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Rayleigh waves in isotropic microstretch thermoelastic diffusion solid half space

Abstract

This paper is devoted to the study of propagation of Rayleigh waves in a homogeneous isotropic microstretch generalized thermoelastic diffusion solid half-space. Secular equations in mathematical conditions for Rayleigh wave propagation are derived for stress free, insulated/impermeable and isothermal/isoconcentrated boundaries. The phase velocity, attenuation coefficient, the components of normal stress, tangential stress, tangential couple stress, microstress, temperature change and mass concentration are computed numerically. The path of surface particles is also obtained for the propagation of Rayleigh waves. The computationally stimulated results for the resulting quantities are represented to show the effect of thermally insulated, impermeable boundaries and isothermal, isoconcentrated boundaries alongwith the relaxation times. Some particular cases have also been deduced from the present investigation.

Keywords

Rayleigh waves; Frequency equation; Phase velocity; Attenuation coefficient; Microstretch

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1 INTRODUCTION

Eringen [1966, 1968] developed the theory of micromorphic bodies by considering a material point as endowed with three deformable directions. Subsequently, he developed the theory of microstretch elastic solid [1971] which is a generalization of micropolar elasticity [1966]. The material points in microstretch elastic body can stretch and contract independently of the translational and rotational processes. The difference between these solids and micropolar elastic solids stems from the presence of scalar microstretch and a vector first moment. These solids can undergo intrinsic volume change independent of the macro volume change and is accompanied by a non deviatoric stress moment vector.

Eringen [1990] also developed the theory of thermo microstretch elastic solids. The microstretch continuum is a model for Bravias lattice with a basis on the atomic level and a two phase dipolar solid with a core on the macroscopic level. For example, composite materials reinforced with chopped elastic fibres, porous media whose pores are filled with gas or inviscid liquid, asphalt or other elastic inclusions and 'solid-liquid' crystals, etc., should be characterizable by microstretch solids. A comprehensive review on the micropolar continuum theory has been given in his book by Eringen [1999].

Iesan and Pompei [1995] discussed the equilibrium theory of microstretch elastic solids. Propagation of Rayleigh surface waves in microstretch thermoelastic continua under inviscid fluid loadings have been investigated by Sharma et. al.[2008]. Quintanilla [2002] also developed the spatial decay for the dynamic problems of thermomicrostretch elastic solid. The plane waves of generalized thermomicrostretch elastic half space under three theories have been developed by Othman and Lotfy[2010].The propagation of free vibrations in microstretch thermoelastic homogeneous isotropic, thermally conducting plate bordered with layers of inviscid liquid on both sides subjected to stress free thermally insulated and isothermal conditions have been investigated by Kumar and Pratap [2009]. In recent times, Kumar and Kansal [2011] construct the fundamental solution of system of differential equations in the theory of thermomicrostretch elastic diffusive solids in case of steady oscillations in terms of elementary functions.

The thermodiffusion in elastic solids is due to coupling of fields of temperature, mass diffusion and that of strain in addition to heat and mass exchange with the environment. Nowacki [1974,1976] developed the theory of thermoelastic diffusion by using coupled thermoelastic model. Dudziak and Kowalski [1989] and Olesiak and Pyryev [1995] respectively, discussed the theory of thermodiffusion and coupled quasi-stationary problems of thermal diffusion for an elastic layer. Uniqueness and reciprocity theorems for the equations of generalized thermoelastic diffusion problem, in isotropic media, was proved by Sherief et al. [2004] on the basis of the variational principle equations, under restrictive assumptions on the elastic coefficients. Recently, Kumar and Kansal [2008] developed the basic equation of anisotropic thermoelastic diffusion based upon Green-Lindsay model.

Kumar and Kansal [2009] discussed the propagation of Rayleigh waves in a homogeneous transversely isotropic, generalized thermoelastic diffusive half-space. Sharma [2007, 2008] discussed the plane harmonic generalized thermoelastic diffusive waves and elasto thermodiffusive surface waves in heat-conducting solids. In recent times, Singh et. al [2012] discussed the Rayleigh wave in a rotating magneto-thermo-elastic half-plane.

Rayleigh surface waves have been well recognized in the study of earthquakes, seismology, geophysics and geodynamics. These types of surface waves propagate in half space. These waves have been of long standing interest in seismology, Richter [1958]. Keeping in view the above applications of microstretch thermoelastic diffusion processes, in what follows the propagation of Rayleigh waves in a homogeneous, isotropic, generalized microstretch thermoelastic diffusion half-space will be investigated. The phase velocity, attenuation coefficient, normal stress, tangential stress, microrotation, microstress, temperature change, and mass concentration, specific loss and path of surface particles of wave propagation are obtained from the secular equations. The resulting quantities are computed numerically and presented graphically.

2 BASIC EQUATIONS

Following Eringen [1999], Sherief et al. [2004] and Kumar & Kansal [2008], the equations of motion and the constitutive relations in a homogeneous isotropic microstretch thermoelastic diffusion solid in the absence of body forces, body couples, stretch force, and heat sources are given by

$$\left(\lambda+2\mu+K\right)\nabla\left(\nabla.\vec{u}\right)-\left(\mu+K\right)\nabla\times\nabla\times\vec{u}+K\nabla\times\vec{\varphi}+\lambda_O\nabla\varphi^*-\beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla T-\beta_2\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla C=\rho\frac{\partial^2\vec{u}}{\partial t^2}, \quad (1)$$

$$\left(\alpha+\beta+\gamma\right)\nabla\left(\nabla.\vec{\varphi}\right)-\gamma\,\nabla\times\left(\nabla\times\vec{\varphi}\right)+K\nabla\times\vec{u}-2K\vec{\varphi}=\rho j\frac{\partial^2\vec{\varphi}}{\partial t^2}, \quad (2)$$

$$\alpha_0\nabla^2\varphi^*+\nu_1\left(T+\tau_1\dot{T}\right)+\nu_2\left(C+\tau^1\dot{C}\right)-\lambda_1\varphi^*-\lambda_0\nabla.\vec{u}=\frac{\rho j_0}{2}\frac{\partial^2\varphi^*}{\partial t^2} \quad (3)$$

$$K^*\nabla^2T=\beta_1T_0\left(1+\varepsilon\tau_0\frac{\partial}{\partial t}\right)\nabla.\dot{\vec{u}}+\nu_1T_0\left(1+\varepsilon\tau_0\frac{\partial}{\partial t}\right)\dot{\varphi}^*+\rho C^*\left(1+\tau_0\frac{\partial}{\partial t}\right)\dot{T}+aT_0\left(\dot{C}+\gamma_1\ddot{C}\right), \quad (4)$$

$$D\beta_2\nabla^2(\nabla.\vec{u})+D\nu_2\nabla^2\varphi^*+Da\nabla^2\left(T+\tau_1\dot{T}\right)+\left(\dot{C}+\varepsilon\tau^0\ddot{C}\right)-Db\nabla^2\left(C+\tau^1\dot{C}\right)=0, \quad (5)$$

and constitutive relations are

$$t_{ij}=\lambda u_{r,r}\delta_{ij}+\mu\left(u_{i,j}+u_{j,i}\right)+K\left(u_{j,i}-\varepsilon_{ijr}\varphi_r\right)+\lambda_o\delta_{ij}\varphi^*-\beta_1(1+\tau_1\frac{\partial}{\partial t})T\delta_{ij}-\beta_2(1+\tau^1\frac{\partial}{\partial t})C\delta_{ij}, \quad (6)$$

$$m_{ij}=\alpha\varphi_{r,r}\delta_{ij}+\beta\varphi_{i,j}+\gamma\varphi_{j,i}+b_0\varepsilon_{mji}\varphi^*_{,m}, \quad (7)$$

$$\lambda^*_i=\alpha_0\varphi^*_{,i}+b_0\varepsilon_{ijm}\varphi_{j,m}, \quad (8)$$

where

$\lambda,\mu,\alpha,\beta,\gamma,K,\lambda_0,\lambda_1,\alpha_0,b_0$, are material constants ρ, is the mass density, $\vec{u}=\left(u_1,u_2,u_3\right)$ is the displacement vector and $\vec{\varphi}=\left(\varphi_1,\varphi_2,\varphi_3\right)$ is the microrotation vector, φ^* is the scalar microstretch function, T and T_0 are the small temperature increment and the reference temperature of the body chosen such that $|T/T_0| = 1$. C is the concentration of the diffusion material in the elastic body K^* is the coefficient of the thermal conductivity, C^* the specific heat at constant strain, D is the thermoelastic diffusion constant. a, b are, respectively, coefficients describing the measure of thermodiffusion and of mass diffusion effects, $\beta_1=\left(3\lambda+2\mu+K\right)\alpha_{t1}$, $\beta_2=\left(3\lambda+2\mu+K\right)\alpha_{c1}$,

$v_1 = (3\lambda + 2\mu + K)\alpha_{t2}$, $v_2 = (3\lambda + 2\mu + K)\alpha_{c2}, \alpha_{t1}, \alpha_{t2}$ are coefficients of linear thermal expansion and α_{c1}, α_{c2} are the coefficients of linear diffusion expansion. j is the microintertia, j_0 is the microinertia of the microelements, t_{ij} and m_{ij} are components of stress and couple stress tensors respectively, λ^*_i is the microstress tensor, $e_{ij} = \left(\frac{1}{2}\left(u_{i,j} + u_{j,i}\right)\right)$ are components of infinitesimal strain, e_{kk} is the dilatation, δ_{ij} is the Kronecker delta, τ^0, τ^1 are diffusion relaxation times with $\tau^1 \geq \tau^0 \geq 0$ and τ_0, τ_1 are thermal relaxation times with $\tau_1 \geq \tau_0 \geq 0$. Here $\tau_0 = \tau^0 = \tau_1 = \tau^1 = \gamma_1 = 0$ for Coupled Thermoelasitc (CT) model, $\tau_1 = \tau^1 = 0, \varepsilon = 1, \gamma_1 = \tau_0$ for Lord-Shulman (L-S) model and $\varepsilon = 0, \gamma_1 = \tau^0$ where $\tau^0 > 0$ for Green-Lindsay (G-L) model.
In the above equations, a comma followed by a suffix denotes spatial derivative and a superposed dot denotes the derivative with respect to time respectively.

3 FORMULATION OF THE PROBLEM

We consider a homogeneous isotropic microstretch generalized thermoelastic diffusion half-space initially at uniform temperature T_0. The origin of the coordinate system (x_1, x_2, x_3) is taken at any point on the plane horizontal surface with x_3- axis and pointing vertically downward to the half-space, which is thus represented by $x_3 \geq 0$. The surface $x_3 = 0$ is subjected to stress free boundary. We choose the x_1 axis in the direction of wave propagation in such a way that all the particles on a line parallel to the x_2 axis are equally displaced. Therefore, all field quantities are independent of the x_2 coordinate. For the two dimensional problem, we take

$$\vec{u}(x_1,x_3,t) = (u_1,0,u_3),\ \vec{\varphi} = (0,\varphi_2,0), \varphi^*(x_1,x_3,t), \mathrm{T}(x_1,x_3,t),\ C(x_1,x_3,t), \tag{9}$$

We define the following dimensionless quantities

$$\begin{aligned}
&\left(x_1', x_3'\right) = \frac{\omega^*}{c_1}(x_1, x_3), \left(u_1', u_3'\right) = \frac{\rho c_1 \omega^*}{\beta_1 T_o}(u_1, u_3), \varphi_2' = \frac{\rho c_1^2}{\beta_1 T_o}\varphi_2, \varphi^{*'} = \frac{\rho c_1^2}{\beta_1 T_o}\varphi^*, t_{ij}' = \frac{t_{ij}}{\beta_1 T_o}, m_{ij}' = \frac{\omega^*}{c_1 \beta_1 T_o} m_{ij}, \\
&\lambda_i^{*'} = \frac{\lambda_i^* \omega^*}{c_1 \beta_1 T_o}, T' = \frac{T}{T_o}, C' = \frac{\beta_2 C}{\rho^2 c_1^2}, t' = \omega^* t, \tau_o' = \omega^* \tau_o, \tau^{0'} = \omega^* \tau^0, \tau_1' = \omega^* \tau_1, \tau^{1'} = \omega^* \tau^1,
\end{aligned} \tag{10}$$

where

$\omega^* = \frac{\rho C^* c_1^2}{K^*}$, $c_1^2 = \frac{\lambda + 2\mu + K}{\rho}$, ω^* is the characteristic frequency of the medium, Upon introducing the quantities (10) in equations (1)-(5), with the aid of (9) and after suppressing the primes, we obtain

$$\delta^2 \frac{\partial e}{\partial x_1} + \left(1-\delta^2\right)\nabla^2 u_1 - a_1 \frac{\partial \varphi_2}{\partial x_3} + a_2 \frac{\partial \varphi^*}{\partial x_1} - \tau_t^1 \frac{\partial T}{\partial x_1} - a_3 \tau_c^1 \frac{\partial C}{\partial x_1} = \frac{\partial^2 u_1}{\partial t^2}, \tag{11}$$

$$\delta^2 \frac{\partial e}{\partial x_3} + \left(1-\delta^2\right)\nabla^2 u_3 + a_1 \frac{\partial \varphi_2}{\partial x_1} + a_2 \frac{\partial \varphi^*}{\partial x_3} - \tau_t^1 \frac{\partial T}{\partial x_3} - a_3 \tau_c^1 \frac{\partial C}{\partial x_3} = \frac{\partial^2 u_3}{\partial t^2}, \tag{12}$$

$$a_4 \nabla^2 \varphi_2 + a_5 \left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1} \right) - a_6 \varphi_2 = \frac{\partial^2 \varphi_2}{\partial t^2}, \tag{13}$$

$$\left(\delta_1^2 \nabla^2 - a_7\right)\varphi^* - a_8 e + a_9 \tau_t^1 T + a_{10} \tau_c^1 C = \frac{\partial^2 \varphi^*}{\partial t^2}, \tag{14}$$

$$\nabla^2 T = a_{11} \tau_e^0 \frac{\partial e}{\partial t} + a_{12} \tau_e^0 \frac{\partial \varphi^*}{\partial t} + \tau_t^0 \frac{\partial T}{\partial t} + a_{13} \tau_c^0 \frac{\partial C}{\partial t}, \tag{15}$$

$$a_{14} \nabla^2 e + a_{21} \nabla^2 \varphi^* + a_{15} \tau_t^1 \nabla^2 T + \tau_f^0 \frac{\partial C}{\partial t} - a_{16} \tau_c^1 \nabla^2 C = 0, \tag{16}$$

where

$$\left(a_1, a_2\right) = \frac{1}{\rho c_1^2}\left(K, \lambda_0\right), a_3 = \frac{\rho c_1^2}{\beta_1 T_0}, \left(a_4, a_5, a_6\right) = \frac{1}{j\rho}\left(\frac{\gamma}{c_1^2}, \frac{K}{\omega^{*2}}, \frac{2K}{\omega^{*2}} \right), \delta^2 = \frac{\lambda + \mu}{\rho c_1^2}$$

$$\left(a_{11}, a_{12}, a_{13}\right) = \frac{1}{K^* \omega^*}\left(\frac{T_0 \beta_1^2}{\rho}, \frac{\beta_1 T_0 \nu_1}{\rho}, \frac{\rho c_1^4 a}{\beta_2} \right), \left(a_{14}, a_{15}, a_{16}\right) = \frac{D\omega^*}{c_1^2}\left(\frac{\beta_1^2}{\rho c_1^2}, \frac{\beta_2 a}{\beta_1}, b \right)$$

$$\left(a_7, a_8, a_9, a_{10}\right) = \frac{2}{j_0 \omega^{*2}}\left(\frac{\lambda_1}{\rho}, \frac{\lambda_0}{\rho}, \frac{\nu_1 c_1^2}{\beta_1}, \frac{\nu_2 \rho c_1^4}{\beta_1 \beta_2 T_0} \right), \delta_1^2 = \frac{c_2^2}{c_1^2}, c_2^2 = \frac{2\alpha_0}{\rho j_0}, a_{21} = \frac{D \nu_2 \beta_2 \omega^*}{\rho c_1^4}$$

$$\tau_t^1 = 1 + \tau_1 \frac{\partial}{\partial t}, \tau_c^1 = 1 + \tau^1 \frac{\partial}{\partial t}, \tau_f^0 = 1 + \varepsilon \tau^0 \frac{\partial}{\partial t}, \tau_t^0 = 1 + \tau_0 \frac{\partial}{\partial t},$$

$$\tau_e^0 = 1 + \varepsilon \tau_0 \frac{\partial}{\partial t}, \tau_c^0 = 1 + \gamma_1 \frac{\partial}{\partial t}, e = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}, \nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}$$

We introduce the potential functions ϕ *and* ψ through the relations

$$u_1 = \frac{\partial \phi}{\partial x_1} - \frac{\partial \psi}{\partial x_3}, u_3 = \frac{\partial \phi}{\partial x_3} + \frac{\partial \psi}{\partial x_1}, \tag{17}$$

in the equations (11)-(16), we obtain

$$\nabla^2 \phi + a_2 \varphi^* - \tau_t^1 T - a_3 \tau_c^1 C = \ddot{\phi}, \tag{18}$$

$$\left(1-\delta^2\right)\nabla^2 \psi + a_1 \varphi_2 = \ddot{\psi}, \tag{19}$$

$$\left(a_4 \nabla^2 - a_6\right)\varphi_2 - a_5 \nabla^2 \psi = \ddot{\varphi}_2, \tag{20}$$

$$\left(\delta_1^2 \nabla^2 - a_7\right)\varphi^* - a_8 \nabla^2 \phi + a_9 \tau_t^1 T + a_{10} \tau_c^1 C = \ddot{\varphi}^*, \tag{21}$$

$$\nabla^2 T = \tau_e^0 \left(a_{11} \nabla^2 \dot{\phi} + a_{12} \dot{\varphi}^*\right) + \tau_t^0 \dot{T} + a_{13} \tau_c^0 \dot{C}, \tag{22}$$

$$a_{14} \nabla^4 \phi + a_{21} \nabla^2 \varphi^* + a_{15} \tau_t^1 \nabla^2 T - a_{16} \tau_c^1 \nabla^2 C + \tau_f^0 \dot{C} = 0 \tag{23}$$

4 SOLUTION OF THE PROBLEM

We assume the solutions of the form

$$\left\{\phi, \varphi^*, T, C\right\}\left(x_1, x_3, t\right) = \left\{\bar{\phi}, \bar{\varphi}^*, \bar{T}, \bar{C}\right\}\left(x_3\right) e^{i\xi\left(x_1 - ct\right)} \tag{24}$$

where ξ is the wave number, $\omega = \xi c$ is the angular frequency, and c is phase velocity of the wave. Using (24) in equations (18), (21)–(23), we obtain a system of four homogeneous equations in four unknowns $\bar{\phi}, \bar{\varphi}^*, \bar{T}$ and $\bar{C}$ which for the nontrivial solution yields

$$D^8 + A_1^* D^6 + B_1^* D^4 + C_1^* D^2 + D_1^* = 0 \tag{25}$$

where $D = d/dx_3$, and the coefficients A_1^*, B_1^*, C_1^* and D_1^* are given in appendix A.

Let the roots of equation (25) be denoted by m_p^2 $(p = 1,2,3,4)$. Four positive values of c in the descending order will be the velocities of propagation of four possible waves, namely longitudinal displacement wave (LD), mass diffusion wave (MD), thermal wave (T) and longitudinal microstretch wave (LM), respectively. Since we are interested in surface waves only, it is essential that the motion is confined to the free surface of $x_3 = 0$ in the half-space so that the characteristic roots satisfy the radiation conditions $\mathrm{Re}\left(m_p\right) \geq 0, (p = 1,2,3,4)$. Thus, the solution of field equations takes the form

$$\left\{\phi, \varphi^*, T, C\right\} = \sum_{p=1}^{4} \left[A_p \left\{1, n_{1p}, n_{2p}, n_{3p}\right\} e^{-m_p x_3} \right] e^{i\xi\left(x_1 - ct\right)} \tag{26}$$

where A_p $(p = 1,2,3,4)$ are arbitrary constants.

The coupling constants n_{1p}, n_{2p}, n_{3p} are given in appendix B.

Similarly, we assume the solutions of the field equations as

$$\left\{\psi, \varphi_2\right\}\left(x_1, x_3, t\right) = \left\{\bar{\psi}, \bar{\varphi}_2\right\}\left(x_3\right) e^{i\xi\left(x_1 - ct\right)} \tag{27}$$

using (27) in equations (19) and (20), we obtain a system of two homogeneous equations in two unknowns $\bar{\psi}$ and $\bar{\varphi}_2$ which for the nontrivial solution yields

$$D^4 + A_2^* D^2 + B_2^* = 0 \tag{28}$$

where

$$A_2^* = \left(a_4 b_{26} + b_{27}(1-\delta^2) + a_1 a_5\right) \Big/ a_4(1-\delta^2), B_2^* = \left(b_{26} b_{27} - a_1 a_5 \xi^2\right) \Big/ a_4(1-\delta^2),$$

Let the roots of equation (28) be denoted by m_p^2 $(p = 5,6).$ Two positive values of c in the descending order will be the velocities of propagation of two coupled transverse displacement and transverse microrotational waves (CD I, CD II), respectively. Since we are interested in surface waves only, it is essential that the motion is confined to the free surface of $x_3 = 0$ in the half-space so that the characteristic roots satisfy the radiation conditions $\mathrm{Re}\left(m_p\right) \geq 0, (p = 5,6)$. Thus, the solution of field equations takes the form

$$\left\{\psi, \varphi_2\right\} = \sum_{p=5}^{6} \left[A_p \left\{1, n_{4p}\right\} e^{-m_p x_3} \right] e^{i\xi\left(x_1 - ct\right)} \tag{29}$$

where A_p $(p = 5,6)$ are arbitrary constants.

and $n_{4p} = a_5\left(m_p^2 - \xi^2\right) \Big/ \left(b_{27} + a_4 m_p^2\right)$, for $\left(p = 5,6\right)$

5 BOUNDARY CONDITIONS

The appropriate boundary conditions at the surface $x_3 = 0$, are

$$t_{33} = 0, \tag{30}$$

$$t_{31} = 0, \tag{31}$$

$$m_{32} = 0, \tag{32}$$

$$\lambda_3^* = 0, \tag{33}$$

$$\frac{\partial T}{\partial x_3} + h_1 T = 0, \quad \begin{cases} h_1 \to 0 \text{ corresponds to thermally insultated boundary} \\ h_1 \to \infty \text{ corresponds to isothermal boundary} \end{cases} \tag{34}$$

$$\frac{\partial C}{\partial x_3} + h_2 C = 0, \quad \begin{cases} h_2 \to 0 \text{ corresponds to impermeable boundary} \\ h_2 \to \infty \text{ refers to isoconcentrated boundary} \end{cases} \tag{35}$$

where

$$t_{33} = \frac{\partial u_3}{\partial x_3} + b_1 \frac{\partial u_1}{\partial x_1} - \tau_t^1 T - \tau_c^1 C + a_2 \varphi^*, \; t_{31} = b_2 \frac{\partial u_1}{\partial x_3} + b_3 \frac{\partial u_3}{\partial x_1} - a_1 \varphi_2,$$

$$m_{32} = b_4 \frac{\partial \varphi_2}{\partial x_3} + b_5 \frac{\partial \varphi^*}{\partial x_1}, \; \lambda_3^* = b_6 \frac{\partial \varphi^*}{\partial x_3} - b_5 \frac{\partial \varphi_2}{\partial x_1},$$

and $\left(b_1, b_2, b_3\right) = \dfrac{1}{\rho c_1^2}\left(\lambda, \mu + K, \mu\right), \left(b_4, b_5, b_6\right) = \dfrac{\omega^{*2}}{\rho c_1^4}\left(\gamma, b_o, \alpha_0\right)$

6 DERIVATIONS OF THE SECULAR EQUATIONS

Making use of equations (26) and (29) in the equations (30)-(35), we obtain a system of six simultaneous linear equations:

$$\sum_{p=1}^{6} k_{1p} A_p = 0, \; \sum_{p=1}^{6} k_{2p} A_p = 0, \; \sum_{p=1}^{6} k_{3p} A_p = 0, \; \sum_{p=1}^{6} k_{4p} A_p = 0, \; \sum_{p=1}^{6} k_{5p} A_p = 0, \; \sum_{p=1}^{6} k_{6p} A_p = 0 \tag{36}$$

where

$$k_{1p} = \begin{cases} m_p^2 - b_1\xi^2 + a_2 n_{1p} + n_{2p}\left(i\xi c\tau_1 - 1\right) + n_{3p}\left(i\xi c\tau^1 - 1\right), \ for(p=1,2,3,4) \\ i\xi m_p\left(b_1 - 1\right), \ for(p=5,6), \end{cases},$$

$$k_{2p} = \begin{cases} -i\xi m_p\left(b_2 + b_3\right), \ for(p=1,2,3,4) \\ -\left(b_2 m_p^2 + b_3\xi^2 + a_1 n_{4p}\right), \ for(p=5,6) \end{cases}, \quad k_{3p} = \begin{cases} i\xi b_5 n_{1p}, \ for(p=1,2,3,4) \\ -b_4 n_{4p} m_p, \ for(p=5,6) \end{cases},$$

$$k_{4p} = \begin{cases} n_{1p} m_p b_6, \ for(p=1,2,3,4) \\ i\xi b_5 n_{4p}, \ for(p=5,6) \end{cases}, \quad k_{5p} = \begin{cases} \left(h_1 - m_p\right) n_{2p}, \ for(p=1,2,3,4) \\ 0, \ for(p=5,6) \end{cases},$$

$$k_{6p} = \begin{cases} \left(h_2 - m_p\right) n_{3p}, \ for(p=1,2,3,4) \\ 0, \ for(p=5,6) \end{cases},$$

The system of equations (36) has a non-trivial solution if the determinant of amplitudes A_P, (p=1, 2, 3, 4, 5, 6) vanishes which leads to the secular equation

$$\begin{vmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} & k_{46} \\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55} & k_{56} \\ k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & k_{66} \end{vmatrix}_{6\times 6} = 0 \tag{37}$$

Equation (37) is the frequency equation for the propagation of Rayleigh waves in a microstretch thermoelastic diffusion solid. This equation has complete information about the phase velocity, wave number, and attenuation coefficient of the surface waves propagating in such a medium. If we write

$$c^{-1} = v^{-1} + i\omega^{-1} F \tag{38}$$

so that, $\xi = K + iF$, where $K = \dfrac{\omega}{\nu}$, ν and F are real. Also the roots of equations (25) and (28) are, in general complex, and hence we assume that $m_p = p_p + iq_p$, so that the exponent in the plane wave solutions (26) and (29) for the half-space becomes

$$iK\left(x_1 - m_p^I x_3 - \nu t\right) - K\left(\frac{F}{K}x_1 + m_p^R x_3\right), \quad \left(p = 1,2,3,4,5,6\right) \tag{39}$$

where

$$m_p^R = p_p - q_p \frac{F}{K}, m_p^I = q_p + p_p \frac{F}{K}, \quad \left(p = 1,2,3,4,5,6\right) \tag{40}$$

This shows that ν is the propagation velocity and F is the attenuation coefficient of wave.

The equations (26) and (29) can be rewritten as

$$\left\{\phi, \varphi^*, T, C\right\} = \sum_{p=1}^{4} A_p \left\{1, n_{1p}, n_{2p}, n_{3p}\right\} \exp\left(-Fx_1 - \lambda_p^R x_3\right) \exp\left[i\left(K\left(x_1 - \nu t\right) - \lambda_p^I x_3\right)\right], \tag{41}$$

$$\left\{\psi, \varphi_2\right\} = \sum_{p=5}^{6} A_p \left\{1, n_{4p}\right\} \exp\left(-Fx_1 - \lambda_p^R x_3\right) \exp\left[i\left(K\left(x_1 - \nu t\right) - \lambda_p^I x_3\right)\right], \tag{42}$$

with $\lambda_p = K\left(m_p^R + i m_p^I\right) = \lambda_p^R + i\lambda_p^I \left(say\right), \left(p = 1,2,3,4,5,6\right)$. Moreover, it is clear that

$$\left|\lambda_p^R\right|^2 - \left|\lambda_p^I\right|^2 = K^2\left[\left(m_p^R\right)^2 - \left(m_p^I\right)^2\right], \left|\lambda_p^R\right|\left|\lambda_p^I\right| \cos\Phi^* = \frac{1}{2}K^2 m_p^R m_p^I \tag{43}$$

where Φ^* is the angle between the real and imaginary parts of the complex vector λ_p.

Therefore the phase plane (the phase vertical to vector λ_p^R) and the amplitude plane (the phase vertical to vector λ_p^I) are not parallel to each other, and, hence, the maximum attenuation is not along the direction of wave propagation, but along the direction of vector λ_p^R.

7 SURFACE DISPLACEMENTS, MICROROTATION, MICROSTRETCH, TEMPERATURE CHANGE AND MASS CONCENTRATION

The amplitudes of surface displacements, microrotation, microstretch, temperature change, and mass concentration at the surface $x_3 = 0$ during Rayleigh wave propagation in case of stress-free boundaries of half-space are:

$$\begin{aligned} \left\{u_{1s}, \varphi_s^*, T_s, C_s\right\} &= \left(H_1^*, G^*, F^*, I^*\right) Q \exp\left[iK\left(x_1 - \nu t\right)\right], \\ \left\{u_{3s}, \varphi_{2s}\right\} &= \left(L_1^*, M^*\right) R \exp\left[ik\left(x_1 - \nu t\right)\right], \end{aligned} \tag{44}$$

where

$$\begin{aligned} &Q = A_1 \exp(-Fx_1),\ \ R = A_5 \exp(-Fx_1), H_1^* = \left(iK - F\right)(D_1 - D_2 + D_3 - D_4)\big/ D_1, \\ &G^* = (n_{11} D_1 - n_{12} D_2 + n_{13} D_3 - n_{14} D_4)\big/ D_1, F^* = (n_{21} D_1 - n_{22} D_2 + n_{23} D_3 - n_{24} D_4)\big/ D_1, \\ &I^* = (n_{31} D_1 - n_{32} D_2 + n_{33} D_3 - n_{34} D_4)\big/ D_1, L_1^* = \left(iK - F\right)(D_5 - D_6)\big/ D_5, M^* = (n_{45} D_5 - n_{46} D_6)\big/ D_5, \end{aligned} \tag{45}$$

8 PATH OF SURFACE PARTICLES

We shall now discuss the path of the particles at the surface $x_3 = 0$. On the surface $x_3 = 0$, we have

$$u_{1s} = XQ \exp\left\{-i(\alpha - q)\right\}, \quad u_{3s} = YR \exp\left\{-i(\beta - q)\right\} \tag{46}$$

where $H_1^* = X \exp\left\{-i\alpha\right\},\ L_1^* = Y \exp\left\{-i\beta\right\},\ q = K\left(x_1 - \nu t\right)$. Using the Euler representation of complex numbers and simplifying, we obtain from Eq. (46) (retaining only real parts)

$$u_{1s} = XQ\cos(\alpha - q),\ u_{3s} = YR\cos(\beta - q), \tag{47}$$

Eliminating q from Eq. (47), we get

$$\frac{u_{1s}^2}{X^2} + \frac{u_{3s}^2}{Y^2} - 2\frac{u_{1s}u_{3s}}{XY}\cos(\alpha - \beta) = A^2 \sin^2(\alpha - \beta), \tag{48}$$

Since

$$\frac{\cos^2(\alpha - \beta)}{X^2Y^2} - \frac{1}{X^2Y^2} = -\frac{\sin^2(\alpha - \beta)}{X^2Y^2} < 0,$$

Equation (47) represents an ellipse with semimajor axis X^* semiminor axis Y^*, and eccentricity e which are given by

$$X^{*2} = \frac{2A^2X^2Y^2\sin^2(\alpha-\beta)}{X^2+Y^2-\left[\left(X^2-Y^2\right)^2+4X^2Y^2\cos^2(\alpha-\beta)\right]^{1/2}}, \tag{49}$$

$$Y^{*2} = \frac{2A^2X^2Y^2\sin^2(\alpha-\beta)}{X^2+Y^2+\left[\left(X^2-Y^2\right)^2+4X^2Y^2\cos^2(\alpha-\beta)\right]^{1/2}}, \tag{50}$$

$$e^2 = \frac{2\left[\left(X^2-Y^2\right)^2+4X^2Y^2\cos^2(\alpha-\beta)\right]^{1/2}}{X^2+Y^2+\left[\left(X^2-Y^2\right)^2+4X^2Y^2\cos^2(\alpha-\beta)\right]^{1/2}}, \tag{51}$$

If θ^* is the inclination of the major axis to the wave normal, then

$$\tan 2\theta^* = \frac{2XY\cos(\alpha-\beta)}{Y^2-X^2}$$

Thus, the surface particles trace elliptical paths given by Equation (48) in the vertical planes parallel to the direction of wave propagation. The semi-axes depend upon $Q = A_1\exp(-Fx_1)$, $R = A_5\exp(-Fx_1)$ and hence increase or decrease exponentially. The decay of elliptical paths of surface particles is clearly a function of attenuation coefficient F.

9 PARTICULAR CASES

(i) Take $\tau^0 > 0,\ \varepsilon = 0$ and $\gamma_1 = \tau^0$ in equation (37), yield the expression of secular equation for the propagation of Rayleigh wave in microstretch thermoelastic diffusion solid half-spaces with two relaxation times.

(ii) Using $\tau_1 = \tau^1 = 0, \gamma_1 = \tau_0$ and $\varepsilon = 1$ in equations (37), gives the corresponding results for the propagation of Rayleigh wave in microstretch thermoelastic diffusion solid half-spaces with with one relaxation time.

(iii) On taking $\tau_0 = \tau^0 = \tau_1 = \tau^1 = \gamma_1 = 0$ in equations (37), provide the corresponding expression of secular equation for the propagation of Rayleigh wave in microstretch thermoelastic diffusion solid half-spaces with Coupled Thermoelastic (CT) theory.

(iv) In absence of thermal, stretch and diffusion effects, the equation (37) will be reduced to the micropolar elastic medium as obtained by De and Sen-Gupta [1974].

10 NUMERICAL RESULTS AND DISCUSSION

The analysis is conducted for a magnesium crystal-like material. Following Eringen [1984], the values of micropolar parameters are

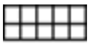
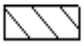
$$\lambda = 9.4\times 10^{10}\mathrm{Nm}^{-2}, \mu = 4.0\times 10^{10}\mathrm{Nm}^{-2},\ K = 1.0\times 10^{10}\, Nm^{-2},$$
$$\rho = 1.74\times 10^{3}\, Kgm^{-3}, j = 0.2\times 10^{-19}\, m^{2}, \gamma = 0.779\times 10^{-9}\, N$$

Thermal and diffusion parameters are given by

$$C^{*} = 1.04\times 10^{3}\, JKg^{-1}K^{-1},\ \mathrm{K}^{*} = 1.7\times 10^{6}\, Jm^{-1}s^{-1}K^{-1}, \alpha_{\mathrm{t1}} = 2.33\times 10^{-5}\mathrm{K}^{-1}, \alpha_{\mathrm{t2}} = 2.48\times 10^{-5}\mathrm{K}^{-1},$$
$$T_{0} = .298\times 10^{3}\mathrm{K}, \tau_{1} = 0.01, \tau_{0} = 0.02, \alpha_{\mathrm{c1}} = 2.65\times 10^{-4}\mathrm{m}^{3}\mathrm{Kg}^{-1}, \alpha_{\mathrm{c2}} = 2.83\times 10^{-4}\mathrm{m}^{3}\mathrm{Kg}^{-1},$$
$$a = 2.9\times 10^{4}\mathrm{m}^{2}\mathrm{s}^{-2}\mathrm{K}^{-1}, b = 32\times 10^{5}\mathrm{Kg}^{-1}\mathrm{m}^{5}\mathrm{s}^{-2}, \tau^{1} = 0.04,, \tau^{0} = 0.03, D = 0.85\times 10^{-8}\, Kgm^{-3}s$$

and, the microstretch parameters are taken as

$$j_{o} = 0.19\times 10^{-19}\, m^{2}, \alpha_{o} = 0.779\times 10^{-9}\, N, b_{o} = 0.5\times 10^{-9}\, N, \lambda_{o} = 0.5\times 10^{10}\, Nm^{-2}, \lambda_{1} = 0.5\times 10^{10}\, Nm^{-2}$$

MATLAB software 7.04 has been used for numerical computation of the resulting quantities. The values of phase velocity and attenuation coefficient with wave number at the stress free boundary with thermally insulated and impermeable boundaries, isothermal and isoconcentrated boundaries alongwith the relaxation times are shown in fig.1 and fig.2. Normal stress, tangential stress, tangential couple stress, microstress, temperature change, and mass concentration with wave number has been determine at the surface $x_3 = 1$, and are shown in figs.3-8. In all figures, the words LSH10 and GLH10 symbolize the graphs of L-S and G-L theories in microstretch thermoelastic diffusion medium for the thermally insulated boundary and impermeable boundary and are represented by  and  respectively, while, the words LSH1N and GLH1N symbolize the graphs of L-S and G-L theories in microstretch thermoelastic diffusion medium for the isothermal boundary and isoconcentrated boundary and represented by  and  respectively.

Phase Velocity

Fig.1 exhibits the variation of phase velocity with wave number ξ. The values of phase velocity at the isothermal boundary and isoconcentrated boundary decrease monotonically for smaller values of ξ, whereas for higher values of ξ the values of phase velocity decrease smoothly and finally become dispersionless, nevertheless, for smaller values ξ, a significant difference in the values of phase velocity for LSH10 and GLH10 are noticed due to thermally insulated and impermeable boundaries when compared with LSH1N and GLH1N related to isothermal boundary and isoconcentrated boundaries, respectively.

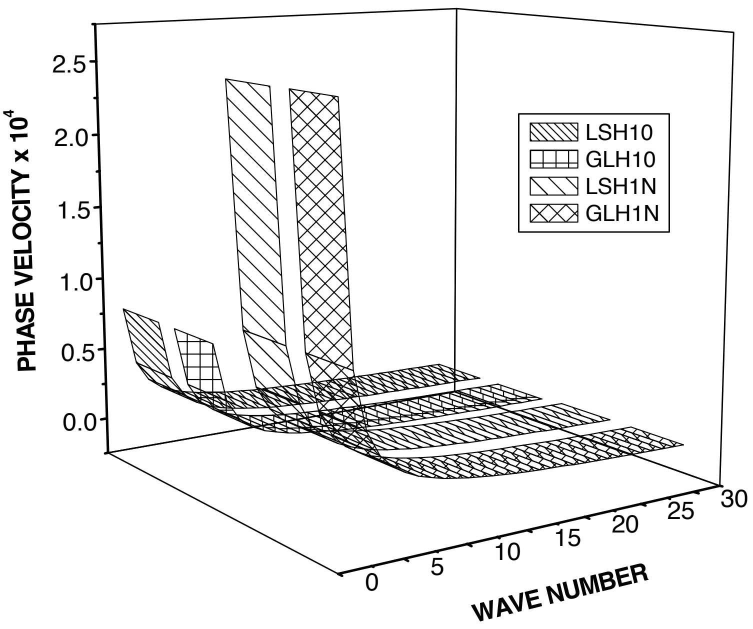


Figure 1 Variation of phase velocity w.r.t wave number

Attenuation

Fig.2 depicts the variation of attenuation with wave number ξ. The trend of variation and behavior of attenuation for LSH10 is opposite to LSH1N and GLH10 is opposite to GLH1N for $0 \le \xi \le 7$, the values of attenuation for smaller values of wave number increase sharply for LSH10 and GLH10, while, decrease strictly for LSH1N and GLH1N which becomes dispersionless for higher values of ξ for all the cases. **Fig.3** shows the variation of normal stress component T_{33} with wave number ξ. The behavior of T_{33} is oscillating for $0 \le \xi \le 12$ and stable for $20 \le \xi \le 30$ attaining maximum at $\xi = 5$ for all the cases while the corresponding values are different in magnitude. The values of T_{33} are more in case of LSH10 and small for LSH1N, the similar behavior can be noticed for GLH10 and GLH1N respectively.

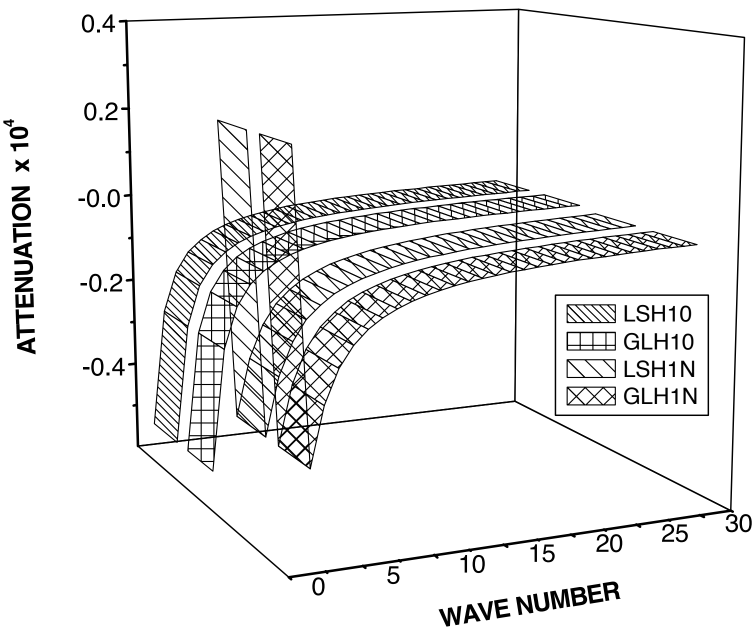


Figure 2 Variation of attenuation w.r.t wave number

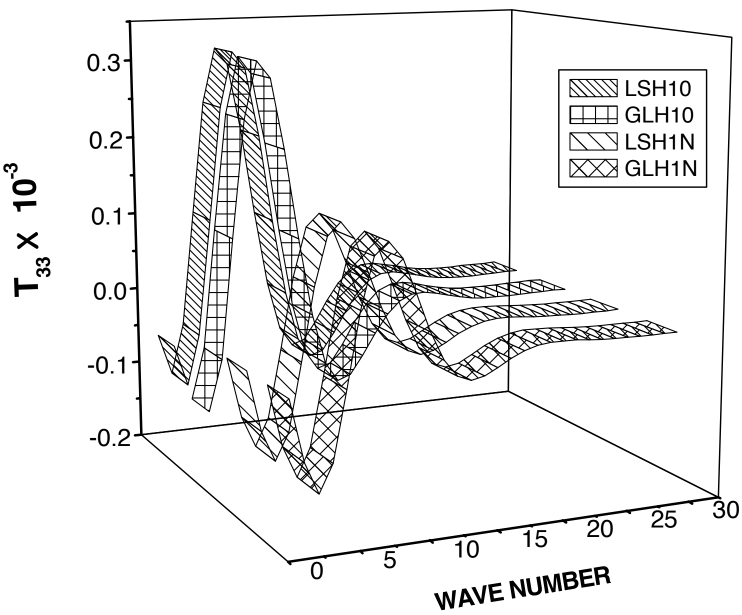


Figure 3 Variation of T_{33} w.r.t wave number

Fig.4 depicts the variation of tangential stress component T_{31} with wave number ξ. The trend of variation and behavior of T_{31} is similar to T_{33}, but, the corresponding values are different in magnitude. An appreciable difference due to various boundaries is noticed for the values of T_{31}. **Figs.5-6** exhibits the variations of couple stress components m_{32} and microstress λ_3^* with wave number. The graph of m_{32} and λ_3^* shows similar behavior, but their corresponding values are different in magnitude. For the higher value of ξ, the values of m_{32} and λ_3^* are convergent, after showing a respectable oscillation for smaller values of ξ. Adequate difference at $\xi = 7$, in the values of m_{32} is noticed corresponding to all the cases, while a major difference can be noticed due to the different boundaries.

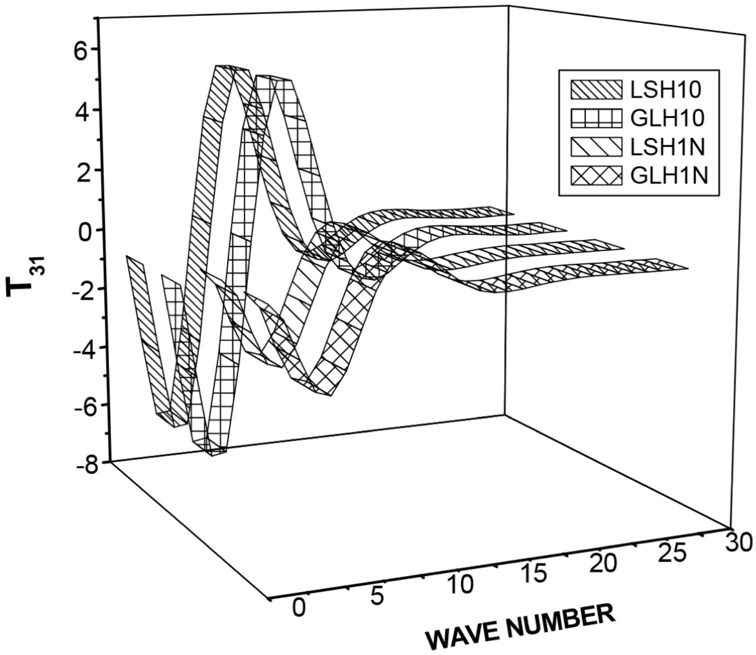


Figure 4 Variation of T_{31} w.r.t wave number

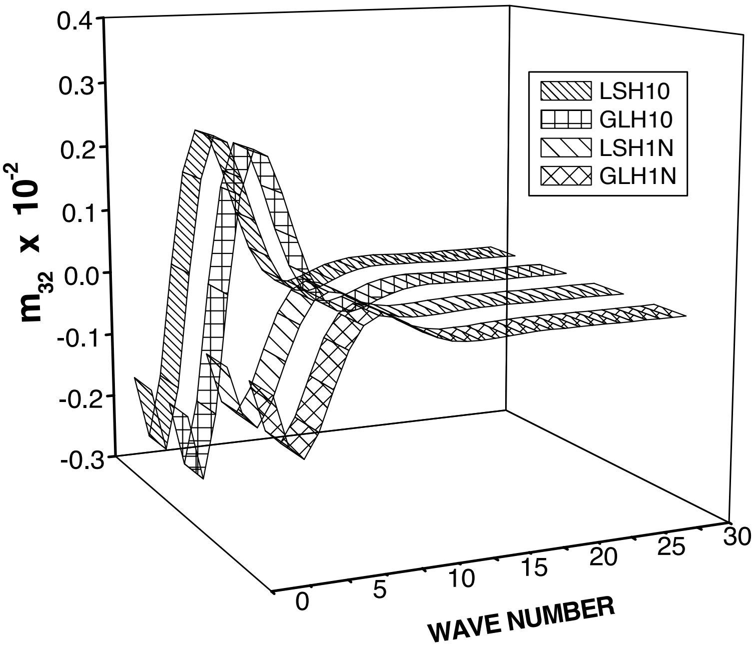


Figure 5 Variation of m_{32} w.r.t wave number

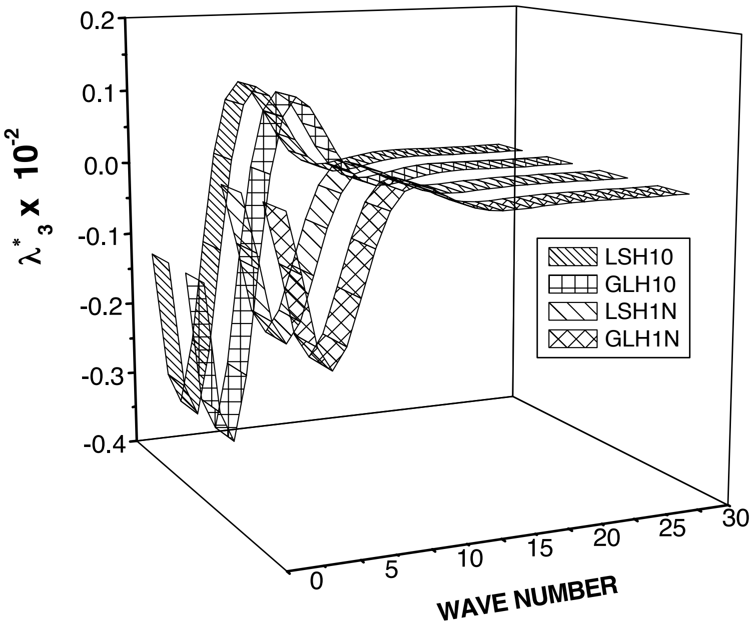


Figure 6 Variation of w.r.t wave number

Fig.7 depicts the variation of concentration C with wave number ξ. The value of C first increase for $0 \le \xi \le 5$, become oscillatory for $5 < \xi \le 15$ and finally got dispersionless for $15 < \xi \le 30$. Concentration attains its minimum value for LSH10, while maximum value for LSH1N, at $\xi = 4$. Similar trend is noticed by GLH10 and LSH1N, however, corresponding values are different in magnitude. **Fig.8** exhibits the variation of temperature change T with wave number ξ. The graph indicates that the values T for LSH10 and GLH10 decreases monotonically for $0 \le \xi \le 6$, increase smoothly for $6 < \xi \le 10$ and becomes stable further, on the other hand, a good difference in the value of T can be noticed in cases of GLH10 when compared to GLH1N, for smaller value of wave number.

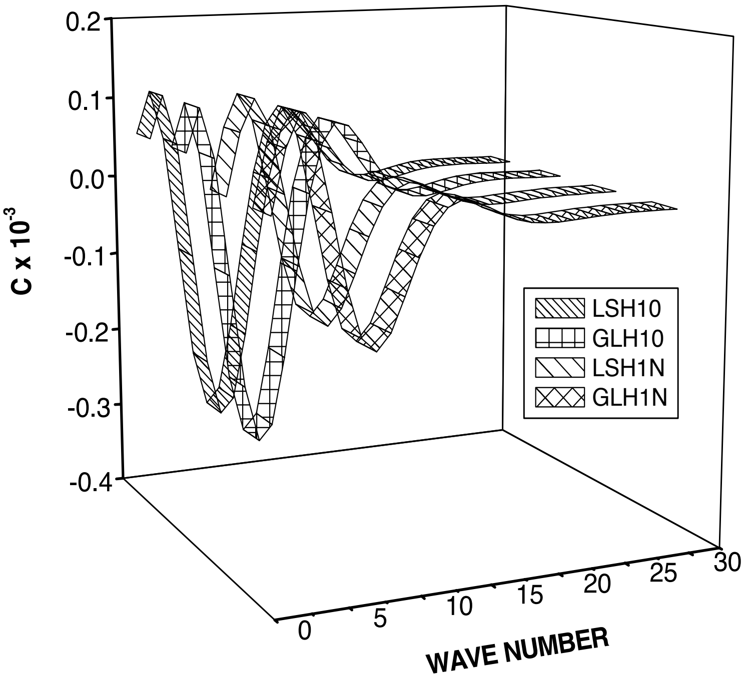


Figure 7 Variation of concentration 'C' w.r.t wave number

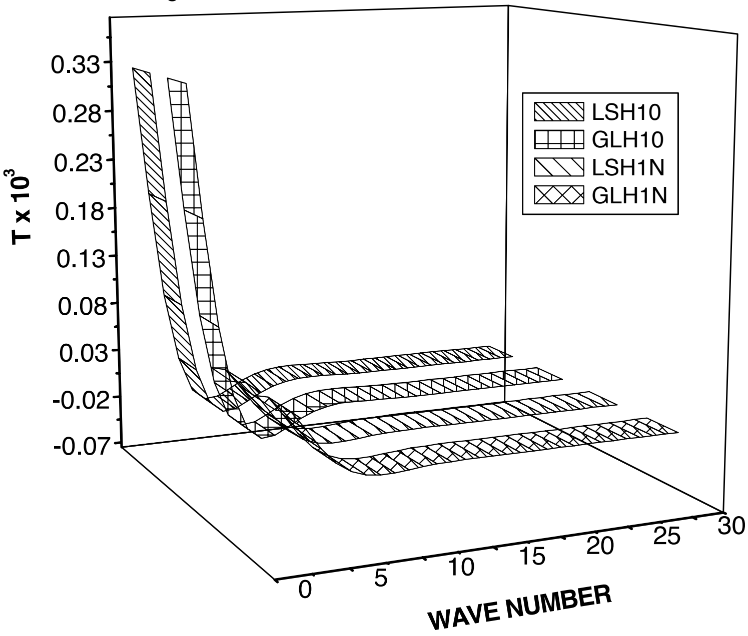


Figure 8 Variation of temperature 'T' w.r.t wave number

11 CONCLUSION

The propagation of Rayleigh waves in a homogeneous isotropic microstretch thermoelastic diffusion solid half-space subjected to stress-free, tangential couple stress, microstress , thermally insulated/isothermal, and impermeable/isoconcentrated boundary conditions has been investigated. Secular equations for surface wave propagation in the considered media are derived. Appreciable effects of relaxation times on the phase velocity, attenuation coefficient, normal stress, tangential stress, couple stress, microstress, temperature change, and mass concentration have been observed. It is observed that the trend of variation and behavior of the all derived components converges towards zero with the increase of wave number. It has also been noticed that the graphs of $T_{33}, T_{31}, m_{32}, \lambda_3^*, C$ and T shows respectable oscillation for $0 \le \xi \le 15$ and finally becomes dispersionless. The magnitude of components in all the graphs, except the phase velocity are more in case of thermally insulated and impermeable boundaries when compared to isothermal and isoconcentrated boundaries, respectively.

References

De, S.N. and Sen-Gupta, P. R.,(1974). Surface waves in micropolar elastic media., Bull. Acad. Pol. Sci. Ser. Sci. Technol. 22:137-146.

Dudziak W. and Kowalski S.J.,(1989).Theory of thermodiffusion for solids.,Int. J. Heat Mass Transfer. 32:2005-2013.

Eringen A.C., (1966).Mechanics of micromorphic materials., Proceedings of the II International Congress of Applied Mechanics. H. Gortler (ed.), Springer, Berlin, 131–138.

Eringen A.C., (1966). Linear theory of micropolar elasticity, J. Math. Mech.15: 909–923.

Eringen A.C., (1968). Mechanics of micromorphic continua. Mechanics of Generalized Continua. E. Kroner (ed.), IUTAM Symposium, Freudenstadt-Stuttgart, Springer, Berlin, 18–35.

Eringen A.C., (1971). Micropolar elastic solids with stretch, Prof Dr Mustafa Inan Anisina, Ari Kitabevi Matbaasi, Istanbul, 1–18

Eringen A.C., (1984). Plane wave in nonlocal micropolar elasticity, Int. J. Eng. Sci. 22:1113–1121.

Eringen A.C., (1990). Theory of thermo-microstretch elastic solids., Int. J. Eng. Sci. 28:1291–1301.

Eringen A.C., (1999). Microcontinuum Field Theories I: Foundations and Solids ,Springer-Verlag, New York.

Iesan D. and Pompei A., (1995).On the equilibrium theory of microstretch elastic solids. Int. J. Eng. Sci. 33:399-410.

Kumar R. and Kansal T., (2008). Propagation of Lamb waves in transversely isotropic thermoelastic diffusive plate., Int. J. Sol. Struc. 45:5890-5913.

Kumar R. and Kansal T.,(2009).Propagation of Rayleigh waves in transversely isotropic generalized thermoelastic diffusion. J. Eng. Phy. Thermophysics. 82:1199-1210.

Kumar R.and Kansal T., (2011).Fundamental solution in the theory of thermomicrostretch elastic diffusive solids, Int. Sch. Res. Net.,Article ID 764632, 15 pages.

Kumar R.and Partap G., (2009). Wave propagation in microstretch thermoelastic plate bordered with layers of inviscid liquid, Multidiscipline Modeling in Mat. Str. 5:171-184.

Nowacki W. (a)., (1974).Dynamical problems of thermodiffusion in solids-I, Bulletin of Polish Academy of Sciences Series, Science and Technology. 22: 55- 64.

Nowacki W. (b)., (1974). Dynamical problems of thermodiffusion in solids-II, Bulletin of Polish Academy of Sciences Series, Science and Technology. 22:129- 135.

Nowacki W. (c).,(1974). Dynamical problems of thermodiffusion in solids-III, Bulletin of Polish Academy of Sciences Series, Science and Technology 22:275- 276.

Nowacki W., (1976).Dynamical problems of diffusion in solids, Eng. Fract. Mech. 8:261-266.

Olesiak Z.S. and Pyryev Y.A., (1995). A coupled quasi-stationary problem of thermodiffusion for an elastic cylinder, Int. J. Eng. Sci. 33:773-780.

Othman M.I.A. and Lotfy K.H., (2010). On the plane waves of generalized thermo-microstretch elastic half-space under three theories, Int. Comm. Heat Mass Transfer 37:192-200.

Quintanilla R., (2002). On the spatial decay for the dynamical problem of thermo-microstretch elastic solids, Int. J. Eng. Sci. 40:109–121.

Richter Ch. F., (1958). Elementary Seismology. W. H. Freeman and Company, San Francisco.

Sharma J. N., (2007). Generalized thermoelastic diffusive waves in heat conducting materials, J. Sound Vib. 301:979– 993.

Sharma J. N., Kumar S. and Sharma Y. D., (2008). Propagation of Rayleigh surface waves in microstretch thermoelastic continua under inviscid fluid loadings, J. Ther. Stress. 31:18-39.

Sharma J. N., Sharma Y. D. and Sharma P. K., (2008).On the propagation of elastothermodiffusive surface waves in heat conducting materials, J. Sound Vib.315:927–938.

Sherief H.H., Hamza F.A. and Saleh H.A., (2004). The theory of generalized thermoelastic diffusion, Int. J.Eng. Sci. 42:591-608.

Singh B., Kumari S. and Singh J., (2012). Rayleigh wave in rotating magneto-thermo-elastic half-plane, J. Ther. Appl. Mech. 42:75-92.

Appendix A

$$A_1^* = d_{12}/d_{11}, B_1^* = d_{13}/d_{11}, C_1^* = d_{14}/d_{11}, D_1^* = d_{15}/d_{11}$$

$$d_{11} = -\left(l_{11} + b_{13}l_{18}\right), d_{12} = -l_{12} + b_{11}l_{11} - b_{12}l_{15} + b_{13}\left(\xi^2 l_{18} - l_{19}\right) - a_2 l_{22},$$
$$d_{13} = -l_{13} + b_{11}l_{12} + b_{12}\left(\xi^2 l_{15} - l_{16}\right) + b_{13}\left(\xi^2 l_{19} - l_{20}\right) - a_2\xi^2 l_{22} - a_2 l_{23},$$
$$d_{14} = -l_{14} + b_{11}l_{13} + b_{12}\left(\xi^2 l_{16} - l_{17}\right) + b_{13}\left(\xi^2 l_{20} - l_{21}\right) + a_2 l_{23}, d_{15} = b_{11}l_{14} + b_{12}\xi^2 l_{17} + b_{13}\xi^2 l_{21} + a_2\xi^2 l_{24} - a_2 l_{24},$$

$$l_{11} = b_{23}\delta_1^2, l_{12} = \delta_1^2(b_{19}b_{22} - g_{12}) + b_{23}g_{14}, l_{13} = -b_{22}\xi^2(b_{19}\delta_1^2 + g_{11}) - g_{12}g_{14} + b_{23}g_{13} + a_2\left(g_{15} - b_{15}\right),$$
$$l_{14} = \xi^2\left(b_{22}g_{11} - a_2 b_{15}\right) - g_{12}g_{13} - a_2\xi^2\left(g_{15} + b_{15}\right), l_{15} = \delta_1^2(a_{14}b_{19} + b_{17}b_{23}),$$
$$l_{16} = \delta_1^2(g_{12}b_{17} - a_{14}b_{19}\xi^2) - a_{14}g_{16} + g_{18}b_{23} + a_2 g_{19}, l_{17} = \xi^2\left(a_{14}g_{16} - a_2 g_{19}\right) - g_{12}g_{18},$$
$$l_{18} = -a_{14}\delta_1^2, l_{19} = \delta_1^2(a_{14}\xi^2 - b_{17}b_{22}) - a_{14}g_{14}, l_{20} = \xi^2(b_{17}b_{22}\delta_1^2 + a_{14}g_{14}) - a_{14}g_{13} + b_{22}g_{18} + a_2 g_{20},$$
$$l_{21} = \xi^2(a_{14}g_{13} - b_{22}g_{18} - a_2 g_{20}), l_{22} = a_8 b_{22} - a_{14}b_{15}, l_{23} = a_{14}(\xi^2 b_{15} - g_{21}) + b_{22}g_{22} - a_8 g_{23} - b_{22}g_{24},$$
$$l_{24} = \xi^2(a_{14}g_{21} - b_{22}g_{22}) + g_{23}g_{24},$$

$$g_{11} = b_{15}b_{20} - b_{16}b_{19}, g_{12} = b_{23}\xi^2 - b_{24}, g_{13} = b_{14}b_{20} - b_{16}b_{18}, g_{14} = b_{16} - b_{18}\delta_1^2, g_{15} = b_{14}b_{19} - b_{15}b_{18},$$
$$g_{16} = b_{15}b_{20} - b_{16}b_{19}, g_{18} = a_8 b_{20} - b_{16}b_{17}, g_{19} = a_8 b_{19} - b_{15}b_{17}, g_{20} = a_8 b_{18} - b_{14}b_{17}, g_{21} = b_{14}b_{19} - b_{15}b_{18},$$
$$g_{22} = a_8 b_{19} - b_{15}b_{17}, g_{23} = (b_{22}\xi^2 - b_{24}), g_{24} = a_8 b_{18} - b_{14}b_{17},$$

$$b_{11} = \xi^2\left(1 - c^2\right), b_{12} = 1 - i\xi c\tau_1, b_{13} = a_3\left(1 - i\xi c\tau^1\right), b_{14} = a_9\left(1 - i\xi c\tau_1\right), b_{15} = a_{10}\left(1 - i\xi c\tau^1\right),$$
$$b_{16} = \xi^2\left(c^2 - \delta_1^2\right) - a_7, b_{17} = a_{11}(i\xi c + \varepsilon\tau_0\xi^2 c^2), b_{18} = \xi^2\left(1 - c^2\tau_0\right) - i\xi c, b_{19} = -a_{13}(i\xi c + \gamma_1\xi^2 c^2),$$
$$b_{20} = -a_{12}(i\xi c + \varepsilon\tau_0\xi^2 c^2), b_{21} = 2\xi^2 a_{14}, b_{22} = -a_{15}\left(1 - i\xi c\tau_1\right), b_{23} = a_{16}\left(1 - i\xi c\tau^1\right), b_{24} = i\xi c(1 - i\varepsilon\tau^0\xi c),$$

Appendix B

$$n_{1p} = -\left[-l_{22}m_p^6 + \left(\xi^2 l_{22} - l_{23}\right)m_p^4 + \left(\xi^2 l_{23} - l_{24}\right)m_p^2 + \xi^2 l_{24}\right] \Big/ \left[l_{11}m_p^6 + l_{12}m_p^4 + l_{13}m_p^2 + l_{14}\right],$$
$$n_{2p} = \left[-l_{15}m_p^6 + \left(\xi^2 l_{15} - l_{16}\right)m_p^4 + \left(\xi^2 l_{16} - l_{17}\right)m_p^2 + \xi^2 l_{17}\right] \Big/ \left[l_{11}m_p^6 + l_{12}m_p^4 + l_{13}m_p^2 + l_{14}\right],$$
$$n_{3p} = -\left[-l_{18}m_p^8 + \left(\xi^2 l_{18} - l_{19}\right)m_p^6 + \left(\xi^2 l_{19} - l_{20}\right)m_p^4 + \left(\xi^2 l_{20} - l_{21}\right)m_p^2 + \xi^2 l_{21}\right] \Big/ \left[l_{11}m_p^6 + l_{12}m_p^4 + l_{13}m_p^2 + l_{14}\right],$$

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ERRATA

The heading of the article **"Rayleigh waves in isotropic microstretch thermoelastic diffusion solid half space"** published in the issue 2, volume 11, 2014 should read: 11(2014) 299 - 319.

The author name S.K.Garg[c] should read: S.K.Garg.

On page 301, $\overset{r}{u} = (u_1, u_2, u_3)$, $\overset{r}{\varphi} = (\varphi_1, \varphi_2, \varphi_3)$, $[T / T_0] = 1$ should read respectively: $\overset{r}{u} = (u_1, u_2, u_3)$, $\overset{r}{\varphi} = (\varphi_1, \varphi_2, \varphi_3)$ and $|T/T_0| = 1$,

Section 10 should read: NUMERICAL RESULTS AND DISCUSSION and not PATH OF SURFACE PARTICLES

We are sorry for these misprints.

The Editor.