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On the crack propagation analysis of rock like Brazilian disc specimens containing cracks under compressive line loading

Abstract
The pre-existing cracks in the brittle substances seem to be the main cause of their failure under various loading conditions. In this study, a simultaneous analytical, experimental and numerical analysis of crack propagation, cracks coalescence and failure process of brittle materials has been performed. Brazilian disc tests are being carried out to evaluate the cracks propagation paths in rock-like Brazilian disc specimens containing single and double cracks (using rock-like specimens which are specially prepared from Portland Pozzolana Cement (PPC), fine sands and water in a rock mechanics laboratory). The failure load of the pre-cracked disc specimens are measured showing the decreasing effects of the cracks and their orientation on the final failure load. The same specimens are numerically simulated by a higher order indirect boundary element method known as displacement discontinuity method. These numerical results are compared with the existing analytical and experimental results proving the accuracy and validity of the proposed numerical method. The numerical and experimental results obtained from the tested specimens are in good agreement and demonstrate the accuracy and effectiveness of the proposed approach.
Keywords: Crack propagation, cracks coalescence, higher order displacement discontinuity method, Brazilian disc specimens, rock-like materials.

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1 INTRODUCTION

The presence of pre-existing cracks may reduce the fracture toughness of brittle materials (Kato and Nishioka, 2005). The mechanical behavior of brittle materials may be affected by the micromechanical behaviors of the cracks. Nevertheless the extension of cracks depends on the properties of cracks such as size, location, orientation and loading condition. The production and propagation of cracks play a vital role in predicting the cyclic failure process of rock specimens (Ichikawa et al., 2001).
In the crack propagation process of brittle materials (such as pre-cracked rock specimens) usually two types of cracks may be observed which are emanating from the original tips of pre-existing cracks (i.e. wing cracks and secondary cracks). Wing cracks are usually produced due to tension while secondary cracks may initiate due to shear. Therefore, initiation of wing cracks in rocks is favored relative to secondary cracks because of the lower toughness of these materials in tension than in shear (Bieniawski, 1967). The pre-existing cracks in rocks are normally under compressive loading rather than under tension, shear or mixed mode loadings (Ke et al., 2008). It is mainly ex-

pected that the crack initiation will follow in the direction (approximately) parallel to the maximum compressive load (Hoek and Bieniawski, 1965).

Many experimental works have been devoted to study the crack initiation, crack propagation path, and coalescence of the pre-existing cracks in specimens made of various brittle materials, including natural rocks or rock-like materials under compressive loading (Ingraffea ,1985; Horii and Nemat-Nasser, 1985; Huang et al., 1990; Shen et al., 1995; Wong and Chau, 2001; Sahouryeh et al., 2002; Li et al., 2005; Park and Bobet, 2006; Park and Bobet, 2007; Park. 2008; Yang et al., 2009; Park and Bobet, 2009; Park and Bobet, 2010; Janeiro and Einstein, 2010; Yang, 2011; Lee and Jeon, 2011; Cheng-zhi and Ping, 2012; Haeri et al., 2013; Reis and Nunes, 2014. Brazilian disc test is one of the most suitable tests in evaluating the static and dynamic fracture toughness of rocks and rock-like specimens containing central pre-existing crack or cracks. These tests may also be used to study the crack initiation, propagation path and cracks coalescence of brittle substances such as rocks under compressive line loadings (Ayatollahi and Aliha, 2008; Wang, 2010; Dai et al., 2010; Dai et al., 2011; Ayatollahi and Sistaninia, 2011; Wang et al., 2011; Wang et al., 2012; Ghazvinian et al., 2012). This testing procedure used extensively to measure the tensile strength, fracture toughness and mixed mode failure process in the un-cracked and pre-cracked disc specimens of various brittle substances under compressive line loading (Awaji and Sato, 1978; Sanchez, 1979; Atkinson et al., 1982; Shetty et al., 1986; Fowell and Xu, 1994; Krishnan et al., 1998; Khan and Al-Shayea, 2000; Al-Shayea et al., 2000; Al-Shayea et al., 2001; Al-Shayea, 2005). It should be noted that in Brazilian disc specimens, the crack initiation and failure process of the specimens often happen very soon under compressive line loading due to the low tensile strength of rocks and rock-like materials. For example Al-Shayea (2005) experimentally studied the crack propagation paths in the Central Straight Through Crack Brazilian Disk (CSCBD) specimens of brittle limestone with different crack inclination angles under mixed mode I/II loading. He also investigated the influence of confining pressure and temperature on the crack initiation and propagation of the rock samples. The experimental results were compared with theoretical predictions of crack initiation angle. Ghazvinian et al. (2012) have carried out analytical, experimental, and numerical studies for a better understanding of crack propagation process in the CSCBD specimens under compressive line loading. The existing experimental and numerical analyses confirmed the effect of crack inclination angle and crack length on the fracturing processes of brittle materials under various loading conditions.

Various numerical methods have been developed for the simulation of crack propagation in brittle substances. These numerical methods include the Finite Element Method (FEM), Boundary Element Method (BEM), Discrete Element Method (DEM) (Iturrioz et al., 2009). Three important fracture initiation criteria were proposed to study the crack propagation mechanism of brittle materials i.e. i) the maximum tangential stress (σ-criterion) (Erdogan and Sih, 1963) ii) the maximum energy release rate (G-criterion) (Hussian and Pu, 1974) and iii) the minimum energy density criterion (S-criterion) (Sih (1974)). Some modified form of the mentioned criteria e.g. F-criterion which is a modified form of energy release rate criterion proposed by Shen and Stephansson (She and Stephansson, 1994) may also be used to study the failure behavior of brittle substances (Marji et al., 2006; Marji and Dehghani, 2010; Barros et al., 2012; Marji, 2013). Several computer codes were used to model the failure mechanism of brittle materials such as rocks, for example, FROCK code (Park, 2008), Rock Failure Process Analysis ($RFPA^{2D}$) code (Wong et al., 2002), 2D Particle Flow Code (PFC^{2D})(Lee and Jeon, 2011; Ghazvinian et al., 2012; Manouchehrian et al., 2013).

In this study, a comprehensive analytical, numerical and experimental approach is developed for the analyses of crack propagation and cracks coalescence in rocks and rock-like materials under compressive loading condition. A typical analytical study is presented first, then several Brazilian tests on disc specimens of rock-like materials containing either single and double cracks in the central part of the specimens are performed for crack propagation analysis of brittle substances. The center single and double cracked disc specimens (prepared from PCC, fine sands and water) tested under compressive line loading. These experimental works are also simulated numerically by a modified higher order displacement discontinuity method and the crack propagation and cracks coalescence in the bridge area are studied based on Mode I and Mode II stress intensity factors (SIFs). Comparing the numerical results with both the analytical and experimental results demonstrate the accuracy and effectiveness of the proposed numerical method.

2 ANALYTICAL STUDY OF A CENTRAL STRAIGHT TROUGH CRACK BRAZILIAN DISC (CSCBD) SPECIMEN

Let consider a Central Straight Through Crack Brazilian Disk (CSCBD) specimen with radius, R=42 mm containing a central straight through crack with a half-length, b=5 mm and inclination angle, φ, changing counterclockwise from the y axis, and the line load, F is acting parallel to the y axis as shown in Fig. 1. The analytical solution of this typical fracture mechanics problem is given in the literature (e.g. Atkinson et al. (1982)). Based on Fig.1, the analytical solution for Mode I and Mode II stress intensity factors (SIFs) in a CSCBD specimen can be estimated from:

$$K_I = \frac{F\sqrt{b}}{\sqrt{\pi}RB}\varpi_I, \qquad K_{II} = \frac{F\sqrt{b}}{\sqrt{\pi}RB}\varpi_{II} \tag{1}$$

where, K_I and K_{II} are Mode I and Mode II stress intensity factors (SIFs), respectively expressed in MPa $m^{1/2}$, F is the compressive load at failure in Newton, B is thickness (length) of the disk in mm, and ϖ_I and ϖ_{II} are the non-dimensional coefficients depending on the crack inclination angle, φ, which can be defined as:

$$\begin{aligned} \varpi_I &= 1-4\sin^2\varphi + 4\sin^2\varphi(1-4\cos^2\varphi)(\frac{b}{R})^2 \\ \varpi_{II} &= [2+(8\cos^2\varphi-5)(\frac{b}{R})^2]\sin 2\varphi \end{aligned} \tag{2}$$

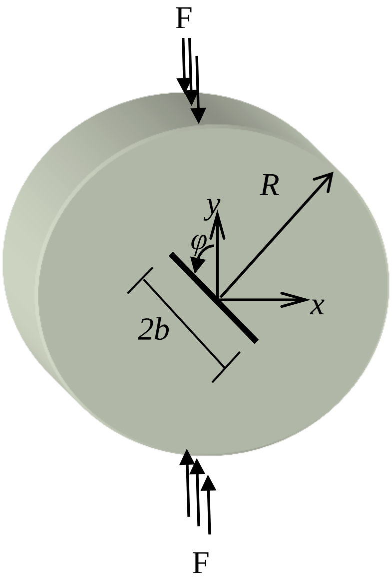


Fig. 1. Schematic view of (CSCBD) specimen containing a center slant crack

As it can be seen in equations (1) and (2), the SIFs of crack tips are affected by the crack geometry such as half crack length (b), radius (R), thickness (B) and crack inclination angle (φ).
In analytical solution, thickness (B) of the disk is essential to estimate the value of SIFs. The value of thickness is assumed to be 25 mm.
Variations of ϖ_I and ϖ_{II} for the assumed CSCBD specimen are illustrated in Fig. 2 considering different φ angles. As shown in this figure, ϖ_I decreases monotonically with increasing φ angle, while ϖ_{II} has a global maximum value at φ=45 degrees. Furthermore, Fig. 2 implies that pure Mode I loading is achieved only at φ=0 (ϖ_I =1), whereas pure Mode II loading is obtained at φ=45 (ϖ_{II} =1.98).

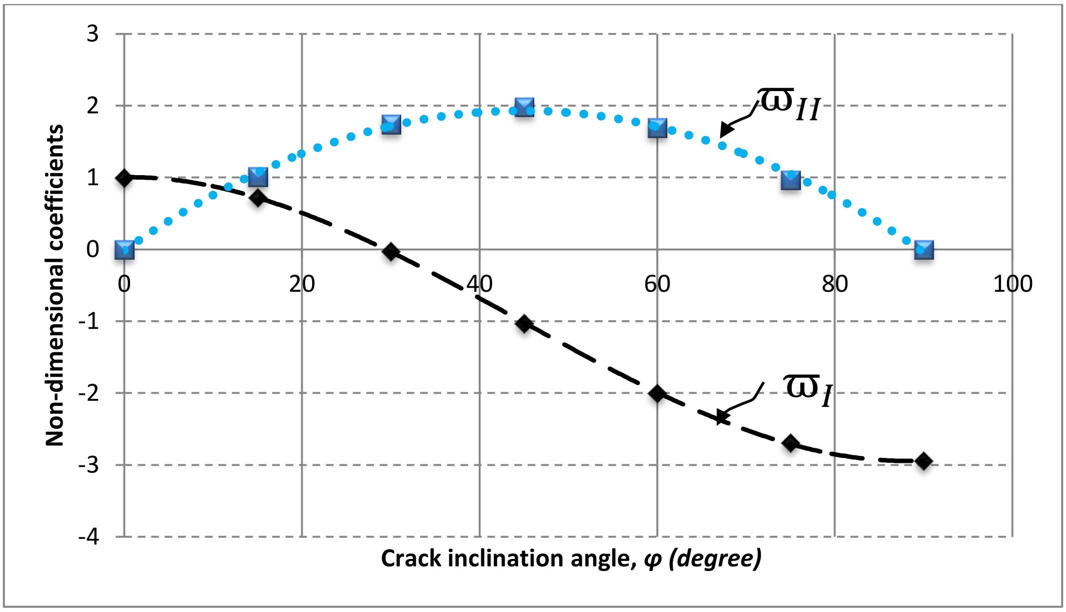


Fig. 2 Variation of ϖ_I and ϖ_{II} with crack inclination angles

3 EXPERIMENTAL ANALYSIS

In this section, some experimental works have been performed in a rock mechanics laboratory on some specially prepared specimens (from rock-like brittle substances).

3.1 SPECIMEN PREPARATION AND TESTING

The pre-cracked rock-like disc specimens with 84 mm, diameters and 25 mm, thickness are specially prepared from a mixture of Portland Pozzolana cement (PPC), fine sands and water. Table 1 gives the mechanical properties of the prepared rock-like specimens tested in the rock mechanics laboratory before inserting the cracks.

Table 1. Some mechanical properties of the un-cracked rock-like disc specimens

Description	Parameter	Value	Unit
Compressive strength	σ_c	28	MPa
Young's modulus	E	15	GPa
Tensile strength	σ_t	3.81	MPa
Poisson's ratio	ν	0.21	-

Various Brazilian tests were conducted on rock-like disc specimens containing either a single center crack or two cracks 1 and 2. These cracks are created by inserting one or two thin metal shims with 10 mm width and 1 mm thickness into the specimens (during the specimens casting in the mold) as shown in Fig. 3.

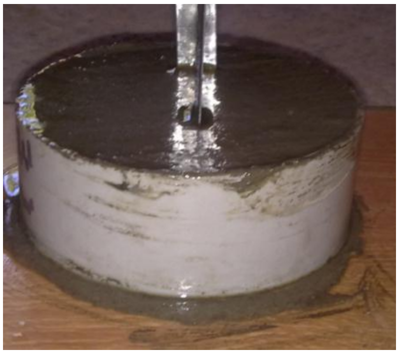


Fig.3. A typical rock-like Brazilian disc specimen

Several Brazilian disc specimens of rock-like materials (with the same crack geometry) were prepared and tested in the laboratory to check out the reproducibility of the test results. Some of the Brazilian disc specimens have a single center crack with different inclination angles. Fig. 4 illustrates the Brazilian disc specimens with double cracks which are prepared in such a manner that the direction of crack 1 is kept constant and the crack 2 is oriented at different angles with respect to the direction of crack 1 i.e. at the angles $\varphi=0°$, 30°, 60° and 90° (in a counterclockwise direction). The compressive line loading, F was applied and the loading rate was kept at 0.5 MPa/s during the tests.

Fig.4 demonstrates a schematic view of the geometry of a specimen with two cracks (i.e. crack 1 and crack 2) of equal lengths, 2b=10 mm and the ratio of half crack length, b to the specimen radius, R is taken as 0.119 (b/R=0.119).

In this research, three specimens were prepared for each experimental work and as a whole, twenty one CSCBD specimens (Brazilian discs with different center crack inclinations) were prepared. Twelve double cracked disc specimens were also prepared with crack 1 and crack 2 located at the centerline of each specimen with the spacing S=25 mm as shown in Fig. 5 (the spacing (S) is taken as the vertical distance between the centers of two cracks expressed in mm).

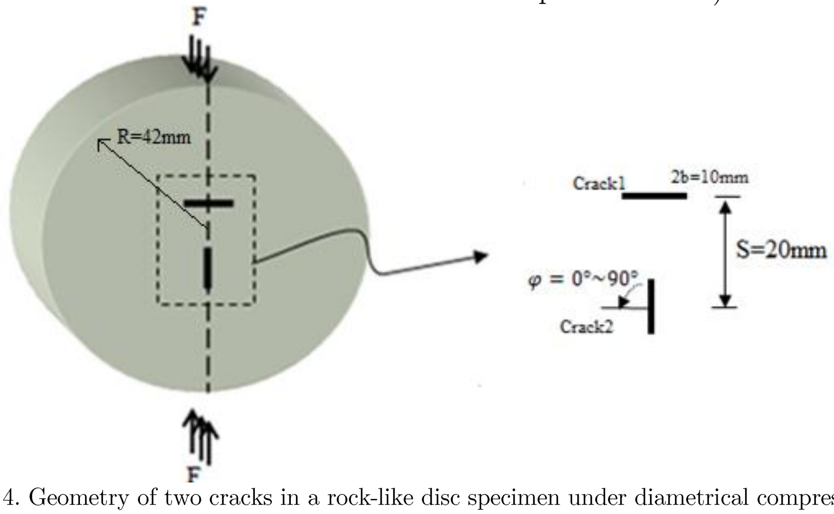
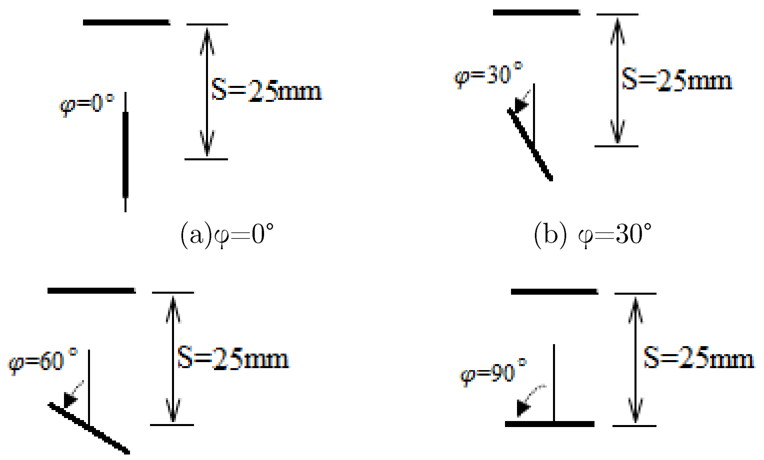


Fig. 4. Geometry of two cracks in a rock-like disc specimen under diametrical compression



(a)φ=0° (b) φ=30°

(c) φ=60° (d) φ=90°

Fig.5. Crack geometries with spacing S=25 mm

3.2 EXPERIMENTAL TESTS AND RESULTS

The rock-like Brazilian disc specimens were tested experimentally and the results were used to analyze the failure loads and the crack propagation process of the pre-cracked disc specimens. The crack propagation process of the disc specimens are discussed considering the two cases of disc specimens with: i) single crack and ii) double cracks.

3.2.1 FRACTURE ANALYSIS OF THE PRE-CRACKED DISC SPECIMENS

It is obvious that the pre-cracked rock-like disc specimens have a lower strength compared to the un-cracked specimens (specimens having no cracks). The failure load analysis of the pre-cracked disc specimens containing either a single crack or two cracks with different orientations is of paramount importance to study the behavior of the brittle materials. Fig. 6 describes a variation of the normalized failure load for single and double cracked disc specimens. The failure load of the pre-cracked disc specimens is normalized by the average failure load of the un-cracked specimens. The average failure load of un-cracked specimens is about 16 KN. In addition, the normalized failure load in the double cracked disc specimens is smaller than those in the single cracked disc specimens.

The normalized failure loads for the single and double cracked disc specimens are usually less than one because the pre-existing crack may decrease the final strength of specimen (Fig. 6). In the single cracked specimens, the normalized failure loads for φ =0°,75°, and 90° is larger than normalized failure load for other inclination angles. In the double cracked specimens, failure loads at different stages of crack propagation process are decreasing for φ=0° to less than about 30° but increasing for φ=30° to 90° above 30°, respectively (Fig. 6). However, the normalized failure load of the double cracked specimen was similar to that of single cracked specimen when φ was 90°. This means that the horizontal center crack may have a little effect on the specimen failure load due to shielding effect of the inclined crack.

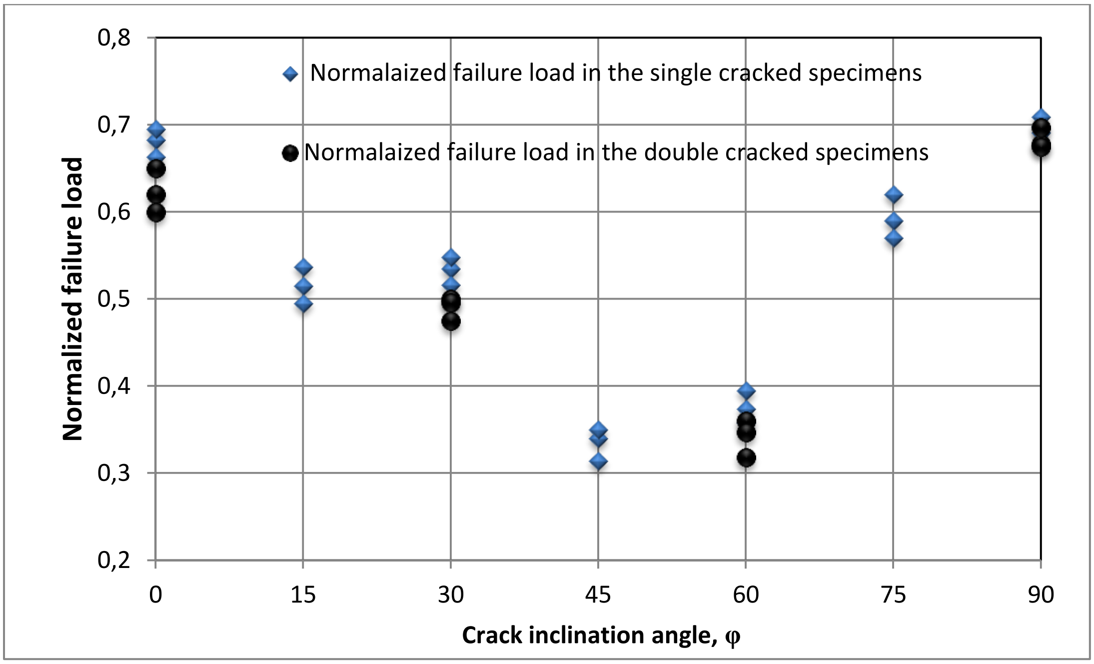


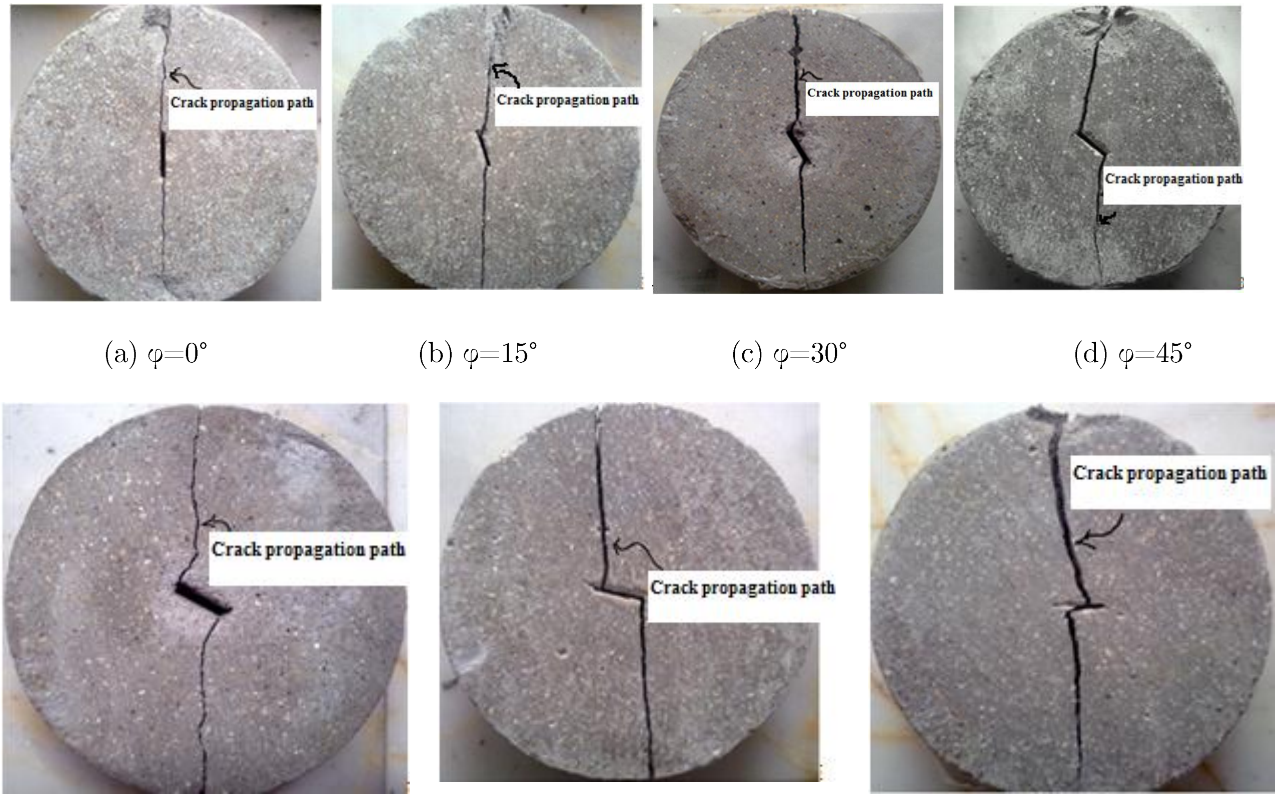
Fig.6. Normalized failure load versus crack inclination angle in the single and double cracked disc specimens

3.3 CRACK PROPAGATION PROCESS OF PRE-CRACKED DISC SPECIMENS

Experimental investigation of pre-cracked rock-like specimens is accomplished considering the two cases: i) specimens containing a single crack and ii) specimens containing double cracks.

3.3.1 PRE-CRACKED DISC SPECIMENS WITH A SINGLE CRACK

In this research, some experimental works have been established to study the mechanism of crack initiation and crack propagation emanating from CSBDC specimens containing different crack inclination angles. In the single cracked disc specimens, the wing cracks propagated in a curved path and continue their growth in a direction (approximately) parallel to the direction of maximum compressive load, as shown in Figs. 7 (a)-(g). These wing cracks are initiated at the original tips of the cracks for all crack inclination angles greater than 15°. It should be noted that (as it is clearly evident from Fig. 7 (g)) the wing cracks may not start their initiation from the original tips of the single crack when the inclination angle are close to 90° (that is at right angle to the direction of applied compressive line load). On the other hand, the specimen may fail away due to the indirect tensile effect (axial splitting) exactly like that of the un-cracked Brazilian disc specimen in a conventional Brazilian test.



(a) φ=0° (b) φ=15° (c) φ=30° (d) φ=45°

(e) φ=60° (f) φ=75° (g) φ=90°

Fig. 7. Experimental tests showing the cracking patterns in the single cracked disc specimens with different crack inclination angles: (a) φ=0°, (b) φ=15°, (c) φ=30°, (d) φ=45°, (e) φ=60°, (f) φ=75° and (g) φ=90°.

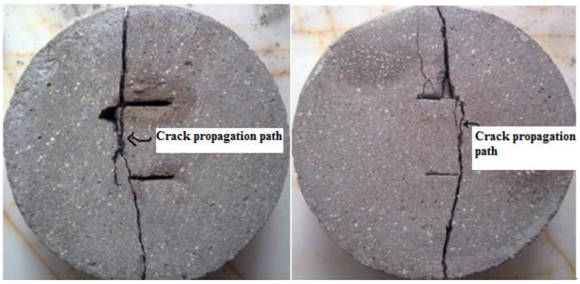
3.3.2 DOUBLE CRACKED BRAZILIAN DISC SPECIMENS

Cracks coalescence phenomenon may occur when the two pre-existing cracks combine due to propagation of wing and/or secondary cracks (originating from the tips of the pre-existing cracks) in brittle materials under various loadings. As shown in Figs. 8, the cracks coalescence in the bridge area may also occur during the crack propagation process. In the current experimental works, the wing cracks are instantaneously initiated quasi-statically (Figs. 7 and 8). The development and coalescence of wing cracks in the bridge area (i.e. the area in-between the two pre-existing cracks) may be the main cause of the fracturing paths in rock-like disc specimens (Figs. 8). The bridge area may be considered as the area starting from the right tip of a horizontal crack (crack 1) to that of the right tip of an inclined crack (crack 2) for the cases shown in Figs. 8 (a)-(c) (φ=0°,30° and 60°). It should be noted that for the case shown in Fig. 8 (d) (φ=90°) the cracks initiated at the tips of inclined crack (crack 2) and then the specimen might fail due to crack propagation process starting from the tips of crack 2 in a tensile splitting mode (i.e. no coalescence might occur at the tips of cracks). It has been experimentally observed (as shown in Figs. 8 (a)-(d)) that the wing cracks emanating from the two original cracks may propagate toward each other and eventually the cracks coalescence may occur in the bridge area. Table 2 shows the required load (in KN) for the crack initiation of single and double cracked Brazilian disc specimens at different cracks inclination angles (the cracks coalescence loads for double cracked specimens are also included).

(a) φ=0°

(b) φ=30°

(c) φ=60°



(d) φ=90°

Fig.8. Experimental results illustrating the coalescence path of rock-like disc specimens containing double cracks ((a) φ=0°, (b) φ=30°, (c) φ=60°, (d) φ=90°)

Table 2 Cracks initiation and coalescence loads in the single and double-cracked specimens

Crack inclination angle	Wing crack initiation load (KN)		Cracks coalescence load (KN)
	Single cracke	Double cracke	Double cracke
φ=0°	6.6	6.2	9.7
φ=15°	5.3	-	-
φ=30°	5.6	4.8	7.5
φ=45°	2.7	-	-
φ=60°	3.2	2.9	5.2
φ=75°	6.3	-	-
φ=90°	7.4	7.1	9.2

4 INDIRECT BOUNDARY ELEMENT SIMULATION OF THE PRE-CRACKED BRAZILIAN DISC SPECIMENS

A displacement based version of the indirect boundary element method known as Displacement Discontinuity Method (DDM) originally proposed by Crouch (1967a) for the solution of elasto-static problems in solid mechanics is modified in this study to simulate the pre-cracked Brazilian disc specimens (Guo et al., 1990; Scavia, 1990; Aliabadi and Rooke, 1991; Haeri et al., 2013; Haeri et al., 2013)

4.1 HIGHER ORDER DISPLACEMENT DISCONTINUITY METHOD

In this research, a higher accuracy of the displacement discontinuities along the boundary of the problem is obtained by using quadratic displacement discontinuity (DD) elements. A quadratic DD element is divided into three equal sub-elements. Each sub-element contains a central node where the nodal DD is numerically evaluated (Marji and Dehghani, 2010; Marji, 2013).

4.2 NUMERICAL SIMULATION OF THE PRE-CRACKED SPECIMENS

The pre-cracked Brazilian disc specimens (prepared from rock-like materials) under compressive line loading can also be simulated numerically by the higher order displacement discontinuity method. The numerical results can be compared with the corresponding analytical and experimental results already obtained in the previous sections of this research to get a better knowledge of the crack propagation mechanism and failure of the brittle materials such as rocks. Based on the mechanical properties of the CSCBD specimen given in Table 3, the Mode I and Mode II stress intensity factors, (K_I and K_{II}), for different crack inclination angles (which can be estimated analytically from Eqs. (1)) are evaluated numerically by means of the higher order displacement discontinuity method.

The normalized Mode I and Mode II SIFs are simplified as:

$$K_I^N = K_I \Big/ \frac{F\sqrt{b}}{\sqrt{\pi}RB}$$
$$K_{II}^N = K_{II} \Big/ \frac{F\sqrt{b}}{\sqrt{\pi}RB} \tag{3}$$

The analytical, experimental and numerical results of K_I^N and K_{II}^N are given in Table 4. As shown in this table the proposed numerical method gives very accurate results and can be effectively used for the crack analysis of pre-cracked Brazilian disc specimens. In the numerical analysis of this problem, 90 quadratic elements have been used along the pre-existing crack, three special crack tip elements have been used for each crack tip and the ratio of crack tip element length, L to b is kept as 0.2 (L/b=0.2).

Table 3. Mechanical properties of CSCBD specimens

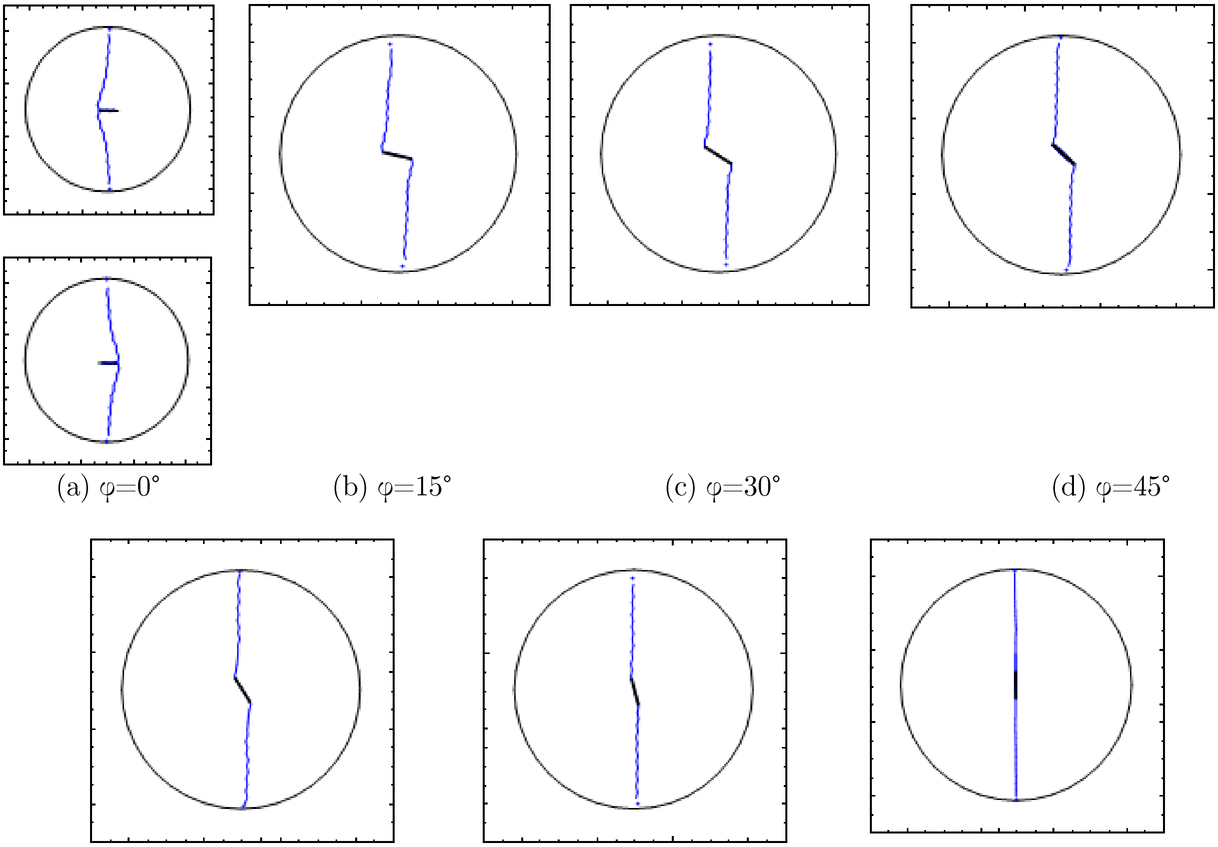
Description	Parameter	Value	Unit
Crack length	2b	10	mm
Compressive load	F	16	KN
Modulus of elasticity	E	15	GPa
Poisson's ratio	ν	0.21	-
Fracture toughness	K_{IC}	2	MPa $m^{1/2}$
Crack tip length	L	0.2	mm
Crack inclination angle	φ	-	Deg.

Table 4. The analytical, numerical and experimental values of K_I^N and K_{II}^N for different crack inclination angles

Crack inclination angle	K_I^N			K_{II}^N		
	Analytical	Experimental	Numerical	Analytical	Experimental	Numerical
φ=0°	1.014	1.000	1.008	0	0	0
φ=15°	0.721	0.715	0.709	1.014	1.017	1.017
φ=30°	-0.028	-0.016	-0.019	1.744	1.778	1.783
φ=45°	-1.028	-1.014	-1.012	1.985	2.040	2.035
φ=60°	-1.999	-2.054	-2.043	1.695	1.712	1.706
φ=75°	-2.693	-2.701	-2.706	0.968	0.947	0.923
φ=90°	-2.943	-2.948	-2.993	0	0	0

Table 4 demonstrates that the proposed numerical method gives very accurate results for CSCBD specimens. Thus this method may be considered as a suitable tool for the analysis of cracks propagation and failure process in brittle materials.

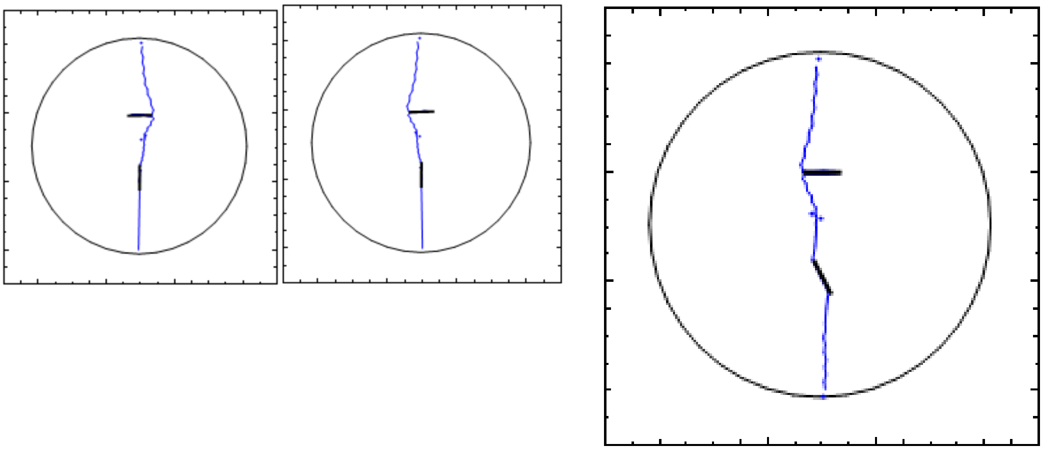
The proposed indirect boundary element method is also used for the simulation of the experimental works (to study the cracks coalescence in the bridge area and crack propagation process of brittle materials under compressive line loading). The experimental works already shown in Figs. 7 and 8 are numerically simulated by the displacement discontinuity method and the results are graphically shown in Figs. 9 and 10, respectively for comparison. The Linear Elastic Fracture Mechanics (LEFM) approach (based on the concept of Mode I and Mode II stress intensity factors (SIFs) proposed by Irwin (1957)) is implemented in the boundary element code and the maximum tangential stress criterion given by Erdogan and Sih (1963) is used in a stepwise procedures to estimate the propagation paths of the propagating wing cracks. The simulated propagation paths are in good agreement with the corresponding experimental results as can be observed by comparing the Figs. 7 and 8 with Figs. 9 and 10, respectively. It should be noted that the numerical results are based on the crack propagation process originating from the cracks tips (but as previously shown in Figs. 7 and 8 some experimental specimens do not include the starting wing cracks from the tip of the horizontal cracks in pre-cracked specimens).



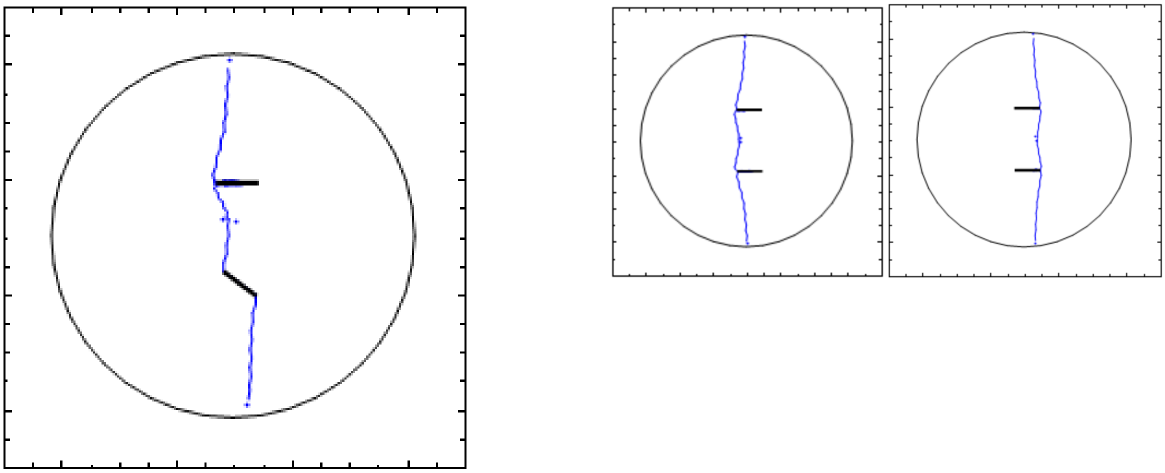
(a) φ=0° (b) φ=15° (c) φ=30° (d) φ=45°

(e) φ=60° (f) φ=75° (g) φ=90°

Fig.9. Numerical simulation of the crack propagation path for single-cracked Brazilian disc specimens



(a) φ=0° (b) φ=30°



(c) φ=60° (d) φ=90°

Fig. 10. Numerical simulation of crack propagation paths and cracks coalescence for double-cracked Brazilian disc specimens

5 DISCUSSIONS AND CONCLUSIONS

The analytical, experimental, and numerical investigations of the crack propagation process in brittle substances have been accomplished on the central Straight through Crack Brazilian Disc (CSCBD) specimens. As it is a complicated process, further research may be devoted to investigate the crack propagation, cracks coalescence in the bridge area and crack propagation paths of the rocks and rock-like materials under compressive line loading. Effects of fracturing on the failure load of the pre-cracked rock-like materials have been discussed. It has been shown that the crack propagation in the brittle substances due to the cracks coalescence phenomenon in the bridge area may occur mainly by propagation of wing cracks emanating from the tips of the pre-existing cracks.

The same experimental specimens have been numerically modeled by an indirect boundary element method and it has been clearly demonstrated that the corresponding numerical results are in good agreement with the experimental and analytical results. The experimental and numerical models well illustrate the production of the wing cracks and the cracks propagation paths produced by the coalescence phenomenon of the two pre-existing cracks in the bridge area (for double cracked Brazilian disc specimens).

As a whole, it may be concluded that the numerical and experimental results presented in this study may be very useful for the recognition of propagation and coalescence mechanism of cracks in the brittle substances such as rocks. Therefore, this framework can be extended to the fracture mechanics study of rock materials under various loading conditions (e.g. triaxial compressive, tensile and shear loading conditions).

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