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Buckling and free vibration analysis of orthotropic plates by using exponential shear deformation theory

Abstract
In the present paper, an exponential shear deformation theory is used to determine the natural frequencies and critical buckling loads of orthotropic plates. The theory accounts for a parabolic distribution of the transverse shear strains across the thickness, and satisfies the zero traction boundary conditions on the top and bottom surfaces of the plate without using shear correction factors. The in-plane displacement field uses an exponential function in terms of thickness coordinate to include the effect of shear deformation and rotary inertia. Governing equations and boundary conditions are derived from the dynamic version of principle of virtual work. The Navier type solution is employed for solving the governing equations of simply supported square orthotropic plates. The results obtained using present higher order shear deformation theory are found to be agree well with those obtained by other several existing higher order theories for analyzing the buckling and free vibration behaviour of orthotropic plates.

Keywords
orthotropic plates, shear correction factor, shear deformation, natural frequencies, uniaxial, biaxial, critical buckling load.

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1 INTRODUCTION

Orthotropic plates are widely used in structural applications because of their advantageous properties such as high stiffness and strength to weight ratios. Main failure mechanisms in orthotropic plates are bending, buckling and free vibration. Unlike any other isotropic plate, the buckling and free vibration analysis of orthotropic plate is more complicated due to inherently anisotropic. Thus, an accurate buckling and free vibration analysis of the orthotropic plates is an important part of the structural design.

Classical plate theory (CPT) which neglects the effect of transverse shear deformation, overestimates natural frequencies and critical buckling loads. The errors in natural frequencies and critical buckling loads are quite significant for plate made out of composite materials. The Reissner (1945) and Mindlin (1951) have developed first order shear deformation plate theories (FSDTs) considering the transverse shear and rotary inertia effects by the way of linear variation of in-plane displacements through the thickness of plate. Since these models violate the equilibrium conditions at the top and bottom surfaces of the plate, shear correction factors are required to correct the unrealistic variation of transverse shear stresses and shear strains through the thickness.

To overcome the limitations of classical plate theory and first order shear deformation theory, a many higher-order shear deformation plate theories which involve the higher-order terms in power series of the coordinate normal to the middle plane, have been proposed. . A critical review of these higher-order shear deformation plate theories has been given by Vasil'ev (1992), Noor and

Burton (1989), Liew et al. (1995), Ghugal and Shimpi (2002), Chen and Zhen (2008), Kreja (2011). Ghugal and Sayyad (2010, 2011a and 2011b) have developed trigonometric shear deformation theory considering effect of transverse shear and transverse normal strain/stress for the static flexure and free vibration analysis of thick plates. Ghugal and Pawar (2011) employed hyperbolic shear deformation theory for the buckling and free vibration analysis of orthotropic plates. Shimpi and Patel (2006a and 2006b) have developed a two variable refined plate theory for static and dynamic analysis of orthotropic plates whereas Kim et al. (2009) extended it for the buckling analysis of orthotropic plates using Navier's solution. Thai and Kim (2011) also employed Levy type solution for the buckling analysis of orthotropic plates based on two variable plate theory.

There exists another class of refined shear deformation theories, wherein use of an exponential function is made to take into account shear deformation effect. Karama et al. (2009) have used exponential function to predict the mechanical behavior multilayered laminated composite plates. Sayyad and Ghugal (2012a and 2012b) has carried out bending, buckling and free vibration analysis of isotropic plates using an exponential function in-terms of thickness coordinates to represent the effect of shear deformation. Sayyad (2013) also applied exponential shear deformation theory for the flexural analysis of orthotropic plates.

The purpose of the present study is to derive the analytical solutions of exponential shear deformation theory (Sayyad and Ghugal, 2012a and 2012b; Sayyad, 2013) for buckling and free vibration analysis of simply supported orthotropic plates. The displacement model contains exponential terms in addition to classical plate theory terms. The number of unknown variables is same as that of first order shear deformation theory. Governing equations and associated boundary conditions are derived from dynamic version of principle of virtual work. The Navier type solution is employed for solving the governing equations of simply supported rectangular orthotropic plates. The Navier type solution for orthotropic plate based on higher order shear deformation theory (HSDT) of Reddy (1984), trigonometric shear deformation theory (TSDT) of Ghugal and Sayyad (2011b), first order shear deformation theory (FSDT) of Mindlin (1951) and classical plate theory are generated for the verification purpose. The natural frequencies and critical buckling loads of orthotropic plates for various modular and aspect ratios are studied and discussed in detail. Exact elasticity solution for vibration of simply supported homogeneous thick rectangular plate is provided by Srinivas et al. (1970) whereas exact elasticity solution for buckling analysis of plates is not available in the literature.

2 Orthotropic Plate under Consideration

Consider a rectangular plate of sides 'a' and 'b' and a constant thickness of 'h'. The plate is subjected to transverse load (q) and in-plane compressive forces (N_{xx}^0, N_{yy}^0 and N_{xy}^0). The co-ordinate system (x, y, z) chosen and the coordinate parameters are such that, the plate occupies a region given by Eq. (1).

$$0 \le x \le a; \qquad 0 \le y \le b; \qquad -h/2 \le z \le h/2 \tag{1}$$

2.1 Assumptions made in theoretical formulation

Assumptions of the exponential shear deformation theory are as follows:

1. The displacements are small in comparison with the plate thickness 'h' and, therefore, strains involved are infinitesimal.
2. The in-plane displacement u in x direction as well as displacement v in y direction each consists of bending and shear components

$$u = u_b + u_s \,;\, v = v_b + v_s \tag{2}$$

a) The bending components u_b and v_b are assumed to be similar to the displacements given by the classical plate theory. Therefore, the expression for u_b and v_b can be given as.

$$u_b = -z\frac{\partial w(x,y,t)}{\partial x}; \qquad v_b = -z\frac{\partial w(x,y,t)}{\partial y} \tag{3}$$

b) Shear components u_s and v_s are assumed to be exponential in nature with respect to thickness coordinate, such that the maximum shear stress occurs at neutral plane. Consequently, the expressions for u_s and v_s can be given as.

$$u_s = z\exp\left[-2\left(\frac{z}{h}\right)^2\right]\phi(x,y,t); \quad v_s = z\exp\left[-2\left(\frac{z}{h}\right)^2\right]\psi(x,y,t) \tag{4}$$

where, ϕ and ψ are the unknown functions associated with the shear slopes.

3. The transverse displacement w in z direction is assumed to be a function of coordinates x and y and time t.

$$w(x,y,z,t) = w(x,y,t) \tag{5}$$

2.2 Kinematics:

Based on the above mentioned assumptions, the displacement field can be obtained using Eqs. (2)–(5) as:

$$u(x,y,z,t) = -z\frac{\partial w(x,y,t)}{\partial x} + z\exp\left[-2\left(\frac{z}{h}\right)^2\right]\phi(x,y,t) \tag{6}$$

$$v(x,y,z,t) = -z\frac{\partial w(x,y,t)}{\partial y} + z\exp\left[-2\left(\frac{z}{h}\right)^2\right]\psi(x,y,t)$$

$$w(x,y,t) = w(x,y,t)$$

This displacement field accounts for zero traction boundary conditions on the top and bottom surfaces of the plate and the parabolic variation of transverse shear strains and stresses through the thickness of plates. The kinematic relations can be obtained as follows:

$$\varepsilon_x = z k_x^b + f(z) k_x^s; \quad \varepsilon_y = z k_y^b + f(z) k_y^s; \quad \varepsilon_z = 0; \tag{7}$$

$$\gamma_{xy} = z k_{xy}^b + f(z) k_{xy}^s; \quad \gamma_{zx} = g(z)\phi; \quad \gamma_{yz} = g(z)\psi.$$

where,

$$k_x^b = -\frac{\partial^2 w}{\partial x^2}; \quad k_x^s = \frac{\partial \phi}{\partial x}; \quad k_y^b = -\frac{\partial^2 w}{\partial y^2}; \tag{8}$$

$$k_y^s = \frac{\partial \psi}{\partial y}; \quad k_{xy}^b = -2\frac{\partial^2 w}{\partial x \partial y}; \quad k_{xy}^s = \frac{\partial \phi}{\partial y} + \frac{\partial \psi}{\partial x};$$

$$f(z) = z\exp\left[-2\left(\frac{z}{h}\right)^2\right]; \quad g(z) = \exp\left[-2\left(\frac{z}{h}\right)^2\right]\left[1-4\left(\frac{z}{h}\right)^2\right]$$

2.3 Constitutive relations

The constitutive relations for orthotropic materials are as follows:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & 0 & 0 & 0 \\ \bar{Q}_{21} & \bar{Q}_{22} & 0 & 0 & 0 \\ 0 & 0 & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & 0 \\ 0 & 0 & 0 & 0 & \bar{Q}_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} \tag{9}$$

where, $\bar{Q}_{ij}$ are the plane stress reduced elastic constants in the material axes of the plate, and are defined as:

$$\bar{Q}_{11} = \frac{E_1}{1-\mu_{12}\mu_{21}}; \quad \bar{Q}_{12} = \frac{\mu_{12}E_2}{1-\mu_{12}\mu_{21}}; \quad \bar{Q}_{22} = \frac{E_2}{1-\mu_{12}\mu_{21}}; \tag{10}$$

$$\bar{Q}_{66} = G_{12}; \quad \bar{Q}_{55} = G_{13}; \quad \bar{Q}_{44} = G_{23}.$$

3 Derivation of Governing Equations and Boundary Conditions

The governing equations and boundary conditions are derived using principle of virtual work. Let δ be the arbitrary variations

$$\iiint_V \left(\delta U - \delta W + \delta T\right) = 0 \tag{11}$$

where the virtual strain energy δU, virtual potential energy δW due to the transverse load $q(x,y)$ and constant inplane compressive and shear forces (N_{xx}^0, N_{yy}^0 and N_{xy}^0) and the virtual kinetic energy δT are given by

$$\delta U = \int_{-h/2}^{h/2}\int_0^b\int_0^a \left(\sigma_x\,\delta\varepsilon_x + \sigma_y\,\delta\varepsilon_y + \tau_{yz}\,\delta\gamma_{yz} + \tau_{zx}\,\delta\gamma_{zx} + \tau_{xy}\,\delta\gamma_{xy}\right) dx\,dy\,dz \tag{12}$$

$$\delta W = \int_0^b\int_0^a q(x,y)\,\delta w\,dx\,dy + \frac{1}{2}\int_0^b\int_0^a \delta\left[N_{xx}^0\left(\frac{\partial w}{\partial x}\right)^2 + N_{yy}^0\left(\frac{\partial w}{\partial y}\right)^2 + 2N_{xy}^0\left(\frac{\partial w}{\partial x\partial y}\right)^2\right]\delta w\,dx\,dy$$

$$\delta T = \rho\int_{-h/2}^{h/2}\int_0^b\int_0^a \left[\frac{\partial^2 u}{\partial t^2}\delta u + \frac{\partial^2 v}{\partial t^2}\delta v + \frac{\partial^2 w}{\partial t^2}\delta w\right] dx\,dy\,dz$$

Substituting Eq. (12) in Eq. (11)

$$\begin{aligned}
&\int_{-h/2}^{h/2}\int_0^b\int_0^a \left(\sigma_x\,\delta\varepsilon_x + \sigma_y\,\delta\varepsilon_y + \tau_{yz}\,\delta\gamma_{yz} + \tau_{zx}\,\delta\gamma_{zx} + \tau_{xy}\,\delta\gamma_{xy}\right) dx\,dy\,dz \\
&-\int_0^b\int_0^a q(x,y)\,\delta w\,dx\,dy - \frac{1}{2}\int_0^b\int_0^a \delta\left[N_{xx}^0\left(\frac{\partial w}{\partial x}\right)^2 + N_{yy}^0\left(\frac{\partial w}{\partial y}\right)^2 + 2N_{xy}^0\left(\frac{\partial w}{\partial x\partial y}\right)^2\right]\delta w\,dx\,dy \\
&+\rho\int_{-h/2}^{h/2}\int_0^b\int_0^a \left[\frac{\partial^2 u}{\partial t^2}\delta u + \frac{\partial^2 v}{\partial t^2}\delta v + \frac{\partial^2 w}{\partial t^2}\delta w\right] dx\,dy\,dz = 0
\end{aligned} \tag{13}$$

Substituting Eqs. (6) - (10) into the Eq. (13) and integrating through the thickness of the plate, the principle of virtual work of the plate can be rewritten as

$$\begin{aligned}
&\int_0^b\int_0^a \left[\begin{matrix} M_x\dfrac{\partial^2\delta w}{\partial x^2} - N_{sx}\dfrac{\partial\delta\phi}{\partial x} + M_y\dfrac{\partial^2\delta w}{\partial y^2} - N_{sy}\dfrac{\partial\delta\psi}{\partial y} + 2M_{xy}\dfrac{\partial^2\delta w}{\partial x\partial y} - N_{sxy}\dfrac{\partial\delta\phi}{\partial y} \\ -N_{sxy}\dfrac{\partial\delta\psi}{\partial x} - N_{Tcx}\,\delta\phi - N_{Tcy}\,\delta\psi \end{matrix}\right] dx\,dy \\
&+\int_0^b\int_0^a \left[q(x,y)\,\delta w + N_{xx}^0\frac{\partial w}{\partial x}\frac{\partial\delta w}{\partial x} + N_{yy}^0\frac{\partial w}{\partial y}\frac{\partial\delta w}{\partial y} + 2N_{xy}^0\frac{\partial w}{\partial x\partial y}\left(\frac{\partial w}{\partial x}\frac{\partial\delta w}{\partial y} + \frac{\partial w}{\partial y}\frac{\partial\delta w}{\partial x}\right)\right]\delta w\,dx\,dy \\
&-\int_0^b\int_0^a \left[\begin{matrix} I_1\dfrac{\partial^2 w}{\partial t^2}\delta w + I_2\dfrac{\partial^3 w}{\partial x\partial t^2}\dfrac{\partial\delta w}{\partial x} - I_3\dfrac{\partial^3 w}{\partial x\partial t^2}\delta\phi - I_3\dfrac{\partial^2\phi}{\partial t^2}\dfrac{\partial\delta w}{\partial x} + I_4\dfrac{\partial^2\phi}{\partial t^2}\delta\phi \\ +I_2\dfrac{\partial^3 w}{\partial y\partial t^2}\dfrac{\partial\delta w}{\partial y} - I_3\dfrac{\partial^3 w}{\partial y\partial t^2}\delta\psi - I_3\dfrac{\partial^2\psi}{\partial t^2}\dfrac{\partial\delta w}{\partial y} + I_4\dfrac{\partial^2\psi}{\partial t^2}\delta\psi \end{matrix}\right] dx\,dy = 0
\end{aligned} \tag{14}$$

where, stress resultants $\left(M_x, M_y, M_{xy}, N_{sx}, N_{sy}, N_{sxy}, N_{Tcx}, N_{Tcy}\right)$ are defined as:

$$\left(M_x, M_y, M_{xy}\right)=\int_{-h/2}^{h/2}\left(\sigma_x, \sigma_y, \tau_{xy}\right) z\, dz; \quad \left(N_{sx}, N_{sy}, N_{sxy}\right)=\int_{-h/2}^{h/2}\left(\sigma_x, \sigma_y, \tau_{xy}\right) f(z)\, dz; \tag{15}$$

$$\left(N_{Tcx}, N_{Tcy}\right)=\int_{-h/2}^{h/2}\left(\tau_{zx}, \tau_{yz}\right) g(z)\, dz; \quad V_x=\frac{\partial M_x}{\partial x}+2\frac{\partial M_{xy}}{\partial y}-N_{xx}^0\frac{\partial w}{\partial x}-N_{xy}^0\frac{\partial w}{\partial y};$$

$$V_y=\frac{\partial M_y}{\partial y}+2\frac{\partial M_{xy}}{\partial x}-N_{yy}^0\frac{\partial w}{\partial y}-N_{xy}^0\frac{\partial w}{\partial x}; \quad \left(I_1, I_2, I_3, I_4\right)=\rho\int_{-h/2}^{h/2}\left(1,\, z^2,\, z\, f(z),\, f^2(z)\right) dz.$$

Substituting Eqs. (7) and (9) into Eq. (15) and integrating through the thickness of the plate, the stress resultants are related to the generalized displacements (w, ϕ and ψ) by the relations

$$\begin{Bmatrix} M_x \\ M_y \\ N_{sx} \\ N_{sy} \end{Bmatrix}=\begin{bmatrix} D_{11} & Bs_{11} & D_{12} & Bs_{12} \\ D_{12} & Bs_{12} & D_{22} & Bs_{22} \\ Bs_{11} & Ass_{11} & Bs_{12} & Ass_{12} \\ Bs_{12} & Ass_{12} & Bs_{22} & Ass_{22} \end{bmatrix}\begin{Bmatrix} k_x^c \\ k_x^s \\ k_y^c \\ k_y^s \end{Bmatrix}, \quad \begin{Bmatrix} M_{xy} \\ N_{sxy} \end{Bmatrix}=\begin{bmatrix} D_{66} & Bs_{66} \\ Bs_{66} & Ass_{66} \end{bmatrix}\begin{Bmatrix} k_{xy}^c \\ k_{xy}^s \end{Bmatrix}, \tag{16}$$

$$\begin{Bmatrix} N_{Tcx} \\ N_{Tcy} \end{Bmatrix}=\begin{bmatrix} Acc_{55} & 0 \\ 0 & Acc_{44} \end{bmatrix}\begin{Bmatrix} \phi \\ \psi \end{Bmatrix}$$

Various stiffnesses used in Eq. (16) are expressed below

$$\left(D_{11}, D_{12}, D_{22}, D_{66}\right)=\left(\bar{Q}_{11}, \bar{Q}_{12}, \bar{Q}_{22}, \bar{Q}_{66}\right)\int_{-h/2}^{+h/2} z^2\, dz; \tag{17}$$

$$\left(Bs_{11}, Bs_{12}, Bs_{22}, Bs_{66}\right)=\left(\bar{Q}_{11}, \bar{Q}_{12}, \bar{Q}_{22}, \bar{Q}_{66}\right)\int_{-h/2}^{+h/2} z\, f(z)\, dz;$$

$$\left(Ass_{11}, Ass_{12}, Ass_{22}, Ass_{66}\right)=\left(\bar{Q}_{11}, \bar{Q}_{12}, \bar{Q}_{22}, \bar{Q}_{66}\right)\int_{-h/2}^{+h/2} f^2(z)\, dz;$$

$$\left(Acc_{44}, Acc_{55}\right)=\left(\bar{Q}_{44}, \bar{Q}_{55}\right)\int_{-h/2}^{+h/2} g^2(z)\, dz.$$

Integrating the Eq. (14) by parts, collecting the coefficients of δw, $\delta\phi$, and $\delta\psi$ the equations of motion and boundary conditions for the orthotropic plate are obtained as follows:

$$\frac{\partial^2 M_x}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial^2 M_y}{\partial y^2}+q(x,y)+N_{xx}^0\frac{\partial^2 w}{\partial x^2}+N_{yy}^0\frac{\partial^2 w}{\partial y^2}+2N_{xy}^0\frac{\partial^2 w}{\partial x\partial y}$$
$$=I_1\frac{\partial^2 w}{\partial t^2}-I_2\left(\frac{\partial^4 w}{\partial x^2\partial t^2}+\frac{\partial^4 w}{\partial y^2\partial t^2}\right)+I_3\left(\frac{\partial^3 \phi}{\partial x\partial t^2}+\frac{\partial^3 \psi}{\partial y\partial t^2}\right) \tag{18}$$

$$\frac{\partial N_{sx}}{\partial x}+\frac{\partial N_{sxy}}{\partial y}-N_{Tcx}=-I_3\frac{\partial^3 w}{\partial x\partial t^2}+I_4\frac{\partial^2 \phi}{\partial t^2} \tag{19}$$

$$\frac{\partial N_{sy}}{\partial y}+\frac{\partial N_{sxy}}{\partial x}-N_{Tcy}=-I_3\frac{\partial^3 w}{\partial y\partial t^2}+I_4\frac{\partial^2 \psi}{\partial t^2} \tag{20}$$

The associated boundary conditions of a plate are as follows:

$x=0$ and $x=a$					$y=0$ and $y=b$				
Either	$V_x=0$	or	w	is prescribed	Either	$V_y=0$	or	w	is prescribed
Either	$M_x=0$	or	$\frac{\partial w}{\partial x}$	is prescribed	Either	$M_y=0$	or	$\frac{\partial w}{\partial y}$	is prescribed
Either	$N_{sx}=0$	or	ϕ	is prescribed	Either	$N_{sxy}=0$	or	ϕ	is prescribed
Either	$N_{sxy}=0$	or	ψ	is prescribed	Either	$N_{sy}=0$	or	ψ	is prescribed

Substituting Eq. (16) into Eqs. (18)-(20), the governing equations of the plate in-terms of generalized displacements are as follows:

$$D_{11}\frac{\partial^4 w}{\partial x^4}+(2D_{12}+4D_{66})\frac{\partial^4 w}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4 w}{\partial y^4}-Bs_{11}\frac{\partial^3 \phi}{\partial x^3}-(Bs_{12}+2Bs_{66})\frac{\partial^3 \phi}{\partial x\partial y^2}$$
$$-Bs_{22}\frac{\partial^3 \psi}{\partial y^3}-(Bs_{12}+2Bs_{66})\frac{\partial^3 \psi}{\partial x^2\partial y}+I_1\frac{\partial^2 w}{\partial t^2}-I_2\left(\frac{\partial^4 w}{\partial x^2\partial t^2}+\frac{\partial^4 w}{\partial y^2\partial t^2}\right) \tag{21}$$
$$+I_3\left(\frac{\partial^3 \phi}{\partial x\partial t^2}+\frac{\partial^3 \psi}{\partial y\partial t^2}\right)=q(x,y)+N_{xx}^0\frac{\partial^2 w}{\partial x^2}+N_{yy}^0\frac{\partial^2 w}{\partial y^2}+2N_{xy}^0\frac{\partial^2 w}{\partial x\partial y}$$

$$Bs_{11}\frac{\partial^3 w}{\partial x^3}+(Bs_{12}+2Bs_{66})\frac{\partial^3 w}{\partial x\partial y^2}-Ass_{11}\frac{\partial^2 \phi}{\partial x^2}-Ass_{66}\frac{\partial^2 \phi}{\partial y^2}+Acc_{55}\,\phi$$
$$-(Ass_{12}+Ass_{66})\frac{\partial^2 \psi}{\partial x\partial y}-I_3\frac{\partial^3 w}{\partial x\partial t^2}+I_4\frac{\partial^2 \phi}{\partial t^2}=0 \tag{22}$$

$$Bs_{22}\frac{\partial^3 w}{\partial y^3}+\left(Bs_{12}+2Bs_{66}\right)\frac{\partial^3 w}{\partial x^2\,\partial y}-Ass_{66}\frac{\partial^2\psi}{\partial x^2}-Ass_{22}\frac{\partial^2\psi}{\partial y^2}+Acc_{44}\,\psi \tag{23}$$

$$-\left(Ass_{12}+Ass_{66}\right)\frac{\partial^2\phi}{\partial x\partial y}-I_3\frac{\partial^3 w}{\partial y\partial t^2}+I_4\frac{\partial^2\psi}{\partial t^2}=0$$

4 Navier solution for simply supported square orthotropic plates

The Navier method is only applied for simply supported boundary conditions on all four edges of the rectangular plate. The following are the boundary conditions of the simply supported orthotropic plates.

$$w=\psi=M_x=N_{sx}=0 \quad \text{at } x=0 \text{ and } x=a \tag{24}$$

$$w=\phi=M_y=N_{sy}=0 \quad \text{at } y=0 \text{ and } y=b \tag{25}$$

In order to solve the governing equations with the prescribed boundary conditions, a generalized Navier's approach is employed to obtain the closed-form solutions.

Example 1: Free vibration analysis of simply supported square orthotropic plates

The governing equations for free vibration analysis of plates can be obtained by setting the applied transverse load (q) and in-plane compressive forces (N_{xx}^0, N_{yy}^0 and N_{xy}^0) equal to zero in Eqs. (21)-(23).

$$D_{11}\frac{\partial^4 w}{\partial x^4}+\left(2D_{12}+4D_{66}\right)\frac{\partial^4 w}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4 w}{\partial y^4}-Bs_{11}\frac{\partial^3\phi}{\partial x^3}-\left(Bs_{12}+2Bs_{66}\right)\frac{\partial^3\phi}{\partial x\partial y^2}-Bs_{22}\frac{\partial^3\psi}{\partial y^3} \tag{26}$$

$$-\left(Bs_{12}+2Bs_{66}\right)\frac{\partial^3\psi}{\partial x^2\partial y}+I_1\frac{\partial^2 w}{\partial t^2}-I_2\left(\frac{\partial^4 w}{\partial x^2\partial t^2}+\frac{\partial^4 w}{\partial y^2\partial t^2}\right)+I_3\left(\frac{\partial^3\phi}{\partial x\partial t^2}+\frac{\partial^3\psi}{\partial y\partial t^2}\right)=0$$

$$Bs_{11}\frac{\partial^3 w}{\partial x^3}+\left(Bs_{12}+2Bs_{66}\right)\frac{\partial^3 w}{\partial x\partial y^2}-Ass_{11}\frac{\partial^2\phi}{\partial x^2}-Ass_{66}\frac{\partial^2\phi}{\partial y^2}+Acc_{55}\,\phi \tag{27}$$

$$-\left(Ass_{12}+Ass_{66}\right)\frac{\partial^2\psi}{\partial x\partial y}-I_3\frac{\partial^3 w}{\partial x\partial t^2}+I_4\frac{\partial^2\phi}{\partial t^2}=0$$

$$Bs_{22}\frac{\partial^3 w}{\partial y^3}+\left(Bs_{12}+2Bs_{66}\right)\frac{\partial^3 w}{\partial x^2\,\partial y}-Ass_{66}\frac{\partial^2\psi}{\partial x^2}-Ass_{22}\frac{\partial^2\psi}{\partial y^2}+Acc_{44}\,\psi \tag{28}$$

$$-\left(Ass_{12}+Ass_{66}\right)\frac{\partial^2\phi}{\partial x\partial y}-I_3\frac{\partial^3 w}{\partial y\partial t^2}+I_4\frac{\partial^2\psi}{\partial t^2}=0$$

The following displacement functions $w(x,y)$, $\phi(x,y)$ and $\psi(x,y)$ are chosen to automatically satisfy boundary conditions in Eqs. (24) and (25).

$$w(x,y,t)=\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} w_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\sin\omega_{mn}t;$$

$$\phi(x,y,t)=\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} \phi_{mn}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\sin\omega_{mn}t;$$

$$\psi(x,y,t)=\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} \psi_{mn}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\sin\omega_{mn}t \tag{29}$$

where w_{mn} is the amplitude of translation and ϕ_{mn},ψ_{mn} are the amplitudes of rotation. ω_{mn} is the natural frequency of m^{th} and n^{th} mode of vibration. Substitution of this solution form into the Eqs. (26)-(28), results in following standard Eigen value problem.

$$\left(\begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} - \omega_{mn}^2 \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}\right)\begin{Bmatrix} w_{mn} \\ \phi_{mn} \\ \psi_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{30}$$

where $\left[K_{ij}\right]$ is the stiffness matrix, $\left[M_{ij}\right]$ is the mass matrix. Elements of $\left[K_{ij}\right]$ and $\left[M_{ij}\right]$ are given as below:

$$K_{11}=D_{11}\frac{m^4\pi^4}{a^4}+\left(2D_{12}+4D_{66}\right)\frac{m^2n^2\pi^4}{a^2b^2}+D_{22}\frac{n^4\pi^4}{b^4}; \tag{31}$$

$$K_{12}=-Bs_{11}\frac{m^3\pi^3}{a^3}-\left(Bs_{12}+2Bs_{66}\right)\frac{mn^2\pi^3}{ab^2};$$

$$K_{13}=-Bs_{22}\frac{n^3\pi^3}{b^3}-\left(Bs_{12}+2Bs_{66}\right)\frac{m^2n\pi^3}{a^2b};$$

$$K_{22}=Ass_{11}\frac{m^2\pi^2}{a^2}+Ass_{66}\frac{n^2\pi^2}{b^2}+Acc_{55};\quad K_{23}=\left(Ass_{12}+Ass_{66}\right)\frac{mn\pi^2}{ab};$$

$$K_{33}=Ass_{22}\frac{n^2\pi^2}{b^2}+Ass_{66}\frac{m^2\pi^2}{a^2}+Acc_{44};\quad K_{21}=K_{12};\quad K_{31}=K_{13};\quad K_{32}=K_{23};$$

$$M_{11}=\left(I_1+I_2\frac{m^2\pi^2}{a^2}+I_2\frac{n^2\pi^2}{b^2}\right);\quad M_{12}=-I_3\frac{m\pi}{a};\quad M_{13}=-I_3\frac{n\pi}{b};M_{22}=I_4;$$

$$M_{23}=0;\quad M_{33}=I_4;\quad M_{21}=M_{12};\quad M_{31}=M_{13};\quad M_{32}=M_{23};$$

From the solution of Eq. (30), lowest natural frequency for all modes of vibration can be obtained. The orthotropic plate has following material properties.

$$E_1 / E_2 = 0.52500, \quad G_{12} / E_2 = 0.26293, \quad G_{13} / E_2 = 0.15991, \tag{32}$$

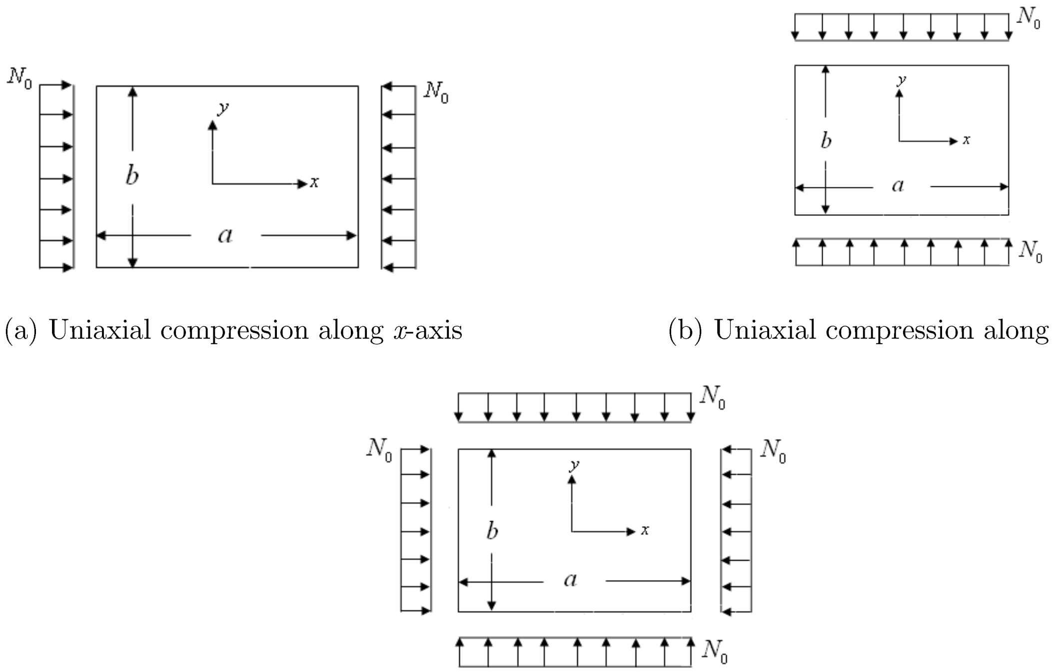
$$G_{23} / E_2 = 0.26681, \quad \mu_{12} = 0.44046, \quad \mu_{21} = 0.23124$$

The bending mode and shear mode frequencies of orthotropic plate are presented in the following non-dimensional form.

$$\bar{\omega}_{mn} = \omega_{mn} h \sqrt{\frac{\rho}{\bar{Q}_{11}}} \qquad \text{where} \quad \bar{Q}_{11} = \frac{E_1}{1 - \mu_{12}\mu_{21}} \tag{33}$$

Example 2: Buckling Analysis of simply supported orthotropic plates subjected in-plane compressive forces

When a plate is subjected to in-plane compressive forces, and if the forces are small enough, the equilibrium of the plate is stable and the plate remains flat until ascertains load is reached. At that load, called the critical buckling load, the stable state of the plate is disturbed and plate seeks an alternative equilibrium configuration accompanied by a change in the load-deflection behavior. A simply supported rectangular plate subjected to the loading conditions, as shown in Fig. 1, is considered to illustrate the accuracy of the present theory in predicting the buckling behavior of the orthotropic plate



(a) Uniaxial compression along x-axis (b) Uniaxial compression along y-axis

(c) Biaxial compression

Fig. 1 The loading conditions of rectangular plate: (a) uniaxial compression along x-axis, (b) uniaxial compression along y-axis (c) biaxial compression

The governing equations of plate in case of static buckling are obtained by setting $q(x,y)=0$, $N_{xx}^0 = k_1 N_0$, $N_{yy}^0 = k_2 N_{xx}^0$ and $N_{xy}^0 = 0$ in Eqs. (21) - (23).

$$D_{11}\frac{\partial^4 w}{\partial x^4} + (2D_{12}+4D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4} - Bs_{11}\frac{\partial^3 \phi}{\partial x^3} - (Bs_{12}+2Bs_{66})\frac{\partial^3 \phi}{\partial x \partial y^2}$$
$$-Bs_{22}\frac{\partial^3 \psi}{\partial y^3} - (Bs_{12}+2Bs_{66})\frac{\partial^3 \psi}{\partial x^2 \partial y} - k_1 N_0 \frac{\partial^2 w}{\partial x^2} - k_2 N_0 \frac{\partial^2 w}{\partial y^2} = 0 \tag{33}$$

$$Bs_{11}\frac{\partial^3 w}{\partial x^3} + (Bs_{12}+2Bs_{66})\frac{\partial^3 w}{\partial x \partial y^2} - Ass_{11}\frac{\partial^2 \phi}{\partial x^2} - Ass_{66}\frac{\partial^2 \phi}{\partial y^2} + Acc_{55}\,\phi$$
$$-(Ass_{12}+Ass_{66})\frac{\partial^2 \psi}{\partial x \partial y} = 0 \tag{34}$$

$$Bs_{22}\frac{\partial^3 w}{\partial y^3} + (Bs_{12}+2Bs_{66})\frac{\partial^3 w}{\partial x^2 \partial y} - Ass_{66}\frac{\partial^2 \psi}{\partial x^2} - Ass_{22}\frac{\partial^2 \psi}{\partial y^2} + Acc_{44}\,\psi$$
$$-(Ass_{12}+Ass_{66})\frac{\partial^2 \phi}{\partial x \partial y} = 0 \tag{35}$$

The following displacement functions $w(x,y)$, $\phi(x,y)$ and $\psi(x,y)$ are chosen to automatically satisfy the boundary conditions in Eqs. (24) and (25).

$$w(x,y) = \sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} w_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b};$$
$$\phi(x,y) = \sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} \phi_{mn}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b};$$
$$\psi(x,y) = \sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty} \psi_{mn}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}. \tag{36}$$

Substituting Eq. (36) into Eqs. (33)-(35), the following system is obtained:

$$\left(\begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} - N_0 \begin{bmatrix} N_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right)\begin{Bmatrix} w_{mn} \\ \phi_{mn} \\ \psi_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{37}$$

where, $\left[K_{ij}\right]$ are given in Eq. (31) and N_{11} as given below:

$$N_{11} = -\left(k_1 \frac{m^2\pi^2}{a^2} + k_2 \frac{n^2\pi^2}{b^2}\right) \tag{38}$$

For nontrivial solution, the determinant of the coefficient matrix in Eq. (37) must be zero. This gives the following equation for buckling load:

$$N_0 = \frac{K_{11}\begin{vmatrix} K_{22} & K_{23} \\ K_{32} & K_{33} \end{vmatrix} - K_{12}\begin{vmatrix} K_{21} & K_{23} \\ K_{31} & K_{33} \end{vmatrix} + K_{13}\begin{vmatrix} K_{21} & K_{22} \\ K_{31} & K_{32} \end{vmatrix}}{N_{11}\begin{vmatrix} K_{22} & K_{23} \\ K_{32} & K_{33} \end{vmatrix}} \tag{39}$$

For each choice of m and n, there is a corresponding unique value of N_0. The critical buckling load is the smallest value of N_0 (m, n). For verification purposes, a simply supported rectangular plate subjected to the uniaxial and biaxial loading conditions, as shown in Fig. 1, is considered to illustrate the accuracy of the present theory in predicting the buckling behavior of the orthotropic plate. The following material properties are used.

$$E_1 / E_2 = \text{open},\ G_{12} / E_2 = G_{13} / E_2 = 0.5,\ G_{23} / E_2 = 0.2,\ \mu_{12} = 0.25 \tag{40}$$

Critical buckling loads are presented in the following non-dimensional form:

$$N_{cr} = \frac{N_0\, a^2}{E_2\, h^3} \tag{41}$$

Table 1: Comparison of non-dimensional natural predominantly bending mode frequencies $\bar{\omega}_w$ of simply-supported orthotropic square plate (b / a = 1, h / a = 0.1)

(m, n)	Exact (Srinivas et al., 1970)	Present	HSDT (Reddy, 1984)	TSDT (Ghugal and Sayyad, 2011b)	RPT (Shimpi and Patel, 2006)	FSDT (Mindlin, 1951)	CPT
(1, 1)	0.0474	0.0474	0.0474	0.0474	0.0477	0.0474	0.0497
(1,2)	0.1033	0.1033	0.1033	0.1031	0.1040	0.1032	0.1120
(1, 3)	0.1888	0.1888	0.1888	0.1793	0.1898	0.1884	0.2154
(1, 4)	0.2969	0.2969	0.2969	0.2932	0.2980	0.2959	0.3599
(2, 1)	0.1188	0.1190	0.1189	0.1196	0.1198	0.1187	0.1354
(2, 2)	0.1694	0.1697	0.1695	0.1696	0.1722	0.1692	0.1987
(2, 3)	0.2475	0.2480	0.2477	0.2478	0.2520	0.2469	0.3029
(2, 4)	0.3476	0.3482	0.3479	0.3468	0.3534	0.3463	0.4480
(3, 1)	0.2180	0.2191	0.2184	0.2199	0.2197	0.2178	0.2779
(3, 2)	0.2624	0.2637	0.2629	0.2671	0.2675	0.2619	0.3418
(3, 3)	0.3320	0.3337	0.3326	0.3326	0.3407	0.3310	0.4470
(4, 1)	0.3319	0.3351	0.3330	0.3346	0.3344	0.3311	0.4773
(4, 2)	0.3707	0.3743	0.3720	0.3727	0.3774	0.3696	0.5415

4.2 Discussion of Results

Example 1: In the present study, free vibration analysis of an orthotropic plate with all edges simply supported is considered. Natural predominantly bending mode ($\bar{\omega}_w$) and thickness shear modes ($\bar{\omega}_\phi$ and $\bar{\omega}_\psi$) frequencies of square plate are obtained for aspect ratio 10. Non-dimensional frequencies of simply supported square plate are presented in Tables 1-3 and compared with higher order shear deformation theory (HSDT) of Reddy (1984), trigonometric shear deformation theory (TSDT) of Ghugal and Sayyad (2011b), refined plate theory (RPT) of Shimpi and Patel (2006), first order shear deformation theory (FSDT) of Mindlin (1951) and classical plate theory (CPT).

Table 1 shows comparison of predominantly bending mode frequencies of orthotropic square plate for various modes of vibration and aspect ratio 10. It is observed that the present theory yields excellent values of frequencies for all modes of vibration. The present theory and HSDT of Reddy predicts exact result of bending frequency for (m = 1, n = 1), (m = 1, n = 2), (m = 1, n = 3) and (m = 1, n = 4) modes of vibration whereas RPT and CPT overestimates the same. From Tables 2-3 it is observed that, thickness shear modes frequencies of orthotropic square plate obtained by the present theory and HSDT of Reddy are in close agreement with the exact values. The FSDT marginally overestimates the value of frequencies of this mode compared to the exact one.

Table 2: Comparison of non-dimensional predominantly thickness shear mode frequencies $\bar{\omega}_\phi$ of simply-supported orthotropic square plate (b / a = 1, h / a = 0.1)

(m, n)	Exact (Srinivas et al., 1970)	Present	HSDT (Reddy, 1984)	TSDT (Ghugal and Sayyad, 2011b)	FSDT (Mindlin, 1951)
(1, 1)	1.3077	1.2999	1.3086	1.3077	1.3159
(1,2)	1.3331	1.3290	1.3339	1.3332	1.3410
(1, 3)	1.3665	1.3638	1.3772	1.3766	1.3841
(1, 4)	1.4372	1.4281	1.4379	1.4371	1.4445
(2, 1)	1.4205	1.4168	1.4216	1.4203	1.4285
(2, 2)	1.4316	1.4277	1.4323	1.4316	1.4393
(2, 3)	1.4596	1.4562	1.4603	1.4598	1.4671
(2, 4)	1.5068	1.5039	1.5076	1.5063	1.5142
(3, 1)	1.5777	1.5744	1.5789	1.5766	1.5857
(3, 2)	1.5651	1.5612	1.5658	1.5644	1.5727
(3, 3)	1.5737	1.5701	1.5744	1.5737	1.5812
(4, 1)	1.7179	1.7119	1.7189	1.7168	1.7265
(4, 2)	1.6940	1.6890	1.6947	1.6942	1.7022

Table 3: Comparison of non-dimensional predominantly thickness shear mode frequencies $\bar{\omega}_\psi$ of simply-supported orthotropic square plate (b / a = 1, h / a = 0.1)

(m, n)	Exact (Srinivas et al., 1970)	Present	HSDT (Reddy, 1984)	TSDT (Ghugal and Sayyad, 2011b)	FSDT (Mindlin, 1951)
(1, 1)	1.6530	1.6448	1.6550	1.6530	1.6647
(1,2)	1.7160	1.7105	1.7209	1.7145	1.7307
(1, 3)	1.8115	1.8052	1.8210	1.8044	1.8307
(1, 4)	1.9306	1.9249	1.9466	1.9121	1.9562
(2, 1)	1.6805	1.6728	1.6827	1.6817	1.6922
(2, 2)	1.7509	1.7462	1.7562	1.7513	1.7657
(2, 3)	1.8523	1.8418	1.8622	1.8458	1.8717
(2, 4)	1.9749	1.9701	1.9912	1.9524	2.0004
(3, 1)	1.7334	1.7274	1.7361	1.7373	1.7452
(3, 2)	1.8195	1.8068	1.8255	1.8255	1.8343
(3, 3)	1.9289	1.9203	1.9395	1.9301	1.9418
(4, 1)	1.8458	1.8437	1.8583	1.7163	1.7267
(4, 2)	1.9447	1.9351	1.9514	1.9568	1.9588

Example 2: Present study also deals with the, buckling analysis of an orthotropic square and rectangular plate with all edges simply supported. Three different in-plane loading conditions are used in this numerical study: (1) uniaxial compression along the x-axis; (2) uniaxial compression along the y-axis; and (3) biaxial compression. For the comparison studies, results are also generated using higher order shear deformation theory (HSDT) of Reddy (1984), trigonometric shear deformation theory (TSDT) of Ghugal and Sayyad (2011b), first order shear deformation theory (FSDT) of Mindlin (1951) and classical plate theory.

Tables 4-5 shows the comparison of non-dimensional critical buckling load for simply supported square plate subjected to uniaxial and biaxial compression with the variation of modular and aspect ratios. It can be seen that the present theory gives excellent results of critical buckling load for all aspect ratios and modular ratio 3, whereas TSDT overestimate and FSDT underestimate the same. The differences between present theory and HSDT of Reddy will slightly increases with respect to increase in modular ratios. Classical plate theory overestimates the value of critical buckling load for all aspect ratios and modular ratios. Tables 6-8 shows the comparison of non-dimensional critical buckling load for simply supported rectangular plate. Examination of these Tables reveals that, non-dimensional critical buckling load decreases with increase in 'b/a' ratios when subjected to uniaxial compression along x-axis whereas increases when subjected to uniaxial compression along y-axis and biaxial buckling. Critical buckling load for rectangular plate increases with increase.

Table 4: Comparison of non-dimensional buckling load factors (N_{cr}) for simply supported orthotropic square plate under uniaxial compression ($b/a = 1, k_1 = -1,\ k_2 = 0,\ m = n = 1$)

a/h	Model	Non-dimensional Critical Buckling Load Factor (N_{cr})				
		Modular Ratio (E_1 / E_2)				
		3	10	20	30	40
5	Present	3.9650	6.3014	8.0946	9.2166	10.049
	HSDT (Reddy, 1984)	3.9434	6.2072	7.8292	8.7422	9.3472
	TSDT (Ghugal and Sayyad, 2011b)	4.0572	6.3212	7.9324	8.8418	9.4502
	RPT (Kim *et al.*, 2009)	---	6.3478	---	---	10.579
	FSDT (Mindlin, 1951)	3.9386	6.1804	7.7450	8.5848	9.1084
	CPT	5.4248	11.163	19.383	27.606	35.830
10	Present	4.9612	9.2998	14.080	17.748	20.676
	HSDT (Reddy, 1984)	4.9568	9.2772	14.001	17.577	20.386
	TSDT (Ghugal and Sayyad, 2011b)	5.0128	9.3646	14.116	17.711	20.534
	RPT (Kim *et al.*, 2009)	---	9.3732	---	---	22.258
	FSDT (Mindlin, 1951)	4.9562	9.2734	13.982	17.532	20.304
	CPT	5.4248	11.163	19.383	27.606	35.830
20	Present	5.3004	10.625	17.681	24.146	30.094
	HSDT (Reddy, 1984)	5.2994	10.621	17.664	24.108	30.025
	TSDT (Ghugal and Sayyad, 2011b)	5.3194	10.653	17.714	24.175	30.107
	RPT (Kim *et al.*, 2009)	---	10.653	---	---	31.069
	FSDT (Mindlin, 1951)	5.2994	10.620	17.662	24.102	30.014
	CPT	5.4248	11.163	19.383	27.606	35.830
50	Present	5.4044	11.072	19.087	26.982	34.758
	HSDT (Reddy, 1984)	5.4040	11.072	19.085	26.976	34.748
	TSDT (Ghugal and Sayyad, 2011b)	5.4116	11.081	19.098	26.993	34.769
	RPT (Kim *et al.*, 2009)	---	11.078	---	---	34.972
	FSDT (Mindlin, 1951)	5.4046	11.072	19.085	26.976	34.748
	CPT	5.4248	11.163	19.383	27.606	35.830
100	Present	5.4196	11.400	19.308	27.447	35.554
	HSDT (Reddy, 1984)	5.4192	11.139	19.307	27.466	35.553
	TSDT (Ghugal and Sayyad, 2011b)	5.4250	11.145	19.314	27.453	35.562
	RPT (Kim *et al.*, 2009)	---	11.142	---	---	35.612
	FSDT (Mindlin, 1951)	5.4206	11.142	19.309	27.448	35.554
	CPT	5.4248	11.163	19.383	27.606	35.830

Table 5: Comparison of non-dimensional buckling load (N_{cr}) for simply supported orthotropic square plate under biaxial compression ($b / a = 1$, $k_1 = -1$, $k_2 = -1$, $m = n = 1$)

a/h	Model	Non-dimensional Critical Buckling Load (N_{cr}) Modular Ratio (E_1 / E_2) 3	10	20	30	40
5	Present	1.9825	3.1507	4.0473	4.6083	5.0246
	HSDT (Reddy, 1984)	1.9717	3.1036	3.9146	4.3711	4.6736
	TSDT (Ghugal and Sayyad, 2011b)	2.0281	3.1606	3.9662	4.4209	4.7251
	RPT (Kim *et al.*, 2009)	---	3.1739	---	---	5.2895
	FSDT (Mindlin, 1951)	1.9693	3.0902	3.8725	4.2924	4.5542
	CPT	2.7124	5.5814	9.6917	13.8034	17.9154
10	Present	2.4806	4.6499	7.0402	8.8741	10.3380
	HSDT (Reddy, 1984)	2.4784	4.6386	7.0002	8.7885	10.1929
	TSDT (Ghugal and Sayyad, 2011b)	2.5064	4.6823	7.0582	8.8558	10.2674
	RPT (Kim *et al.*, 2009)	---	4.6866	---	---	11.1290
	FSDT (Mindlin, 1951)	2.4781	4.6367	6.9910	8.7662	10.1522
	CPT	2.7124	5.5814	9.6917	13.8034	17.9154
20	Present	2.6502	5.3125	8.8405	12.0731	15.0470
	HSDT (Reddy, 1984)	2.6497	5.3101	8.8320	12.0540	15.0127
	TSDT (Ghugal and Sayyad, 2011b)	2.6597	5.3266	8.8574	12.0875	15.0537
	RPT (Kim *et al.*, 2009)	---	5.2265	---	---	15.5345
	FSDT (Mindlin, 1951)	2.6497	5.3100	8.8311	12.0513	15.0070
	CPT	2.7124	5.5814	9.6917	13.8034	17.9154
50	Present	2.7022	5.5364	9.5437	13.4911	17.3791
	HSDT (Reddy, 1984)	2.7020	5.5360	9.5424	13.4884	17.3744
	TSDT (Ghugal and Sayyad, 2011b)	2.7058	5.5407	9.5490	13.4969	17.3849
	RPT (Kim *et al.*, 2009)	---	5.5390	---	---	17.4860
	FSDT (Mindlin, 1951)	2.7023	5.5362	9.5425	13.4885	17.3745
	CPT	2.7124	5.5814	9.6917	13.8034	17.9154
100	Present	2.7098	5.5700	9.6542	13.7238	17.7779
	HSDT (Reddy, 1984)	2.7096	5.5697	9.6533	13.7230	17.7767
	TSDT (Ghugal and Sayyad, 2011b)	2.7124	5.5727	9.6571	13.7269	17.7811
	RPT (Kim *et al.*, 2009)	---	5.5710	---	---	17.8060
	FSDT (Mindlin, 1951)	2.7103	5.5710	9.6544	13.7241	17.7772
	CPT	2.7124	5.5814	9.6917	13.8034	17.9154

Table 6: Comparison of non-dimensional critical buckling load (N_{cr}) of simply supported orthotropic rectangular plates subjected to uniaxial compression along x-axis ($a / h = 5$, $k_1 = -1$, $k_2 = 0$, $m = n = 1$)

E_1 / E_2	Model	Non-dimensional Critical Buckling Load (N_{cr}) (b / a) 1.0	1.5	2	2.5	3.0	3.5	4.0
10	Present	6.3014	5.3026	5.0148	4.8939	4.8317	4.7953	4.7723
	HSDT (Reddy, 1984)	6.2072	5.2245	4.9412	4.8223	4.7611	4.7253	4.7026
	TSDT (Ghugal and Sayyad, 2011b)	6.3212	5.2923	4.9940	4.8682	4.8033	4.7654	4.7412
	RPT (Kim *et al.*, 2009)	6.3478	---	5.0109	---	---	---	---
	FSDT (Mindlin, 1951)	6.1804	5.2025	4.9205	4.8021	4.7412	4.7056	4.6831
	CPT	11.163	9.3549	8.8428	8.6270	8.5154	8.4500	8.4083
25	Present	8.7062	7.8373	7.6007	7.5047	7.4562	7.4281	7.4109
	HSDT (Reddy, 1984)	8.3394	7.4929	7.2631	7.1701	7.1231	7.0961	7.0792
	TSDT (Ghugal and Sayyad, 2011b)	8.4398	7.5414	7.2929	7.1909	7.1391	7.1091	7.0905
	RPT (Kim *et al.*, 2009)	9.1039	---	7.5409	---	---	---	---
	FSDT (Mindlin, 1951)	8.2199	7.3805	7.1530	7.0610	7.0154	6.9883	6.9713
	CPT	23.495	21.690	21.179	20.964	20.854	20.783	20.744
40	Present	10.049	9.2310	9.0145	8.9282	8.8853	8.8608	8.8454
	HSDT (Reddy, 1984)	9.3472	8.5541	8.3455	8.2628	8.2217	8.1983	8.1837
	TSDT (Ghugal and Sayyad, 2011b)	9.4502	8.6015	8.3719	8.2791	8.2324	8.2056	8.1888
	RPT (Kim *et al.*, 2009)	10.579	---	8.7587	---	---	---	---
	FSDT (Mindlin, 1951)	9.1084	8.3237	8.1178	8.0363	7.9958	7.9728	7.9585
	CPT	35.830	34.027	33.516	33.300	33.189	33.124	33.082

Table 7: Comparison of non-dimensional critical buckling load (N_{cr}) of simply supported orthotropic rectangular plates subjected to uniaxial compression along *y*-axis (a / $h = 5$, $k_1 = 0$, $k_2 = -1$, $m = n = 1$)

E_1/E_2	Model	Non-dimensional Critical Buckling Load (N_{cr}) (b / a) 1.0	1.5	2	2.5	3.0	3.5	4.0
10	Present	6.3014	11.930	20.059	30.587	43.485	58.743	76.356
	HSDT (Reddy, 1984)	6.2072	11.755	19.765	30.139	42.849	57.885	75.242
	TSDT (Ghugal and Sayyad, 2011b)	6.3212	11.907	19.975	30.426	43.229	58.375	75.859
	RPT (Kim *et al.*, 2009)	6.3478	---	20.044	---	---	---	---
	FSDT (Mindlin, 1951)	6.1804	11.705	19.682	30.013	42.670	57.644	74.929
	CPT	11.163	21.048	35.371	53.918	76.638	103.51	134.53
25	Present	8.7062	17.634	30.403	46.904	67.107	90.999	118.57
	HSDT (Reddy, 1984)	8.3394	16.859	29.052	44.813	64.110	86.931	113.27
	TSDT (Ghugal and Sayyad, 2011b)	8.4398	16.968	29.171	44.943	64.253	87.089	113.44
	RPT (Kim *et al.*, 2009)	9.1039	---	30.164	---	---	---	---
	FSDT (Mindlin, 1951)	8.2199	16.606	28.611	44.131	63.132	85.604	111.54
	CPT	23.495	48.803	84.716	131.02	187.66	254.63	331.92
40	Present	10.049	20.769	36.058	55.801	79.968	108.55	141.42
	HSDT (Reddy, 1984)	9.3472	19.246	33.382	51.642	73.995	100.42	130.93
	TSDT (Ghugal and Sayyad, 2011b)	9.4502	19.353	33.487	51.744	74.092	100.52	131.02
	RPT (Kim *et al.*, 2009)	10.579	---	35.034	---	---	---	---
	FSDT (Mindlin, 1951)	9.1084	18.728	32.471	50.226	71.962	97.667	127.33
	CPT	35.830	76.560	134.06	208.12	298.69	405.76	529.31

Table 8: Comparison of non-dimensional critical buckling load (N_{cr}) of simply supported orthotropic rectangular plates subjected to biaxial compression (a / $h = 5$, $k_1 = -1$, $k_2 = -1$, $m = n = 1$)

E_1/E_2	Model	Non-dimensional Critical Buckling Load (N_{cr}) (b / a) 1.0	1.5	2	2.5	3.0	3.5	4.0
10	Present	3.1507	3.6710	4.0118	4.2189	4.3485	4.4334	4.4915
	HSDT (Reddy, 1984)	3.1036	3.6170	3.9530	4.1571	4.2849	4.3687	4.4260
	TSDT (Ghugal and Sayyad, 2011b)	3.1606	3.6639	3.9952	4.1967	4.3230	4.4057	4.4623
	RPT (Kim *et al.*, 2009)	3.1739	---	4.0087	---	---	---	---
	FSDT (Mindlin, 1951)	3.0902	3.6017	3.9364	4.1398	4.2671	4.3505	4.4076
	CPT	5.5814	6.4765	7.0743	7.4371	7.6638	7.8122	7.9137
25	Present	4.3531	5.4258	6.0806	6.4696	6.7107	6.8678	6.9750
	HSDT (Reddy, 1984)	4.1697	5.1874	5.8105	6.1811	6.4110	6.5609	6.6631
	TSDT (Ghugal and Sayyad, 2011b)	4.2199	5.2210	5.8343	6.1991	6.4253	6.5728	6.6734
	RPT (Kim *et al.*, 2009)	4.5519	---	6.0327	---	---	---	---
	FSDT (Mindlin, 1951)	4.1099	5.1096	5.7224	6.0870	6.3132	6.4607	6.5613
	CPT	11.747	15.016	16.943	18.072	18.767	19.217	19.524
40	Present	5.0246	6.3907	7.2116	7.6967	7.9968	8.1920	8.3251
	HSDT (Reddy, 1984)	4.6736	5.9221	6.6764	7.1231	7.3995	7.5796	7.7023
	TSDT (Ghugal and Sayyad, 2011b)	4.7251	5.9549	6.6975	7.1372	7.4092	7.5863	7.7071
	RPT (Kim *et al.*, 2009)	5.2895	---	7.0069	---	---	---	---
	FSDT (Mindlin, 1951)	4.5542	5.7626	6.4942	6.9278	7.1963	7.3711	7.4903
	CPT	17.915	23.557	26.813	28.707	29.870	30.623	31.136

5 CONCLUSION

An exponential shear deformation theory (Sayyad and Ghugal, 2012a and 2012b; Sayyad, 2013) has been extended in this paper for buckling and free vibration analysis of orthotropic plates. The theory takes into account of transverse shear effects and parabolic distribution of the transverse shear strains through the thickness of the plate. From the numerical results and discussion it can be concluded that, the frequencies obtained by the present theory are accurate as seen from the comparison with exact results specially in case of natural predominately bending mode. Also, an exponential shear deformation theory can accurately predict the critical buckling loads of the orthotropic plates with various plate aspect ratios and modular ratios.

References

Chen, W., Zhen, W. (2008). A selective review on recent development of displacement-based laminated plate theories. Recent Patents on Mechanical Engineering 1:29-44.

Ghugal, Y.M., Shimpi, R.P. (2002). A review of refined shear deformation theories for isotropic and anisotropic laminated plates. Journal of Reinforced Plastics and Composites 21: 775-813.

Ghugal, Y.M., Sayyad, A.S. (2010). A flexure of thick isotropic plate using trigonometric shear deformation theory. Journal of Solid Mechanics 2(1): 79-90.

Ghugal, Y.M., Sayyad, A.S. (2011a).Free vibration of thick isotropic plates using trigonometric shear deformation theory. Journal of Solid Mechanics 3(2): 172-182.

Ghugal, Y.M., Sayyad, A.S. (2011b).Free vibration analysis of thick orthotropic plates using trigonometric shear deformation theory. Latin American Journal of Solids and Structures 8: 229-243.

Ghugal, Y.M., Pawar, M.D. (2011). Flexural analysis of thick plates by hyperbolic shear deformation theory. Journal of Experimental & Applied Mechanics 2(1):1-21.

Karama, M., Afaq, K.S., Mistou, S. (2009). A new theory for laminated composite plates. Proceeding of Institution of Mechanical Engineers, Series L: Design and Applications 223: 53–62.

Kim, S.E., Thai, H.T., Lee, J. (2009). Buckling analysis of plates using the two variable refined plate theory. Thin-Walled Structures 47: 455–462.

Kreja, I. (2011). A literature review on computational models for laminated composite and sandwich panels. Central European Journal of Engineering 1(1): 59-80.

Liew, K.M., Xiang, Y., Kitipornchai, S. (1995). Research on thick plate vibration: a literature survey. Journal of Sound and Vibration 180: 163-176.

Mindlin, R.D. (1951). Influence of rotatory inertia and shear on flexural motions of isotropic, elastic plates. ASME Journal of Applied Mechanics 18: 31-38.

Noor, A.K., Burton, W.S. (1989). Assessment of shear deformation theories for multilayered composite plates. Applied Mechanics Reviews 42: 1-13.

Reddy, J.N. (1984). A simple higher order theory for laminated composite plates. ASME Journal of Applied Mechanics 51: 745-752.

Reissner, E. (1945). The effect of transverse shear deformation on the bending of elastic plates. ASME Journal of Applied Mechanics 12(2):69-77.

Sayyad, A.S., Ghugal, Y.M. (2012a). Bending and free vibration analysis of thick isotropic plates by using exponential shear deformation theory. Applied and Computational Mechanics 6(1): 65–82.

Sayyad, A.S., Ghugal, Y.M. (2012b). Buckling analysis of thick isotropic plates by using exponential shear deformation theory. Applied and Computational Mechanics 6(2): 185–196.

Sayyad, A.S. (2013).Flexure of thick orthotropic plates by exponential shear deformation theory. Latin American Journal of Solids and Structures 10: 473 – 490.

Shimpi, R.P., Patel, H.G. (2006a). A two variable refined plate theory for orthotropic plate analysis. International Journal of Solids and Structures 43:6783-6799.

Shimpi, R.P., Patel, H.G. (2006b). Free vibrations of plate using two variable refined plate theory. Journal of Sound and Vibration 296:979–999.

Srinivas, S., Rao,C.V.J., Rao, A.K. (1970). An exact analysis for vibration of simply supported homogeneous and laminated thick rectangular plates. Journal of sound and vibration 12(2):187-199.

Thai, H., Kim, S. (2011). Levy-type solution for buckling analysis of orthotropic plates based on two variable refined plate theory. Composite Structures 93: 1738-1746.

Vasil'ev, V.V. (1992). The theory of thin plates.Mechanics of solids (Mekhanika Tverdogo Tela) 27:22-42, (English translation from original Russian by Allerton Press, New York)