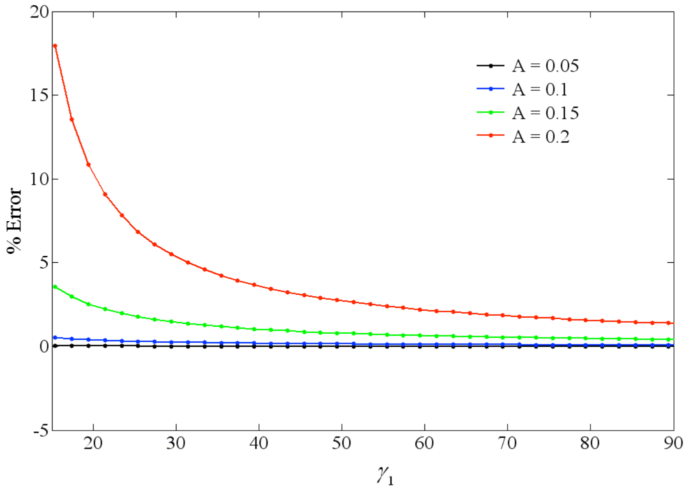


Figure 9 The percent of error in approximating natural frequency of cubic nonlinear beam as a function of γ_1

5 CONCLUSION

In this research, a modern powerful analytical method was employed to solve the governing equation of quintic nonlinear vibrating beams. It demonstrated that the fundamental frequency based upon linear theory and cubic nonlinear beam can be different from the natural frequency of quintic nonlinear beam at large vibration amplitudes. An excellent first-order analytical solution using modern asymptotic approach was obtained. The accuracy of the obtained analytical solutions is verified by numerical methods. These methods can be potentiality used for the analysis of strongly nonlinear oscillation problems.

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