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Latin American Journal of Solids and Structures 2 (2005) 89–101

BEM applied to damage models with emphasis on the localization and regularization techniques

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Abstract

In this work, the implicit BEM formulation, initially developed in the context of plasticity analysis is extended to incorporate damage mechanics models. The algebraic equations adopted for the formulation are obtained either using displacement or traction representations, for boundary nodes, and stress equations for internal nodes. The formulation is modified to incorporate the regularization technique based on the non-local integral. The consistent tangent operator has been obtained for local and non-local formulations. The arc-length concept developed for BEM formulations is adopted to analyse problems exhibiting the snap-back effects.

1 Introduction

Analysis of non-linear problems by means of BEM can be found since the end of seventies [4]. The non-linear formulations used during a quite long time were all based on the initial stress and strain procedures, where constant matrix schemes are employed.

The consistent tangent operator has been introduced into BEM non-linear formulations only recently [2, 15]. Even more recently, Benallal et al. [1, 7] have extended the formulation to deal with localization problems in plasticity. Those authors have derived the complete implicit BEM formulation for gradient plasticity, which was adopted to regularize the solution and to avoid the mesh dependency. They have shown the accuracy of the implicit formulation to compute very large deformation developed over a very narrow and localized bandwish, in comparison with the solution obtained by using an explicit model.

So far, only limited applications of BEM to damage mechanics have been reported in the literature [8]. Damage mechanics is a complete different problem to be dealt with BEM. The Young's modulus is no longer constant, being its new value given by the adopted model, which controls the degradation of the material, making the formulation even more complex when compared with elasto-plastic BEM schemes.

* Corresponding author E-mail: alexbott@sc.usp.br Received 23 March 2005
From *Recent Developments in the Modelling of Rupture in Solids Conference*, ed. A. Benallal & S.P.B. Proença.

In this paper, the non-linear BEM formulation is extended to solids governed by damage models, particularly proposed to deal with brittle material. First, the boundary algebraic equations are derived and then transformed appropriately leading to an incremental solution scheme with tangent predictor. The algebraic equations can be obtained from the singular or hypersingular integral representation, while the domain densities are all approximated using only internal values. A non-local BEM formulation based on the non-local integral concept is also derived to avoid the mesh dependency and to guarantee obtaining a unique solution.

2 Continuun damage model

Continuum Damage Mechanics (CDM), proposed to deal with the load carrying capacity of solids without majors cracks, but where the material itself is damaged due to the presence of microscopic defects such as microcracks and microvoids [9, 11, 13], has been increasingly used to model solids and structures [10, 12].

In the last decade, the researchers have proposed many models trying to simulate the actual behaviour of materials, particularly the quasi-brittle materials. Herein, we have chosen a particular isotropic damage model to deal mainly with concrete materials proposed by Comi & Perego [5]. In this model, the behaviours in tension and in compression are differently represented by damage scalar variables D_t and D_c, respectively. As a consequence, two surfaces, F_t and F_c, are defined in the stress space to give the limit of the elastic zone. The isotropic damage model chosen for this work is derived from the following free energy potential:

$$\psi = \frac{1}{2}\left\{ \begin{array}{l} 2\mu_0(1-D_t)(1-D_c)e : e + K_0(1-D_t)(tr^{+}\varepsilon)^2 \\ +K_0(1-D_c)(tr^{-}\varepsilon)^2 \end{array} \right\} \tag{1}$$

where ε and e are the strain tensor and its deviatoric part, respectively, μ_0 and K_0 are the shear and bulk moduli, $tr^{+}\varepsilon = \langle tr\,\varepsilon \rangle$ and $tr^{-}\varepsilon = -\langle -tr\,\varepsilon \rangle$ represent the contributions of the positive and negative parts of the volumetric strain, and D_t and D_c are the damage parameters in tension and compression.

To complete the model definition one has to find the stress tensor σ and define the loading functions f_t and f_c, and then specify the Kuhn-Tucker and the consistency conditions, which are expressed respectively by:

$$f_t \leq 0,\ \dot{D}_t \geq 0,\ f_t\dot{D}_t = 0,\ f_c \leq 0,\ \dot{D}_c \geq 0,\ f_c\dot{D}_c = 0 \tag{2a}$$

$$\dot{f}_t = 0 \text{ and } \dot{f}_c = 0 \tag{2b,c}$$

Other expressions to define the model can be found in the original work where the model was proposed [5].

In order to regularize the solution and avoid mesh dependency when solving numerically the problem, the non-local integral concept is adopted [14]. The variables will be replaced by their weighted values computed over the whole domain Ω or part of it. Thus,

$$\langle J_\varepsilon(x)\rangle = \int_\Omega W(x-s)\, J_\varepsilon(s)\, d\Omega(s) \tag{3a}$$

$$\left\langle tr^+\varepsilon(x)\right\rangle = \int_\Omega W(x-s)\, tr^+\varepsilon(s)\, d\Omega(s) \tag{3b}$$

$$\left\langle tr^-\varepsilon(x)\right\rangle = \int_\Omega W(x-s)\, tr^-\varepsilon(s)\, d\Omega(s) \tag{3c}$$

where $J_\varepsilon(s)$ is the second invariant of tensor $e(s)$ (deviatoric part of strain tensor $\varepsilon(s)$) i.e., $J_\varepsilon(s) = 1/2\ e(s) : e(s)$, and $tr^+\varepsilon(s)$ and $tr^-\varepsilon(s)$ are the positive and negative parts of the strain trace, while $W(x-s)$is the weigthing function containg the material characteristic length ℓ.

After replacing the local variables by the non-local ones given in Eqs.(3a,b,c) in the loading functions f_t and f_c, their non-local expressions are obtained:

$$\begin{aligned} F_t = F_t(\varepsilon, D_t, D_c) = 4\mu^2 \left\langle J_\varepsilon\right\rangle_t - a_t \left(K_+ \left\langle tr^+\varepsilon\right\rangle_t + K_- \left\langle tr^-\varepsilon\right\rangle_t\right)^2 \\ +b_t r_t(D_t)\ \left(K_+ \left\langle tr^+\varepsilon\right\rangle_t + K_- \left\langle tr^-\varepsilon\right\rangle_t\right) - k_t r_t^2(D_t)\,(1-\alpha D_c) \end{aligned} \tag{4a}$$

$$\begin{aligned} F_c = F_c(\varepsilon, D_t, D_c) = 4\mu^2 \left\langle J_\varepsilon\right\rangle_c + a_c \left(K_+ \left\langle tr^+\varepsilon\right\rangle_c + K_- \left\langle tr^-\varepsilon\right\rangle_c\right)^2 \\ +b_c r_c(D_c)\ \left(K_+ \left\langle tr^+\varepsilon\right\rangle_c + K_- \left\langle tr^-\varepsilon\right\rangle_c\right) - k_c r_c^2(D_c) \end{aligned} \tag{4b}$$

3 Integral representations for damage problems

For a damaged body defined in the domain Ω with boundary Γ the following integral representation of displacements, conveniently written in terms of rates, can be derived [3],

$$c_{ik}\dot{u}_k = \int\limits_\Gamma u_{ik}^*\dot{p}_k d\Gamma - \int\limits_\Gamma p_{ik}^*\dot{u}_k d\Gamma + \int\limits_\Omega u_{ik}^*\dot{b}_k d\Omega + \int\limits_\Omega \varepsilon_{ijk}^*\dot{\sigma}_{lm}^D d\Omega \tag{5}$$

where u_k and p_k are the displacement and the traction components at boundary points, respectively, b_k gives the body forces, the free term c_{ik} is dependent upon the boundary geometry; while the symbol * indicates the well-known Kelvin fundamental solutions and $\dot{\sigma}_{mk}^D$ is the damage stress, $\boldsymbol{\sigma}_{ij}^D = DE_{ijkm}\boldsymbol{\varepsilon}_{km}$, being E_{ijkm} and D the elastic tensor and the damage parameter, respectively.

The integral representation of stresses can be obtained by differentiating (5) with respect to space co-ordinates at an internal point and then applying the Hooke's law, as follows:

$$\dot{\sigma}_{ij} = \int\limits_\Gamma u_{ijk}^*\dot{p}_k d\Gamma - \int\limits_\Gamma p_{ijk}^*\dot{u}_k d\Gamma + \int\limits_\Omega u_{ijk}^*\dot{b}_k d\Omega + \int\limits_\Omega \varepsilon_{ijmk}^*\dot{\sigma}_{mk}^D d\Omega + g_{ij}\left(\dot{\sigma}_{mk}^D\right) \tag{6}$$

where the new kernels are obtained appropriately from the corresponding ones given in Eq.(5); $g_{ij}\left(\dot{\sigma}_{mk}^D\right)$ is a free-term resulting from differentiating the strong singular domain integral.

Equation (6) can be used to compute the stress field at internal points. It can also be written for boundary points, however particular attention is required when writing representations for collocations near the singular point.

Equations (5) and (6) are the exact equilibrium representations of the body under consideration. To solve the boundary value problem governed by these integral equilibrium equations and subjected to the constraints represented by the damage criterion, a space discretization followed by displacement and strain (or stress) field approximations must be assumed. For the example presented in this work, only linear shape functions were used to describe the boundary values along the elements. Moreover, continuous and discontinuous boundary elements can be adopted simultaneously if convenient to approximate the boundary values. The discontinuity is always introduced by defining the collocation along the element or at any outside point near the boundary (collocations near the boundary are recommended to assure the system stability).

To approximate the domain values, strain or stress components, triangular cells with linear approximation have been adopted with nodes defined at corners, but when convenient placed along the bisector lines to introduce discontinuous approximation. Using discontinuous cells is convenient to avoid using nodal boundary values, therefore keeping the domain approximation based only on domain stress components.

To have a reliable set of non-linear equations, the integrals over elements and cells have to be carried out as accurate as possible. For polynomial shape functions, finding the exact analytical expression to evaluate boundary integrals is not a difficult task, however the numerical scheme based on the element sub-division adopted here has proved to give very accurate results. To increase the accuracy when integrating the internal cells we have transformed the domain integral to the cell boundary and then adopted the same sub-element scheme to carry out numerically the integrals along each cell side reaching accuracy of order of 10^{-9}.

For a collocation point s, algebraic equilibrium relations are obtained from Eqs.(5) and (6) performing properly the integrals over boundary elements and along internal cell sides:

$$[c(s)]\{\dot{u}(s)\} + \left[\widehat{H}(s)\right]\{\dot{u}\} - [G(s)]\{\dot{p}\} - [Q(s)]\left\{\dot{\sigma}^D\right\} = 0 \tag{7a}$$

$$\{\dot{\sigma}(s)\} + \left[H'(s)\right]\{\dot{u}\} - \left[G'(s)\right]\{\dot{p}\} - \left[Q'(s)\right]\left\{\dot{\sigma}^D\right\} = 0 \tag{7b}$$

where $\{\dot{u}\}$ and $\{\dot{p}\}$ are vectors containing the boundary displacement and traction rates, respectively; $\left\{\dot{\sigma}^D\right\}$ gives the damage stress rate vector (corrector vector) defined at internal points selected to approximate the stress field, $\{\dot{\sigma}\}$ is the stress rate vector computed at the collocation s; $\left[\widehat{H}(s)\right]$, $[G(s)]$, $[Q(s)]$, $[H'(s)]$, $[G'(s)]$ and $[Q'(s)]$ are matrices obtained by performing the appropriate integrals over boundary elements and internal cell sides.

Equations (7a,b) were obtained for any selected collocation point s that may be placed at an internal position, on the boundary or at an external position. In order to have a closed system of algebraic equations to solve the problem, one must choose a number of equations equal to (or larger than) the number of problem unknowns, i.e., boundary displacements and tractions plus the stresses at the internal nodes defined by the discretization. In practice, Eq.(7a) is usually adopted to build the block of algebraic equations relating boundary values leading to what is called singular formulation. We have also adopted Eq.(7b) for this purpose, but transforming it into a traction representation (we took only the algebraic representations of normal and shear stresses). For this case, we have simplified the problem choosing only outside collocations to avoid dealing with Hadamard's finite parts and also to avoid writing algebraic equation at points where the stress may not be unequally defined.

Thus, a block of equations relating boundary values can be obtained writing algebraic relations taken from Eqs.(7a) and (7b), either independently or from both simultaneously, leading to the following matrix equation:

$$[H]\{\dot{u}\} - [G]\{\dot{p}\} - [Q]\left\{\dot{\sigma}^D\right\} = 0 \tag{8}$$

For the numerical analysis to be shown in this work, only algebraic relations taken from Eq.(7a) was used to build the block (8) relating boundary values, while Eq.(7b) was adopted to compute stress values at internal points. An alternative formulation where the system of algebraic relations, Eq.(8), was built with algebraic relations taken from Eq.(7b) (hyper-singular formulation) has also been implemented and experimented. In this situation only stress representations were used to write boundary value relations and also to compute stresses at internal points. No advantage, regarding the accuracy of the numerical algorithm and its stability, has been noted when this alternative system of algebraic relations was adopted, mainly when fine meshes were used as often required for non-linear analysis.

For the incremental process to be discussed in the next section, Eqs.(8) and (7b) will be written into their incremental form after performing the time integral along a time step Δt, as follows:

$$[H]\{\Delta u\} - [G]\{\Delta p\} - [Q]\left\{\Delta\sigma^D\right\} = 0 \tag{9a}$$

$$\{\Delta\sigma\} + \left[H'\right]\{\Delta u\} - \left[G'\right]\{\Delta p\} - \left[Q'\right]\left\{\Delta\sigma^D\right\} = 0 \tag{9b}$$

Equations (9a,b) represent the approximated equilibrium of the solid during a time increment Δt. In the next section, they will be used to derive an implicit BEM algorithm to deal with problem in the damage mechanics context.

4 BEM implicit formulation for damage mechanics

After applying the boundary conditions Eqs.(9a,b) became:

$$[A]\ \{\Delta X_n\} = \{\Delta f_n\} + [Q]\ \left\{\Delta\sigma_n^D\right\} \tag{10a}$$

$$\{\Delta\sigma_n\} = -\left[A'\right]\ \{\Delta X_n\} + \{\Delta g_n\} + \left[Q'\right]\ \left\{\Delta\sigma_n^D\right\} \tag{10b}$$

where $\{\Delta X_n\}$ contains all boundary unknowns, $\{\Delta f_n\}$ and $\{\Delta g_n\}$ give the prescribed boundary value influences and Δt_n is the considered time increment.

Then, Eqs.(10a,b) can be solved to give:

$$\{\Delta X_n\} = \{\Delta M_n\} + [R]\left\{\Delta\sigma_n^D\right\} \tag{11a}$$
$$\{\Delta\sigma_n\} = \{\Delta N_n\} + [S]\left\{\Delta\sigma_n^D\right\} \tag{11b}$$

where

$$\{\Delta M_n\} = [A]^{\,-1}\{\Delta f_n\} \tag{12a}$$
$$\{\Delta N_n\} = -\left[A'\right]\,[A]^{\,-1}\{\Delta f_n\} + \{\Delta g_n\} \tag{12b}$$
$$[R] = [A]^{\,-1}[Q] \tag{12c}$$
$$[S] = -\left[A'\right]\,[A]^{\,-1}[Q] + \left[Q'\right] \tag{12d}$$

It is important noting that the equilibrium algebraic representation is now reduced to Eq.(11b), conveniently arranged in terms of the stress increment $\Delta\sigma_n$. Equation (11b) can be properly modified writing the stress increment $\Delta\sigma_n$ in terms of the strain increment, $\Delta\varepsilon_n = \varepsilon_{n+1} - \varepsilon_n$ (being $\Delta\boldsymbol{\varepsilon}_n = \Delta\varepsilon_n^D + \Delta\varepsilon_n^e$, the damaged and elastic strain parts, respectively) and the increments of the n internal variables: $\Delta a_{k_n} = a_{k_{n+1}} - a_{k_n}$. Thus, rearranging properly the stress terms Eq.(11b) can be written as [3]:

$$\begin{aligned}&\{Y(\Delta\varepsilon_n, \Delta a_{k_n})\} = \\ &-[E]\,\{\Delta\varepsilon_n\} + \{\Delta N_n\} + \left[\bar{S}\right]\,\{[E]\,\{\Delta\varepsilon_n\} - \{\Delta\sigma(\Delta\varepsilon_n, \Delta a_{k_n})\}\} = \{0\}\end{aligned} \tag{13}$$

where $\bar{S} = S + I$, being $[\,I\,]$ the identity matrix.

Equation (13) is the final non-linear equilibrium equation for the increment Δt_n that now can be solved iteratively. From the damage model constitutive equations, one may obtain explicitly the stress increment in terms of the strain increment, i.e. $\{Y(\Delta\varepsilon_n, \Delta a_{k_n})\} = \{Y(\Delta\varepsilon_n)\}$.

Equation (13) can now be solved by applying the Newton-Raphson's scheme. An iterative process may be required to achieve the equilibrium. For an time increment, Δt_n, the strain increment is computed by cumulating additive corrections $\delta\varepsilon_n^i$ from an interaction i to the next $i+1$, i.e.,

$$\left\{\,\Delta\varepsilon_n^{i+1}\,\right\} = \left\{\,\Delta\varepsilon_n^i\,\right\} + \left\{\,\delta\varepsilon_n^i\,\right\} \tag{14}$$

the determination of $\delta\varepsilon_n^i$ is obtained from the linearized form of Eq.(13), considering only the first term of the Taylor's expansion:

$$\left\{\,Y\left(\Delta\varepsilon_n^i\right)\,\right\} + \frac{\partial\,\{\,Y(\,\Delta\varepsilon_n^i\,)\,\}}{\partial\,\{\,\Delta\varepsilon_n^i\,\}}\;\left\{\,\delta\Delta\varepsilon_n^i\,\right\} + \cdots = 0 \tag{15}$$

Thus, the equilibrium equation (13) now reads:

$$\left\{\,Y\left(\,\Delta\varepsilon_n^i\,\right)\,\right\} = \left[\,[\,E\,] - [\,\bar{S}\,]\,[\,[\,E\,] - \left[E_n^{CTO}\right]\,]\,\right]\;\left\{\,\delta\Delta\varepsilon_n^i\,\right\} \tag{16}$$

where the algorithmic consistent tangent operator, $E_n^{CTO} = \partial\Delta\sigma_n^i / \partial\Delta\varepsilon_n^i$ is explicited. For the model adopted for this work the consistent tangent operator is given by the following expression:

$$\begin{aligned} \frac{\partial\Delta\sigma_n}{\partial\{\Delta\varepsilon_n\}} =& \tilde{E} - A \otimes \begin{bmatrix} \dfrac{\partial\Delta F_{t_n}}{\partial\Delta D_{c_n}} \left(\dfrac{\partial^2 \Delta F_{c_n}}{\partial\{\Delta\varepsilon_n\}^2} : \{\Delta\varepsilon_n\} + \dfrac{\partial\Delta F_{c_n}}{\partial\{\Delta\varepsilon_n\}} \right) /h \\ -\dfrac{\partial\Delta F_{c_n}}{\partial\Delta D_{c_n}} \left(\dfrac{\partial^2 \Delta F_{t_n}}{\partial\{\Delta\varepsilon_n\}^2} : \{\Delta\varepsilon_n\} + \dfrac{\partial\Delta F_{t_n}}{\partial\{\Delta\varepsilon_n\}} \right) /h \end{bmatrix} \\ & - B \otimes \begin{bmatrix} \dfrac{\partial\Delta F_{c_n}}{\partial\Delta D_{t_n}} \left(\dfrac{\partial^2 \Delta F_{t_n}}{\partial\{\Delta\varepsilon_n\}^2} : \{\Delta\varepsilon_n\} + \dfrac{\partial\Delta F_{t_n}}{\partial\{\Delta\varepsilon_n\}} \right) /h \\ -\dfrac{\partial\Delta F_{t_n}}{\partial\Delta D_{t_n}} \left(\dfrac{\partial^2 \Delta F_{c_n}}{\partial\{\Delta\varepsilon_n\}^2} : \{\Delta\varepsilon_n\} + \dfrac{\partial\Delta F_{c_n}}{\partial\{\Delta\varepsilon_n\}} \right) /h \end{bmatrix} \end{aligned} \tag{17}$$

where the partial derivatives should be properly derived from the loading functions, Eq.(4a,b), h is a scalar given by:

$$h = \frac{\partial\Delta F_{t_n}}{\partial\Delta D_{t_n}} \cdot \frac{\partial\Delta F_{c_n}}{\partial\Delta D_{c_n}} - \frac{\partial\Delta F_{t_n}}{\partial\Delta D_{c_n}} \cdot \frac{\partial\Delta F_{c_n}}{\partial\Delta D_{t_n}} \tag{18a}$$

and the tensors A and B are:

$$A = 2\mu_0(1 - D_c)\, e + K_0 tr^{+}\varepsilon\, I \tag{18b}$$

$$B = 2\mu_0(1 - D_t)\, e + K_0 tr^{-}\varepsilon\, I \tag{18c}$$

Although the presented scheme is similar to the one proposed for plasticity [1], there are some strong differences between them. In plasticity, the first try is always based on the initial elastic constants, which makes the algorithm simpler. For damage problems, the rigidity at damaged regions is so severely reduced that initial elastic tries are always an inconvenient choice, leading to a very large number of iterations for convergence. Thus, one has to replace the first elastic matrix by the damaged elastic matrix or the last tangent matrix.

In order to use reduced rigidity matrix, one has to note that the elastic increment of stresses $\{\Delta N_n\}$ has been derived for the isotropic and homogeneous domain, which is no longer the case. Thus, this quantity must be modified properly to take into account anisotropy induced by the damage. We can use Eq.(16) to find the modified elastic matrix and also to compute the modified elastic increment of stresses $\{\Delta N_n^D\}$, which is now given by:

$$\{\Delta N_n^D\} = \{E - [\bar{S}]\, [E - E^T]\}^{-1} \{\Delta N_n\} \tag{19}$$

As can be seen in the example shown in the next section, brittle materials often exhibit reduction of rigidity over small and localized regions. In this situation, the structure is strongly size dependent and often *snap-back* effects appear. The dissipation zone is so reduced that both referenced displacement and the total applied load reduce simultaneously when strains over the dissipation area increase. Loading can be applied up to the limit point, characterized by the singularity of the tangent matrix. From this point, only prescribed displacements can be chosen as the guide variable, even if a regularization technique were adopted to guarantee a single solution beyond the bifurcation point. For the situation where the dissipation zone is so narrow, leading to the presence of snap-back effects, displacements or equivalent values can be chosen as the guide variable. For this situation the simpler techniques to solve the problem are based on displacement gradients or other equivalent values. Despite of several possibilities (controlling the largest non-linear strain, for instance), we have already extended the non-linear BEM formulation presented in this work to incorporate the arc-length procedure [6], which has demonstrated to be efficient to analyse problems where the dissipation zone is very narrow and consequently exhibiting large gradients [3]. This scheme is adopted here together with the formulation presented so far.

5 Solution technique

The numerical solution of the proposed damage problem requires approximations in space and time. The BEM has been adopted to perform the first one. The time approximation is given by dividing the time in increments or load increments, as already describe in the previous sections. At each time increment the problem has to be solved iteratively for the variable $\Delta\varepsilon_n$. Next, we will give the main steps to carry out the iterative solution within an increment Δt_n, i.e., the prevision and correction steps.

a) Prevision

1. At the time t_n the following values are known: strains ε_n, damage parameters D_{t_n} and D_{c_n} and stresses σ_n.

2. Compute the elastic increment ΔN_n, or its corrected values ΔN_n^D when the deteriorated elastic model is assumed, i.e., to take into account the elastic solution corresponding to the anisotropic damaged solid.

b) Correction

1. From the previous try $\Delta\varepsilon_n^i$, compute the strain tensor making $\varepsilon_{n+1}^i = \varepsilon_n + \Delta\varepsilon_n^i$ and then all the damage model parameters, loading functions for instance: $F_t^1 = F_t\left(\varepsilon_{n+1}^1\,,\, D_{t_n}\,,\, D_{c_n}\right)$ and $F_c^1 = F_c\left(\varepsilon_{n+1}^1\,,\, D_{t_n}\,,\, D_{c_n}\right)$.

2. Depending on the Kuhn-Tucker conditions, one of the following alternatives has to be identified to define the damage parameter increment in tension and in compression: a) $F_t^1 \leq 0$ and $F_c^1 \leq 0$ (unloading situation); b) $F_t^1 > 0$ and $F_c^1 \leq 0$, (progressing damage in tension); c) $F_t^1 \leq 0$ and $F_c^1 > 0$ (progressing damage in compression); $F_t^1 > 0$ and $F_c^1 > 0$ (progressing damage in compression and in tension). Then, the damage parameter increments are computed accordingly.

3. From $\Delta\varepsilon_n^1$ and the damage parameter increments ΔD_t^1 and ΔD_c^1, one computes the stress increment$\Delta\sigma_n^1$.

4. Computing the residual forces $\left\{ Y\left(\Delta\varepsilon_n^1\right) \right\}$ to check the convergence. If it is less the tolerance previously chosen, all variables are up dated for t_{n+1} and return to the next prevision. Otherwise, go back to a new iteration, starting by up dating the tangent matrix.

6 Numerical example

The example presented here consists of carrying out numerically the Brazilian concrete tensile strength test (Figure 1). The compressive damage has been neglected for simplicity. Thus, D_t was the only damage parameter assumed for this analysis. Local and non-local models were adopted to simulate this classical test.

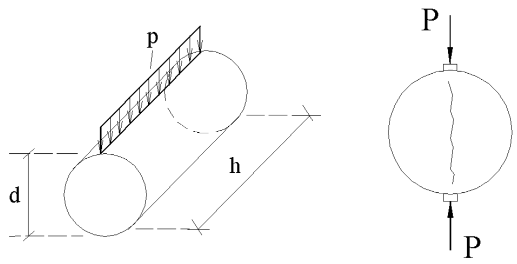

Figure 1: Brazilian concrete tensile strength test.

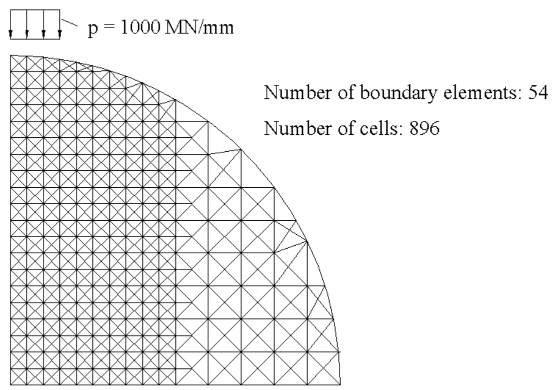

Figure 2: Discretization of a quarter of the cylinder cross-section.

The parameters chosen to conduct the analysis are: The Young's modulus E = *31000Mpa*; the Poisson's ratio 0.1; cylinder length h = *30cm*; diameter d = *15cm;* a_t= *0.333;* b_t = *4.0Mpa;* k_t= *20Mpa*^{2}; $c_t = 5.0; D_{0t} = 0.1; (\sigma_c/\sigma_0) = 0.8$; and ℓ_t=*0.1cm.*

The problem is analysed by discretizing only a quarter of the cylindrical cross-section shown in Figure 2. Consequently, vertical and horizontal displacements along the symmetrical axes are assumed equal zero. The guide traction is applied along three upper elements as shown in Figure 2.

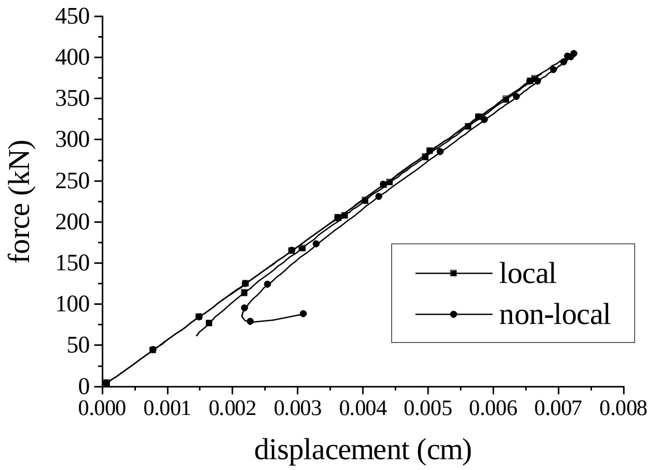

Figure 3: Force x displacement curves for local and non-local models

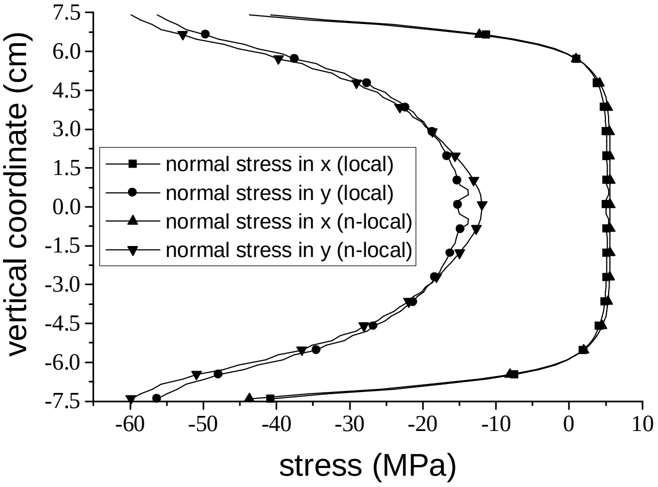

Figure 4: Computed stresses along the cylinder diameter.

The results in terms of vertical displacement x traction resultant computed for the top node are illustrated in Figure 3, while the stresses along the cylinder diameter are depicted in Figure 4. The damaged areas, after the total load has been applied, are illustrated in Figures 5 and 6 for the local and non-local models, respectively.

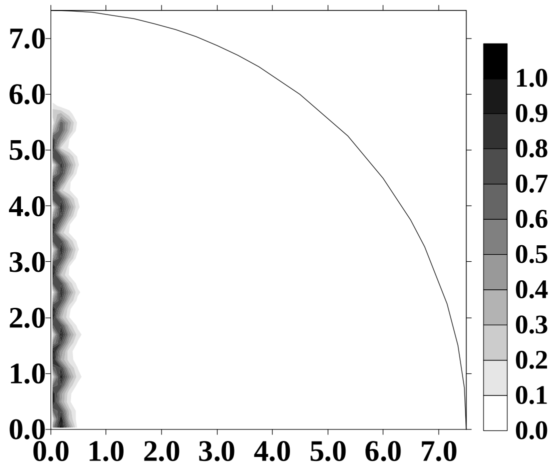

Figure 5: Damaged regions. Local model

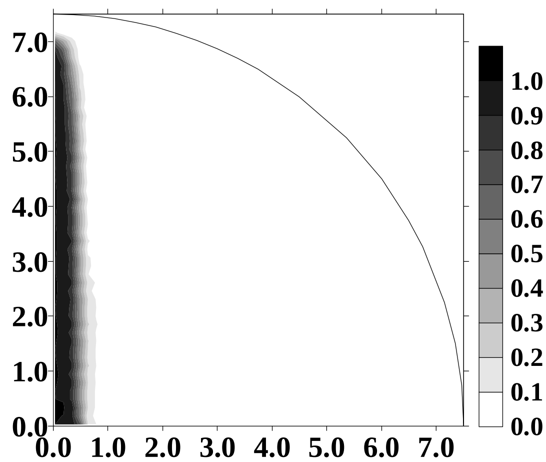

Figure 6: Damaged regions. Non-local model

7 Conclusions

The non-linear boundary element formulation has been extended to analyse damage mechanics problems. Very accurate integration along elements and over cells, together with the consistent tangent operator – CTO to solve non-linear problems have demonstrated to lead to reliable and accurate results, allowing capturing very complex behaviours present in solids with strongly deteriorated materials. The damage models particularly appropriate to represent concrete behaviour together with the proposed BEM formulation are able to indicate the crack initiation in the analysed example and other already tested. Besides that, the results obtained by using that scheme have pointed out the performance of that coupling technique.

Acknowledgements: To FAPESP – São Paulo State Research Foundation for the support given to this work.

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